Chapter Four

The Low of Motion

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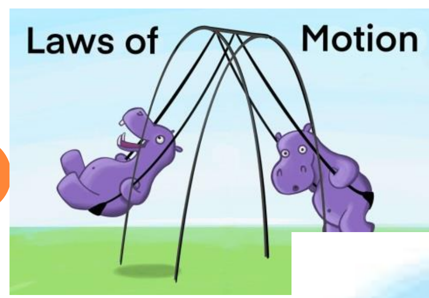
2023-2024



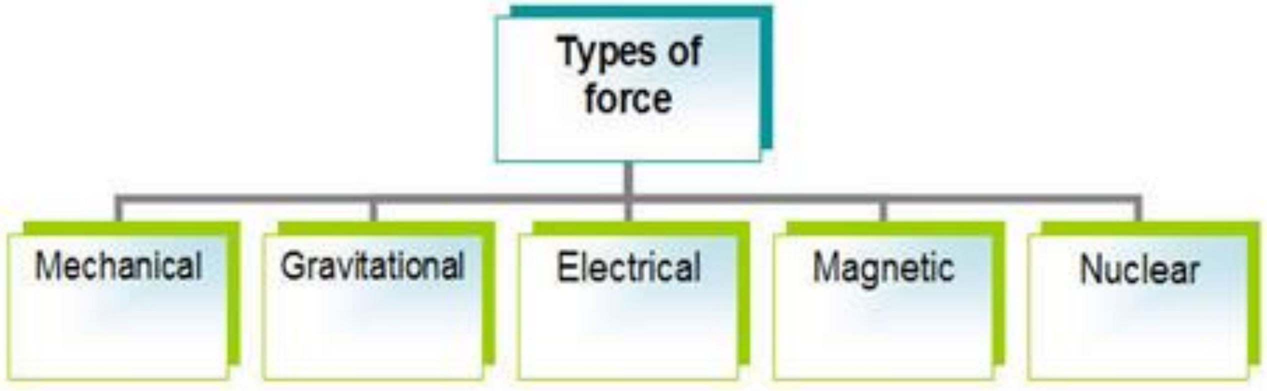
The concept of force

Force

Force is a vector quantity. The magnitude of the total force $|\vec{\mathbf{F}}|$ acting on the object is the product of the mass m_s with the magnitude of the acceleration $|\vec{\mathbf{a}}|$. The direction of the total force on the standard body is defined to be the direction of the acceleration of the body. Thus

$$\vec{\mathbf{F}} \equiv m_s \, \vec{\mathbf{a}}$$

The SI units for force are $[\mathrm{kg \cdot m \cdot s^{-2}}]$. This unit has been named the newton [N] and $1\,\mathrm{N} = 1\,\mathrm{kg \cdot m \cdot s^{-2}}$.



Newton’s laws of motion

- ***<u>Newton's first law,</u>*** **(*Law of Inertia*)** *the law of equilibrium* states that an object at rest will remain at rest and an object in motion will remain in motion with a constant velocity unless acted on by a *net external force.*
- ***<u>Newton's second law</u>*****,** **(*Fundamental Law of Dynamics*)** *the law of acceleration*, states that the acceleration of an object is directly proportional to the net force acting on it and inversely proportional to its mass.
- ***<u>Newton's third law</u>*****,** **(*The Law of Reaction*)** “To every action there is always an equal and opposite reaction.”

Newton's laws in Mathematical forms

$\sum \vec{F} = 0$ *Newton's first law*

$\sum \vec{F} = m\vec{a}$ *Newton's second law*

$\vec{F}_{12} = -\vec{F}_{21}$ ***Newton's third law***

Gravitational Force near the Surface of the Earth

$$\left|\vec{\mathbf{F}}_{\text{grav}}\right| = m_{\text{grav}}\, g$$

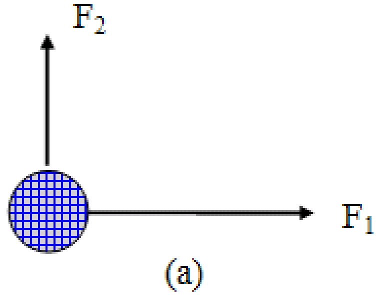
The International Committee on Weights and Measures has adopted as a standard value for the acceleration of a body freely falling in a vacuum $g = 9.80665\ \text{m}\cdot\text{s}^{-2}$. The actual value of g varies as a function of elevation and latitude. If ϕ is the latitude and h the elevation in meters then the acceleration of gravity in SI units is

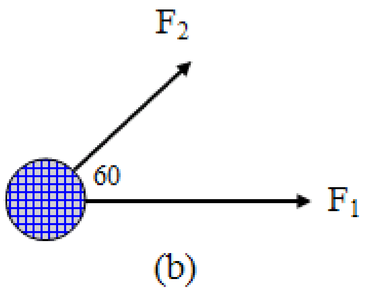
$$g = \left(9.80616 - 0.025928\cos(2\phi) + 0.000069\cos^2(2\phi) - 3.086\times10^{-4} h\right)\text{m}\cdot\text{s}^{-2}$$

This is known as Helmert's equation. The strength of the gravitational force on the standard kilogram at 42° latitude is $9.80345\ \text{N}\cdot\text{kg}^{-1}$, and the acceleration due to gravity at sea level is therefore $g = 9.80345\ \text{m}\cdot\text{s}^{-2}$ for all objects. At the equator, $g = 9.78\ \text{m}\cdot\text{s}^{-2}$ (to three significant figures), and at the poles $g = 9.83\ \text{m}\cdot\text{s}^{-2}$. This difference is primarily due to the earth's rotation, which introduces an apparent repulsive force that affects the determination of g and also flattens the spherical shape earth (the distance from the center of the earth is larger at the equator than it is at the poles by about $26.5\ \text{km}$).

Example

Two forces, F_1 and F_2, act on a 5-kg mass. If F_1 =20 N and F_2 =15 N, find the acceleration in (a) and (b) of the Figure





Solution

(a)

$\sum F = F_1 + F_2 = (20i + 15j)$ N

$\sum F = ma,\ \ 20i + 15j = 5\ a$

$a = (4i + 3j)$ m/s^2

or $|a| = 5$m/s^2

(b)

$F_{2x} = 15 \cos 60 = 7.5$ N

$F_{2y} = 15 \sin 60 = 13$ N

$F_2 = (7.5i + 13j)$ N

$\sum F = F_1 + F_2 = (27.5i + 13j) = ma = 5\ a$

$a = (5.5i + 2.6j)$ m/s^2

or$|a|\ = 6.08$m/s^2

WEIGHT AND MASS

In fact, the terms *mass* and *weight* are often confused with one another. However, in physics their meanings are quite distinct.

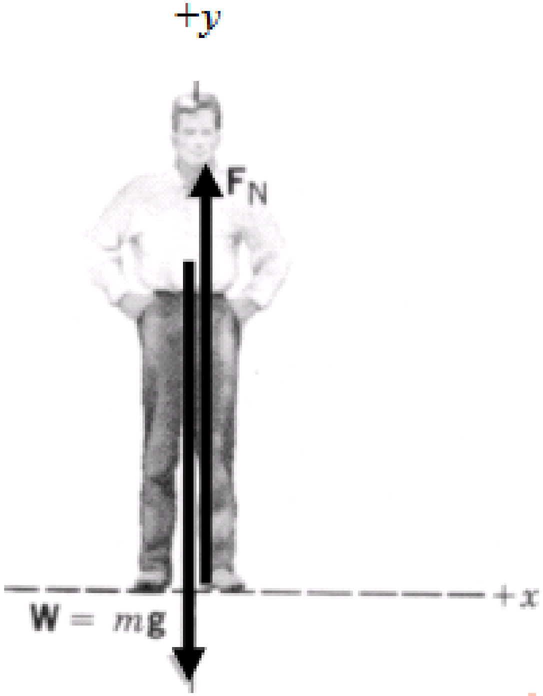
$$W = mg$$

$$\sum \vec{F} = F_N - mg = ma$$

where *a* is the acceleration of the elevator and the person.

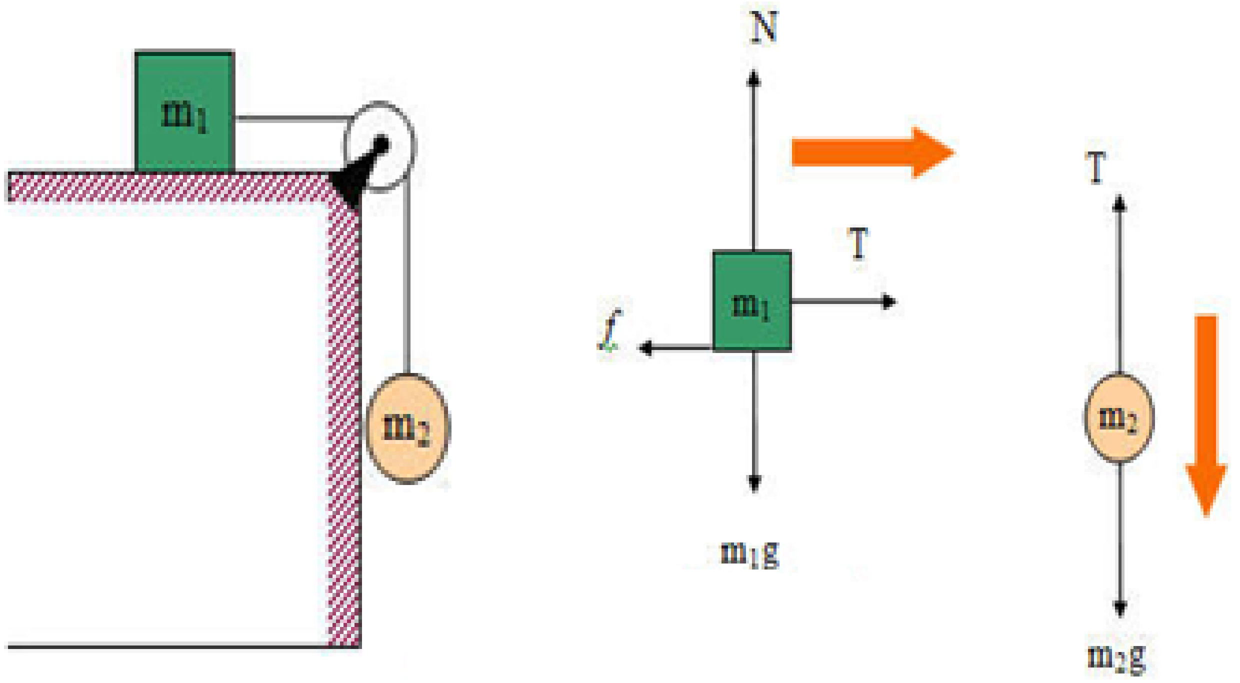
$F_N = mg + ma$ when the elevator moves upward

$F_N = mg - ma$ when the elevator moves downward



Tension

Two blocks are connected by a light string over a frictionless pulley as shown in Figure. The coefficient of sliding friction between m_1 and the surface is m. Find the acceleration of the two blocks and the tension in the string.



Consider the motion of m_1. Since its motion to the right, then $T > f$. If T were less than f, the blocks would remain stationary.

$$\sum F_x \text{ (on } m_1) = T - f = m_1 a$$
$$\sum F_y \text{ (on } m_1) = N - m_1 g = 0$$
since $f = \mu N = \mu m_1 g$, then
$$T = m_1(a+\mu g) \qquad (1)$$

For m_2, the motion is downward, therefore $m_2 g > T$. Note that T is uniform through the rope. That is the force which acts on the right is also the force which keeps m_2 from free falling. The equation of motion for m_2 is:

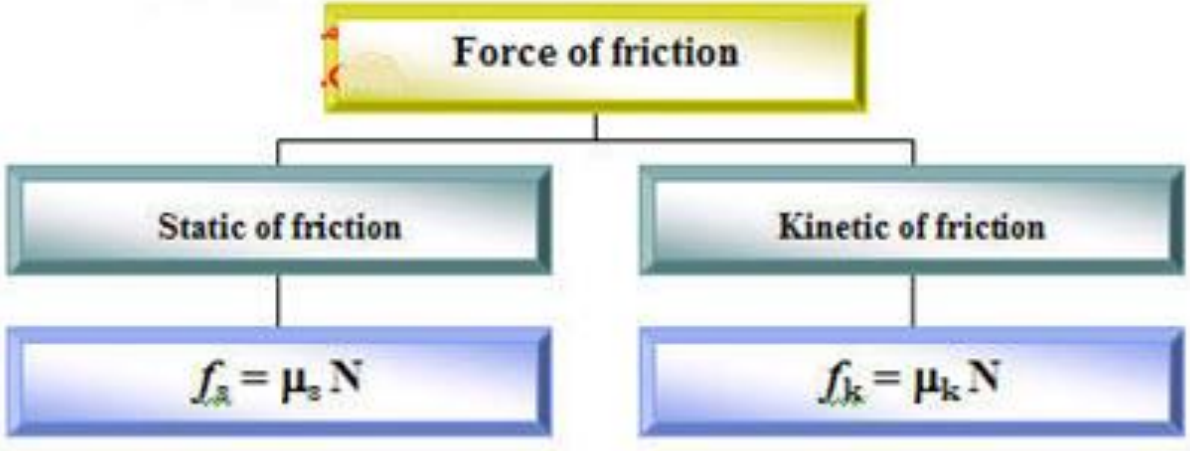
$$\sum F_y \text{ (on } m_2) = T - m_2 g = - m_2 a$$
$$T = m_2(g-a) \qquad (2)$$
Solving the equations 1 and 2

$$m_1(a+\mu g) - m_2(g-a) = 0$$

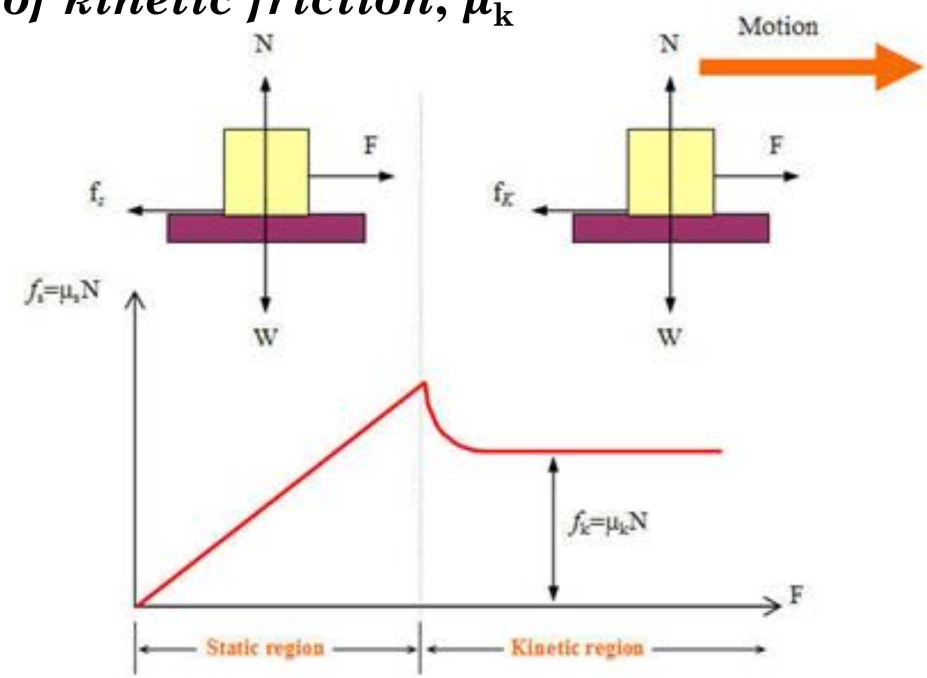
$$a = \left(\frac{m_2 - \mu m_1}{m_1 + m_2}\right) g$$

$$T = m_2\left(1 - \frac{m_2 - \mu m_1}{m_1 + m_2}\right) g = \frac{m_1 m_2 (1+\mu) g}{m_1 + m_2}$$

Force of Friction



- ***Coefficient of static friction ، μ_s***
- ***Coefficient of kinetic friction, μ_k***



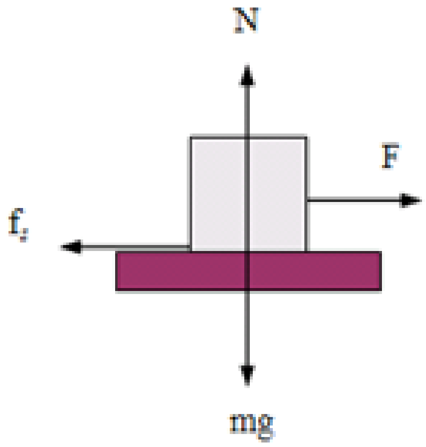
Evaluation of the force of friction

Case (1) when a body slides on a horizontal surface

$$f_k = \mu_k N$$

Since $N = mg$

$$f_k = \mu_k mg$$

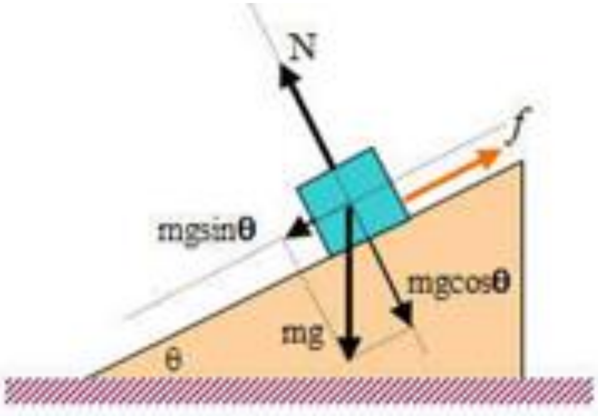


Case (2) when a body slides on an inclined surface

$$f_k = \mu_k N$$

Since $N = mg\cos\theta$

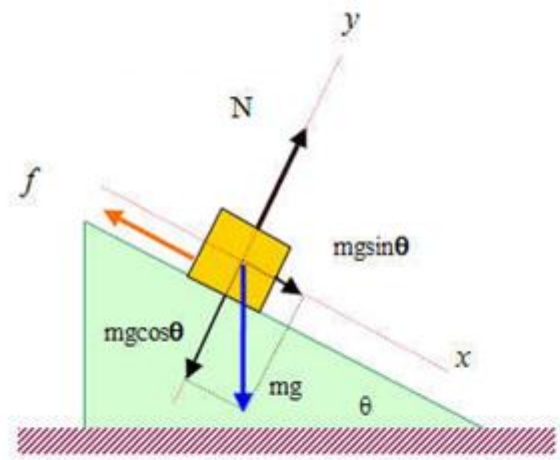
$$f_k = \mu_k mg\cos\theta$$



Example

A 3kg block starts from rest at the top of 30° incline and slides a distance of 2m down the incline in 1.5s. Find (a) the acceleration of the block, (b) the coefficient of kinetic friction between the block and the plane, (c) the friction force acting on the block, and (d) the speed of the block after it has slid 2m.

Solution:



Given $m = 3\text{kg}$, $\theta = 30^{o}$, $x = 2\text{m}$, $t = 1.5\text{s}$

$x = 1/2at^{2}$, $2 = 1/2a\,(1.5)^{2}$, $a = 1.78\text{m/s}^{2}$

$mg\sin30 - f = ma$, $f = m\,(g\sin30 - a)$

$f = 9.37\text{N}$

$N - mg\cos30 = 0$, $N = mg\cos30$

$f = 9.37\text{N}$

$\mu_{k} = f / \text{N} = 0.368$

$\boldsymbol{v^{2} = v_{o}^{2} + 2a\,(x - x_{o})}$

$\boldsymbol{v^{2} = 0 + 2(1.78)(2) = 7.11}$

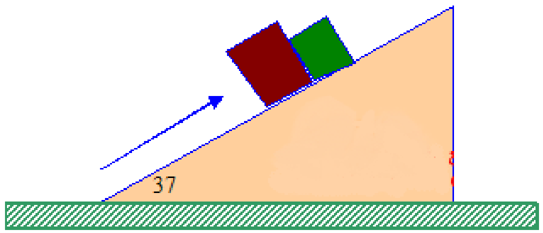
then

$\boldsymbol{v = 2.67\text{m/s}}$

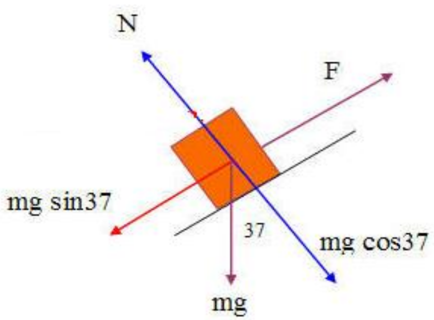
Example

Two blocks having masses of 2 kg and 3 kg are in contact on a fixed smooth inclined plane as in Figure.

(a) Treating the two blocks as a composite system, calculate the force F that will accelerate the blocks up the incline with acceleration of $2 m/s^2$,



Solution



We can replace the two blocks by an equivalent 5 kg block as shown in Figure. Letting the x axis be along the incline, the resultant force on the system (the two blocks) in the x direction gives

$$\sum F_x = F - W \sin(37^o) = m\, a_x$$
$$F - 5\,(0.6) = 5(2)$$
$$F = 39.4 \text{ N}$$

H.W.: Given an incline with angle 30 degrees which has a mass of 2kg placed upon it. It is attached by a rope over a pulley to a mass of 3kg which hangs vertically. Taking downward as the positive direction for the hanging mass find:

1- Acceleration of the system.

2- With this acceleration, find the tension in the rope and the weight for the hanging mass.