Chapter Five

Work and Energy

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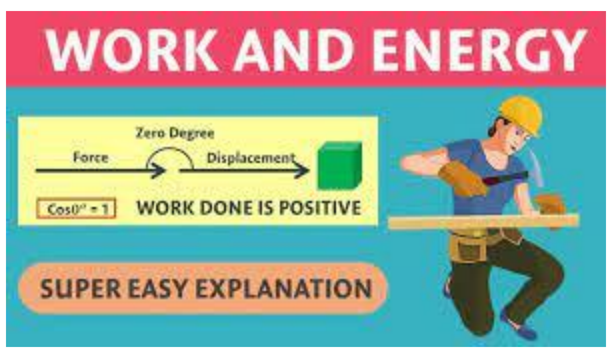
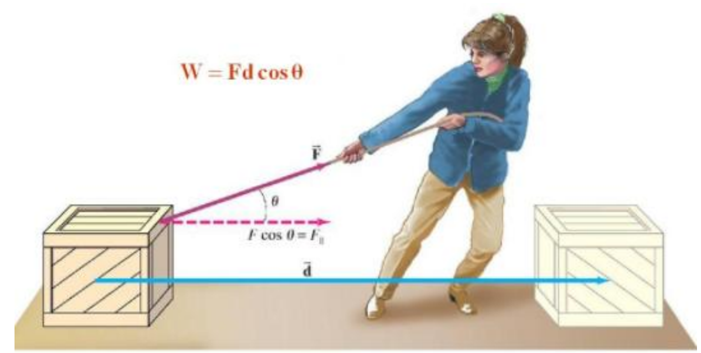
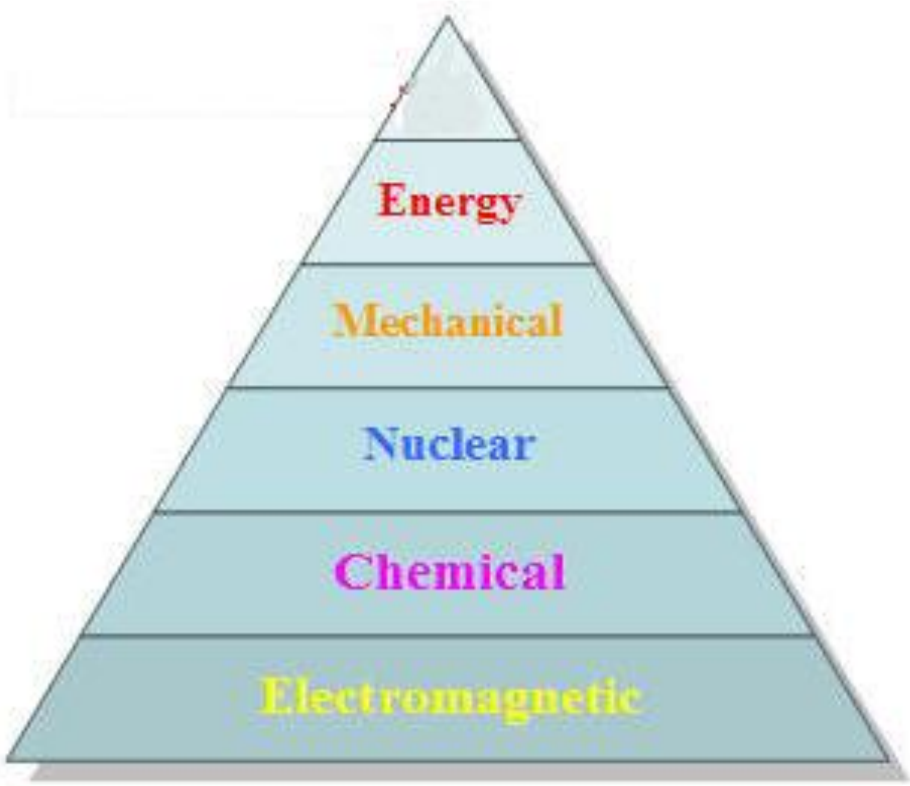


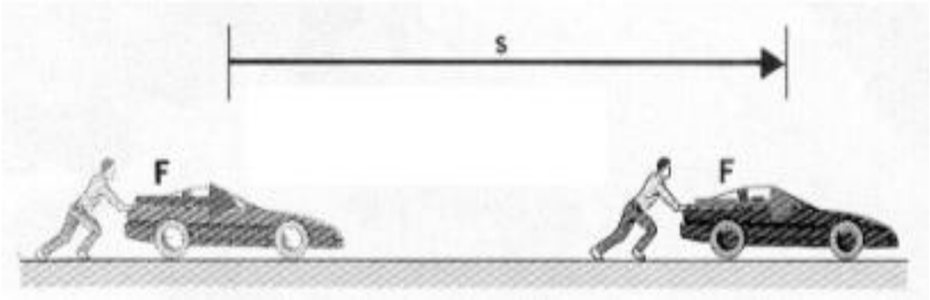



Energy
Mechanical
Nuclear
Chemical
Electromagnetic


Force
Work
Energy

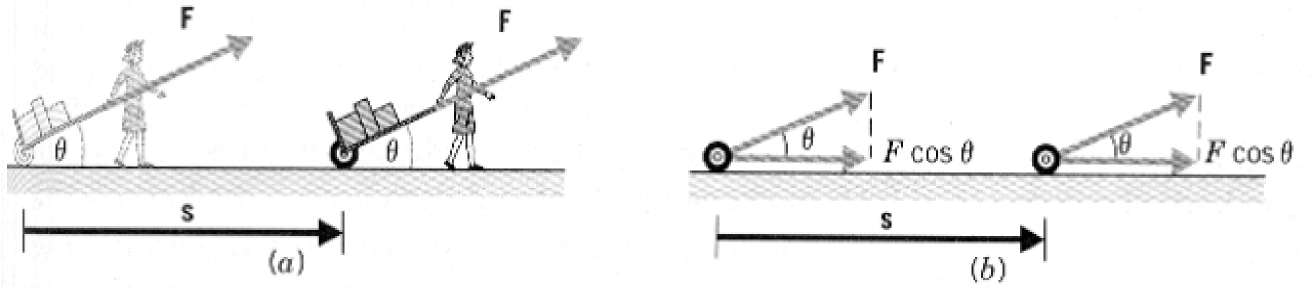
WORK DONE BY A *CONSTANT FORCE*:

$$W = FS$$



$$W = F\,S\cos\theta$$

The above equation can be written in the directional form as dot product $W = \vec{F} \cdot \vec{s}$



It is clearly seen that if $\theta < 90$, the work is positive, in which case, the force contributes the effect of speeding up the object; if $\theta > 90$, the work is negative, and the force contributes the effect of slowing down the object; the work is zero when $\theta=90$.

The unit of the work is N.m which is called Joule (J).

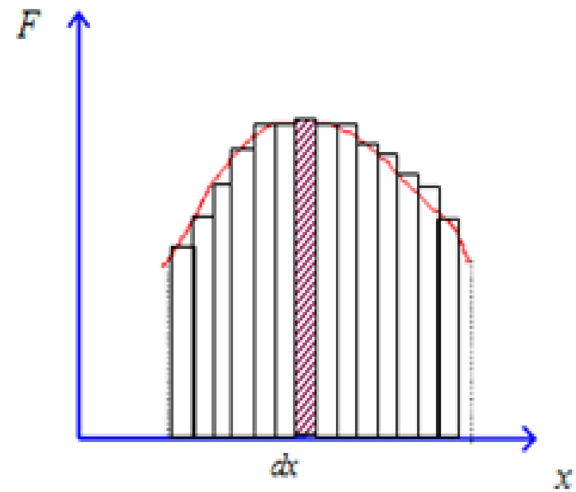
Important Notes

- *The object must undergo a displacement s.*
- *F must have a non-zero component in the direction of s.*
- *Work is zero when there is no displacement.*
- *Work is zero when the force is perpendicular to the displacement.*
- *Work is positive when F is indirection of displacement or when $0 \le \theta < 90$*
- *Work is negative when F is in opposite direction of displacement or when $90 < \theta \le 180$*

WORK DONE BY A VARYING FORCE:

$$\Delta W = F_x \Delta x$$

- By dividing the work curve into very small parts and by calculating the work done through each part we can express it in the following terms:



$$W = \sum_{x_i}^{x_f} F_x \Delta x$$

$$W = \int_{x_i}^{x_f} F_x dx$$

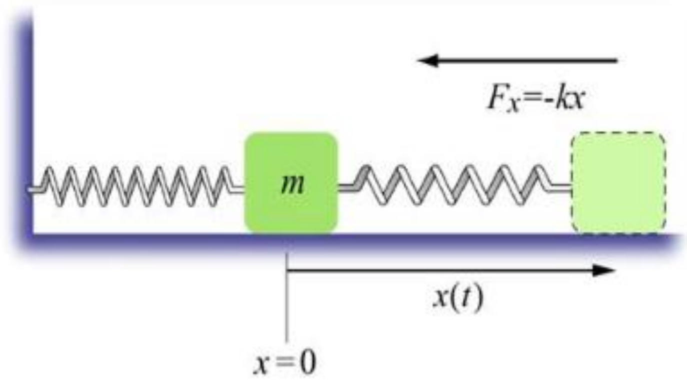
As $\Delta x \to 0$, the summation will be an integral between points where the motion starts and ends:

$$W = \int_{x_i}^{x_f} F_x dx$$

Work done by a spring

$$F = kx$$

$$F_s = -kx$$



- **The minus sign here implies that the direction of the force is always the opposite to the displacement *x*.**

Work done by a spring force from x to its normal position ($x=0$) is given by

$$W = \int_{x_1}^{x_2} F(x)\,dx = \int_0^x -kx\,dx = -k\int_0^x x\,dx = -k\left(\frac{x^2}{2}\right]_0^x = -\frac{1}{2}kx^2 .$$

$$W = \int_{x=x_0}^{x=x_f} F_x dx = \int_{x=x_0}^{x=x_f} (-kx)dx$$

$$W = \int_{x=x_0}^{x=x_f} (-kx)dx = -\frac{1}{2}k(x_f^{\,2} - x_0^{\,2})$$

When the absolute value of the final distance is less than the absolute value of the initial distance, $|x_f| < |x_0|$, the work done by the spring force is positive. This means that if the spring is less stretched or compressed in the final state than in the initial state, the work done by the spring force is positive. The spring force does positive work on the body when the spring goes from a state of 'greater tension' to a state of 'lesser tension'.

Work and Energy relation

- The ***kinetic energy*** *K* of a non-rotating body of mass *m* moving with speed *v* is defined to be the positive scalar quantity

$$K \equiv \frac{1}{2} m v^2$$

The kinetic energy is proportional to the square of the speed. The SI units for kinetic energy are $[\mathrm{kg} \cdot \mathrm{m}^2 \cdot \mathrm{s}^{-2}]$. This combination of units is defined to be a joule and is denoted by $[\mathrm{J}]$, thus $1\ \mathrm{J} \equiv 1\ \mathrm{kg} \cdot \mathrm{m}^2 \cdot \mathrm{s}^{-2}$.

We know the Newton second law that defines the acceleration of a body:

$$\vec{a}_x = \frac{\sum \vec{F}_x}{m}$$

This is the one-dimensional acceleration of a body. If the object moves with a constant acceleration from x_i to x_f with the velocities v_i to v_f, then we can write

$$v_f^2 - v_i^2 = 2a_x(x_f - x_i)$$

and

$$a_x(x_f - x_i) = \frac{1}{2}v_f^2 - \frac{1}{2}v_i^2$$

Let us calculate the net work done on the object by the net force applied on it:

$$W_{net} = \left(\sum F_x\right)\left(x_f - x_i\right),$$

inserting the net force term into this equation, we get

$$W_{net} = \left(ma_x\right)\left(x_f - x_i\right) = m\left[a_x\left(x_f - x_i\right)\right] = m\left(\frac{1}{2}v_f^2 - \frac{1}{2}v_i^2\right)$$

$$W_{net} = m\frac{1}{2}v_f^2 - m\frac{1}{2}v_i^2.$$

If we define $K = m\frac{1}{2}v^2$, then we get

$$W_{net} = K_f - K_i\ .$$

$$W = \Delta K$$

- **This is the work-energy theorem:** The work done by the net force on an object is equal the change of its kinetic energy.

- **Note:** the kinetic energy always is positive but the change of kinetic energy may be positive, negative or zero.

Example: Change in Kinetic Energy of a Car

(a) Suppose car A increases its speed from 10 to 20 mph and car B increases its speed from 50 to 60 mph. Both cars have the same mass m. What is the ratio of the change of kinetic energy of car B to the change of kinetic energy of car A? Which car has a greater change in kinetic energy?

Solution: The ratio of the change in kinetic energy of car B to car A is

$$\frac{\Delta K_B}{\Delta K_A} = \frac{\frac{1}{2}m(v_{B,f})^2 - \frac{1}{2}m(v_{B,0})^2}{\frac{1}{2}m(v_{A,f})^2 - \frac{1}{2}m(v_{A,0})^2} = \frac{(v_{B,f})^2 - (v_{B,0})^2}{(v_{A,f})^2 - (v_{A,0})^2}$$

$$= \frac{(60\text{ mph})^2 - (50\text{ mph})^2}{(10\text{ mph})^2} = 11/3$$

(b) What is the ratio of the change in kinetic energy of car B to car A as seen by an observer moving with the initial velocity of car A?

Solution: Car A now increases its speed from rest to 10 mph and car B increases its speed from 40 to 50 mph. The ratio is now

$$\frac{\Delta K_B}{\Delta K_A} = \frac{\frac{1}{2}m(v_{B,f})^2 - \frac{1}{2}m(v_{B,0})^2}{\frac{1}{2}m(v_{A,f})^2 - \frac{1}{2}m(v_{A,0})^2} = \frac{(v_{B,f})^2 - (v_{B,0})^2}{(v_{A,f})^2 - (v_{A,0})^2}$$

$$= \frac{(50\text{ mph})^2 - (40\text{ mph})^2}{(10\text{ mph})^2} = 9$$

Power

- ***Power is defined as the time rate of energy transfer :***

$$P_{\text{ave}} = \frac{\Delta W}{\Delta t}$$

The instantaneous power is given by

$$P = \lim_{\Delta t \to 0} \frac{\Delta W}{\Delta t} = \frac{dW}{dt}$$

$$P = \frac{dW}{dt} = F.\frac{ds}{dt}$$

$$\therefore P = F.v$$

The unit of power is *the joule per second* which is more commonly called a “***Watt***” (Watt is the surname of *James Watt* (1735-1819) who has spent his life for studying on the steam machines). Another unit commonly used to measure power, especially in everyday situations, is “***the horsepower***”, which is equivalent to about ***746 Watts***.

Time Rate of Change of Kinetic Energy and Power

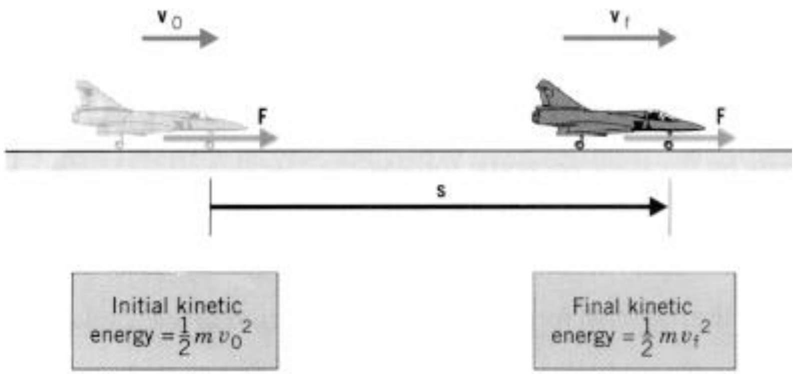
$$\frac{dK}{dt} = \frac{d}{dt}\left(\frac{1}{2} m v_x^{\,2}\right) = m \frac{dv_x}{dt} v_x = m a_x v_x \, .$$

By Newton's Second Law, $F_x = m a_x$

$$\frac{dK}{dt} = F_x v_x = P$$

Example:

A fighter-jet of mass 5×10^4kg is travelling at a speed of v_i=1.1×10^4m/s as showing in the Figure. The engine exerts a constant force of 4×10^5N for a displacement of 2.5×10^6m. Determine the final speed of the jet.



Solution:

According to equation of work, the work done on the engine is

$$W = (F\cos\theta)\, s = 4\times10^5 \cos 0^o \times 2.5\times10^6 = 1\times10^{12}\text{J}$$

The work is positive, because the force and displacement are in the same direction as shown in the Figure. Since $W = K_f - K_i$ the final kinetic energy of the fighter jet is

$$K_f = W + K_i$$

$$= (1\times10^{12}\text{J}) + ½\,(5\times10^4\text{kg})\,(1\times10^4\text{m/s})^2 = 4.031\times10^{12}\text{J}$$

The final kinetic energy is $K_f = ½\, m v_f^2$, so the final speed is

$$v_f = \sqrt{\frac{2K_f}{m}} = \sqrt{\frac{2(4.03\times10^{12})}{5\times10^4}} = 1.27\times10^4 \text{ m/s}$$

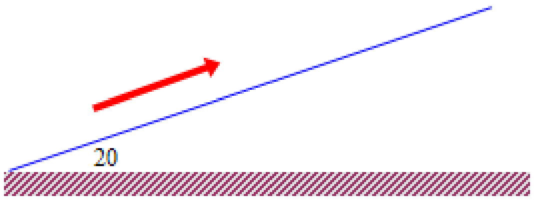
Example:

A 65-kg athlete runs a distance of 600 m up a mountain inclined at 20° to the horizontal. He performs this feat in 80s. Assuming that air resistance is negligible, (a) how much work does he perform and (b) what is his power output during the run?

Solution:

Assuming the athlete runs at constant speed, we have

$W_A + W_g = 0$



Where W_A is the work done by the athlete and W_g is the work done by gravity. In this case,

$W_g = - m\, g\, s\, (\sin\Theta)$

So

$W_A = -W_g = + m\, g\, s\, (\sin\Theta)$

$= (65\text{kg})(9.80\text{m/s}^2)(600\text{m}) \sin 20^{\circ}$

(b) His power output is given by

$$P_A = \frac{W_A}{\Delta t} = \frac{1.31 \times 10^5\, J}{80s} = 1.63\text{kW}$$

Question: Suppose you are initially standing and you start walking by pushing against the ground with your feet and your feet do not slip. How much work is done on you by the static friction force?

Answer: When you apply a contact force against the ground, the ground applies an equal and opposite contact force on you. The tangential component of this constant force is the force of static friction acting on you. Since your foot is at rest while you are pushing against the ground, there is no displacement of the point of application of this static friction force. Therefore static friction does zero work on you while you are accelerating. You may be surprised by this result but if you think about energy transformation, chemical energy stored in your muscle cells is being transformed into kinetic energy of motion and thermal energy.

Example:

Push a cup of mass 0.2 kg along a horizontal table with a force of magnitude 2.0 N for a distance of 0.5 m. The coefficient of friction between the table and the cup is $\mu_k = 0.1$. Calculate the work done by the pushing force and the work done by the friction force.

Answer: The work done by the pushing force is

$$W_{\text{applied}} = F_{\text{applied},x} \Delta x = (2.0\text{ N})(0.5\text{ m}) = 1.0\text{ J}.$$

The work done by the friction force is

$$W_{\text{friction}} = -\mu_k mg \Delta x = -(0.1)(0.2\text{ kg})(9.8\text{ m}\cdot\text{s}^{-2})(0.5\text{ m}) = -0.10\text{ J}.$$

Example:

A person pushes a cup of mass 0.2 kg along a horizontal table with a force of magnitude 2.0 N at an angle of 30^{o} with respect to the horizontal for a distance of 0.5 m as in Example 7.4.2. The coefficient of friction between the table and the cup is $\mu_k = 0.1$. If the cup was initially at rest, what is the final kinetic energy of the cup after being pushed 0.5 m? What is the final speed of the cup?

Solution:

$$W_{\text{total}} = W_{\text{applied}} + W_{\text{friction}} = (F_{\text{applied},x} - \mu_k N)(x_f - x_0)$$

$$= (1.7\ \text{N} - 9.6\times10^{-2}\ \text{N})(0.5\ \text{m}) = 8.0\times10^{-1}\ \text{J}$$

According to our work-kinetic energy theorem,

$$W_{\text{total}} = \Delta K = \frac{1}{2}mv_f^{\,2} - \frac{1}{2}mv_0^{\,2}$$

The initial velocity is zero so the change in kinetic energy is just

$$\Delta K = \frac{1}{2}mv_{y,f}^{\,2} - \frac{1}{2}mv_{y,0}^{\,2} = \frac{1}{2}mv_{y,f}^{\,2}$$

Thus the work-kinetic energy theorem enables us to solve for the final kinetic energy,

$$K_f = \frac{1}{2}mv_f^{\,2} = \Delta K = W_{\text{total}} = 8.0\times10^{-1}\ \text{J}$$

We can solve for the final speed,

$$v_{y,f} = \sqrt{\frac{2K_f}{m}} = \sqrt{\frac{2W_{\text{total}}}{m}} = \sqrt{\frac{2(8.0\times10^{-1}\ \text{J})}{0.2\ \text{kg}}} = 2.9\ \text{m}\cdot\text{s}^{-1}$$

H.W:

Suppose a ball of mass $m = 0.2\,\text{kg}$ starts from rest at a height $y_0 = 15\,\text{m}$ above the surface of the earth and falls down to a height $y_f = 5.0\,\text{m}$ above the surface of the earth. What is the average power exerted by the gravitation force? What is the instantaneous power when the ball is at a height $y_f = 5.0\,\text{m}$ above the surface of the Earth? Make a graph of power vs. time. You may ignore the effects of air resistance.

Ex.: A box of $M = 40$-kg mass is pulled for a distance of $s = 5.0$ m along a horizontal floor with a constant horizontal force of $F = 140$ N. The kinetic coefficient of friction between the box and the floor is $\mu_k = 0.30$.

a) Calculate the work done by the applied force.

b) Calculate the work done by the friction force.

c) When the force of $F = 140$ N is applied initially, the box was moving at $v_i = 2.0$ m/s in the same direction with the force. Calculate the final speed of the box.

d) Calculate the instantaneous power delivered by the force F, at the moment when it is applied initially.

Problem Solution Method:

Five Steps:

1) **Focus the Problem**
   - **draw a picture – what are we asking for?**
2) **Describe the physics**
   - **what physics ideas are applicable**
   - **what are the relevant variables known and unknown**
3) **Plan the solution**
   - **what are the relevant physics equations**
4) **Execute the plan**
   - **solve in terms of numbers**
5) **Evaluate the answer**
   - **are the dimensions and units correct?**
   - **do the numbers make sense?**