Chapter Two

Mechanics

Motion in one Dimension

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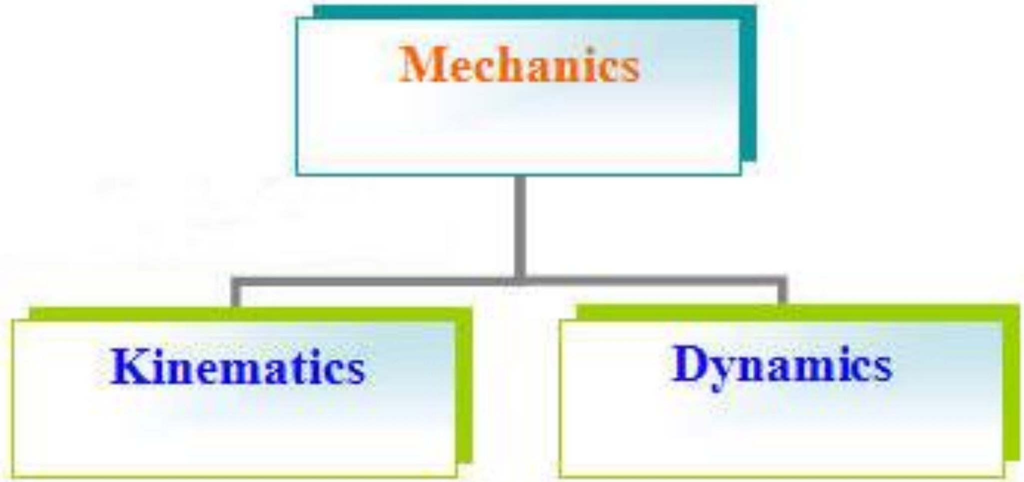
“To understand motion is to understand nature”

Leonardo Da Vinci

- In *nature*, the easiest changes to observe are those of motion: An object is moved from one position in space to another.
- In this chapter we will discuss motion in one dimension. The oldest one of the Physics subjects is the ***Mechanics*** that investigates the motion of a body. It deals not only with a football, but also with the path of a spacecraft that goes from the Earth to the Mars!
- The Mechanics can be divided into two parts: ***Kinematics*** and ***Dynamics***.



- **The kinematics:** It is important for the kinematics that which the path body follows. It answers the questions such that: Where the motion started? Where the motion stopped? What time has taken for the complete of motion? What the velocity body had?.
- **The dynamics** deals with the effects that create the motion or change the motion or stop the motion. It takes into account the forces and the properties of the body that can affect the motion.

- After that point, we will enter into the world of kinematics, first. The One-Dimensional
- Motion is the starting point for the kinematics. We will introduce some definitions like ***displacement, velocity*** and ***acceleration***, and derive equations of motion for bodies moving in one-dimension with ***constant acceleration***. We will also apply these equations to the situation of a body moving under the influence of gravity alone.
- We first need some definitions to identify the motion. It begins by defining the change in position of a particle. We call it "displacement".

Position, Time Interval, Displacement

- Consider an object moving in one dimension. We denote the *position coordinate* of the center of mass of the object *with respect to the choice of origin*. The position coordinate is a function of time and can be positive, zero, or negative, depending on the location of the object.

$$\vec{\mathbf{x}}(t) = x(t)\,\hat{\mathbf{i}}\,.$$

Time Interval

Consider a closed interval of time $[t_1, t_2]$. We characterize this time interval by the difference in endpoints of the interval such that

$$\Delta t = t_2 - t_1 .$$

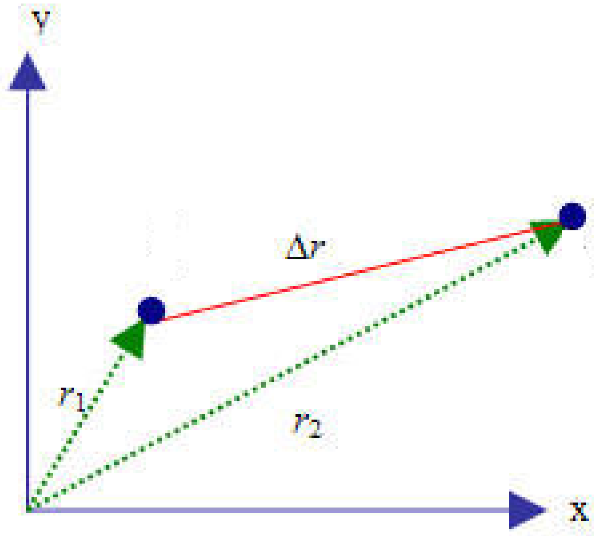
Displacement: is defined to be the change in position or distance that an object has moved and is given by the equation;

$$\Delta \vec{r} = \vec{r_2} - \vec{r_1}$$

where $\vec{r_2}$ is the final position and $\vec{r_1}$ is the initial position.

$$\Delta t = t_f - t_i$$

where the i and f subscripts depict **initial** and **final**, respectively. Generally, $t_i = t_0 = 0$.



$\mathbf{r}_1 = x_1 i + y1j$

$\mathbf{r}_2 = x_2 i + y_2 j$

$\Delta\mathbf{r} = \mathbf{r}_2 - \mathbf{r}_1$

Δr is called the displacement vector which represents the change in the position vector.

Example: 1) Write the position vector for a particle in the rectangular coordinate (x, y, z) for the points P1 (5, -6, 0), P2 (5, -4), and P3 (-1, 3, 6). 2) Also, find the resultant position vector of P1-P2, and P1-P3.

Solution:

1)

For the point (5, -6, 0) the position vector is r = 5i - 6j

For the point (5, -4) the position vector is r = 5i -4j

For the point (-1, 3, 6) the position vector is r = -i + 3j +6k

2)

P1-P2= ?

P1-P3=?

The average velocity and Instantaneous velocity

- The ***average velocity*** of a particle is defined as the ratio of the displacement to the time interval.

$$\vec{v}_{ave} = \frac{\Delta \vec{r}}{\Delta t}$$

- The ***instantaneous velocity*** of a particle is defined as the limit of the average velocity as the time interval approaches zero.

$$\vec{v} = \lim_{\Delta t \to 0} \frac{\Delta \vec{r}}{\Delta t}$$

$$v_x(t) \equiv \lim_{\Delta t \to 0} \overline{v_x} = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \lim_{\Delta t \to 0} \frac{x(t+\Delta t) - x(t)}{\Delta t} \equiv \frac{dx}{dt}.$$

$$\therefore \vec{v} = \frac{d\vec{r}}{dt}$$

The average acceleration and Instantaneous acceleration

Acceleration is the quantity that measures a change in velocity over a particular time interval. Suppose during a time interval Δt a body undergoes a change in velocity

The ***average acceleration*** of a particle is defined as the ratio of the change in the instantaneous velocity to the time interval.

$$\bar{a} = \frac{\Delta \vec{v}}{\Delta t}$$

- The ***instantaneous acceleration*** is defined as the limiting value of the ratio of the average velocity to the time interval as the time approaches zero.

$$\vec{a} = \lim_{\Delta t \to 0} \frac{\Delta \vec{v}}{\Delta t} = \frac{d\vec{v}}{dt}$$

$$a_x(t) \equiv \lim_{\Delta t \to 0} \overline{a_x} = \lim_{\Delta t \to 0} \frac{(v_x(t+\Delta t) - v_x(t))}{\Delta t} = \lim_{\Delta t \to 0} \frac{\Delta v_x}{\Delta t} \equiv \frac{dv_x}{dt}$$

One-dimensional motion with *constant acceleration*

Instantaneous acceleration = Average acceleration

$$a = a_{ave} = \frac{v - v_o}{t - t_o}$$

Let t_o = 0 then the acceleration

$$a = \frac{v - v_o}{t}$$

Or $\quad v = v_o + at$

Since the velocity varies linearly with time we can express the average velocity as

$$v_{ave} = \frac{v + v_o}{2}$$

- **To find the displacement Δx $(x-x_o)$ as a function of time**

$$\Delta x = v_{ave}\,\Delta t = \left(\frac{v+v_o}{2}\right)t$$

$$x = x_o + \frac{1}{2}\,(v+v_o)\,t$$

Also we can obtain the following equations:

$$x = x_o + v_o\,t + \frac{1}{2}\,a\,t^2$$

$$v^2 = v_o^2 + 2a(x-x_o)$$

$$x - x_o = v_o t$$

$$x - x_o = 1/2\,a\,t^2$$

Free Fall

We call that a freely falling object is an object that moves under the influence of gravity only. By neglecting air resistance, all objects in free fall in the earth's gravitational field have a constant acceleration that is directed towards the earth's center, or perpendicular to the earth's surface, and of magnitude. If motion is straight up and down and we can choose a coordinate system with the positive y-axis pointing up and perpendicular to the earth's surface, then we can describe the motion with $a \rightarrow g$, $x \rightarrow y$. (Negative sign arises because the coordinate system is changed and the acceleration direction is downward.)

- An important example of one-dimensional motion (for both scientific and historical reasons) is an object undergoing **free fall**.
- Suppose you are holding a stone and throw it straight up in the air. For simplicity, we'll neglect all the effects of air resistance. The stone will rise and fall along a line, and so the stone is moving in one dimension.

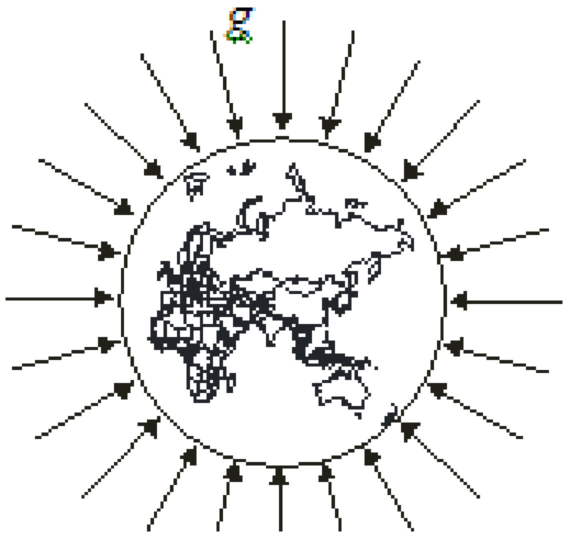
- Galileo Galilee was the first to definitively state that all objects fall towards the earth with a constant acceleration, later measured to be of magnitude $g=9.8$ m/sec^2 .

$$v = v_o - g\,t$$

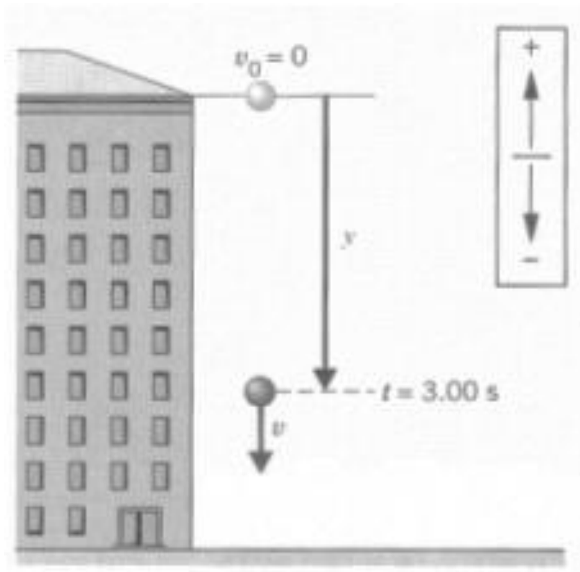
$$y = y_o + 1/2\,(v+v_o)t$$

$$y = y_o + v_o\,t - 1/2\,g\,t^2$$

$$v^2 = v_o^2 - 2g\,(y-y_o)$$



Example: A stone is dropped from rest from the top of a building. After 3s of free fall, what is the displacement y of the stone?



Solution:

From equation

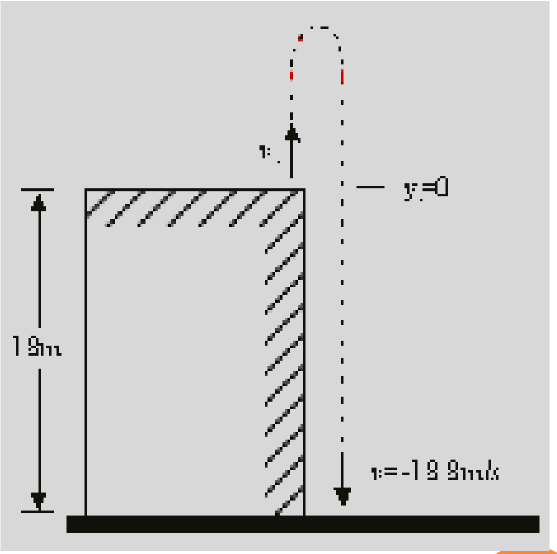
$$y = y_o + v_o\, t - 1/2\, g\, t^2$$

$$y = 0 + 0 - (9.8) \times (3)^2 = -44.1\text{m}$$

Example: A stone is thrown upwards from the edge of a cliff 18m high. It just misses the cliff on the way down and hits the ground below with a speed of 18.8m/s.

(a) With what velocity was it released?

(b) What is its maximum distance from the ground during its flight?



Solution:

Let $y_o = 0$ at the top of the cliff.

(a) From equation

$v^2 = v_o^2 - 2g\ (y - y_o)$

$(18.8)^2 = v_o^2 - 2´9.8´18$

$v_o^2 = 0.8$ **m/s**

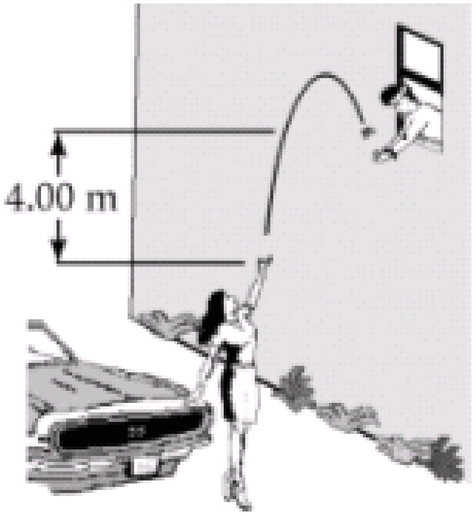
(b) The maximum height reached by the stone is *h*

$$h = \frac{v^2}{2g} = \frac{18}{2 \times 9.8} = 18 \text{ m}$$

Example: A student throws a set of keys vertically upward to another student in a window 4m above as shown in Figure 2.6. The keys are caught 1.5s later by the student.

(a) With what initial velocity were the keys thrown?

(b) What was the velocity of the keys just before they were caught?



Solution:

(a) Let y_o=0 and y=4m at t=1.5s then we find

$y = y_o + v_o\, t - 1/2\, g\, t^2$

$4 = 0 + 1.5\, v_o - 4.9\, (1.5)^2$

$v_o = 10$ m/s

(b) The velocity at any time $t > 0$ is given by

$v = v_o + at$

$v = 10 - 9.8\, (1.5) = -4.68$ m/s

Question

Assume that a car decelerates at $2.0 m/s^2$ and comes to a stop after traveling 25m.

a) Find the speed of the car at the start of the deceleration and

b) Find the time required to come to a stop.

Solution

We are given:

$a = -2.0 m/s^2$

$x = 25m$

$v = ?$

a) From

$$v^2 = v_0^2 + 2ax \Rightarrow v_0^2 = v^2 - 2ax \Rightarrow v_0^2 = 0 - 2(-2)(25) = 100$$

$$v_0 = 10 m/s$$

b) From $v = v_0 + at$ we have $t = \frac{v - v_0}{a} = \frac{-10}{-2} = 5s.$

Question

Assume that a car traveling at a constant speed of 30m/s passes a police car at rest. The policeman starts to move at the moment the speeder passes his car and accelerates at a constant rate of 3.0m/s^2 until he pulls even with the speeding car.

a) Find the time required for the policeman to catch the speeder and
b) Find the distance traveled during the chase.

Solution

We are given, for the speeder:

$v_0^s = 30m/s$, constant speed, then $a^s = 0$

and for the policeman:

$a_0^p = 3.0m/s^2$

a) The distance traveled by the speeder is given as $x^s = v^s t = 30t$. Distance traveled by policeman $x^p = x_0^p + v_0^p t + \frac{1}{2}a_p t^2$. When the policeman catches the speeder $x^s = x^p$ or,

$$x^s = x^s \quad \Rightarrow \quad 30t = 0 + 0 + \frac{3t^2}{2},$$

Solving for t we have $t = 0$ and $t = \frac{2}{3}(30) = 20s$. The first solution tells us that the speeder and the policeman started at the same point at $t = 0$, and the second one tells us that it takes 20 s for the policeman to catch up to the speeder.

b) Substituting back in above we find the distance that the speeder has taken

$x^s = 30(20) = 600m$

And also for the policeman

$$x^p = x_0^p + v_0^p t + \frac{1}{2}a_p t^2 = 0 + 0 + \frac{1}{2}(3)(20)^2 = 600m$$

Question

A rocket moves upward, starting from rest with an acceleration of 29.4m/s^2 for 4 s. At the end of this time, it runs out of fuel and continues to move upward. How high does it go totally?

Solution

For the first stage of the flight we are given:

$$a = 29.4 m/s^2 \quad for \quad t = 4s$$

This gives us the velocity and position at the end of the first stage of the flight:

$$v_1 = v_0 + at = 0 + 29.4(4) = 117.6 m/s$$

and

$$y_1 = y_0 + v_o t + \frac{1}{2}at^2 = 0 + 0 + \frac{1}{2}(29.4)(4)^2 = 235.2m$$

For the second stage of the flight, the rocket will go upward with its velocity till it stops (That is the Newton's Motion Law).So, we start with

$$v_1 = 117.6 m/s$$

$$a = g = -9.8 m/s^2$$

And we end up with $v_2 = 0$. We want to find the distance traveled in the second stage ($y_2 - y_1$). We have,

$$v_2^2 = v_1^2 + 2a(\Delta y) \quad \Rightarrow \quad \Delta y = y_2 - y_1 = \frac{v_2^2 - v_1^2}{2g} = \frac{0^2 - (117.6)^2}{2(-9.8)} = 705.6m.$$

Therefore, the total distance taken by the rocket is $y_2 = y_1 + 705.6m = 235.2 + 705.6 = 940.8m$

Question

A train with a constant speed of 60km/h goes east for 40min. Then it goes 45° north-east for 20min. And finally it goes west for 50min. What is the average velocity of the train?

Question

The motion of a particle is given by $x = t^2 + 3t - 3$, where x is distance in meter and t is time in sec.

a) Find the velocity of the particle after 10 sec.

b) Find also acceleration of the particle. State whether acceleration is uniform of variable.