Chapter Six

Potential Energy & Conservation energy

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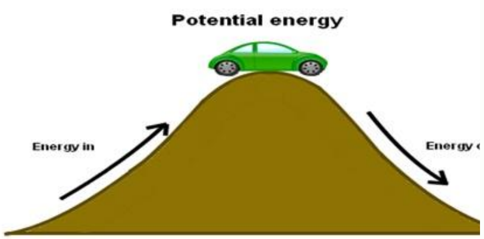
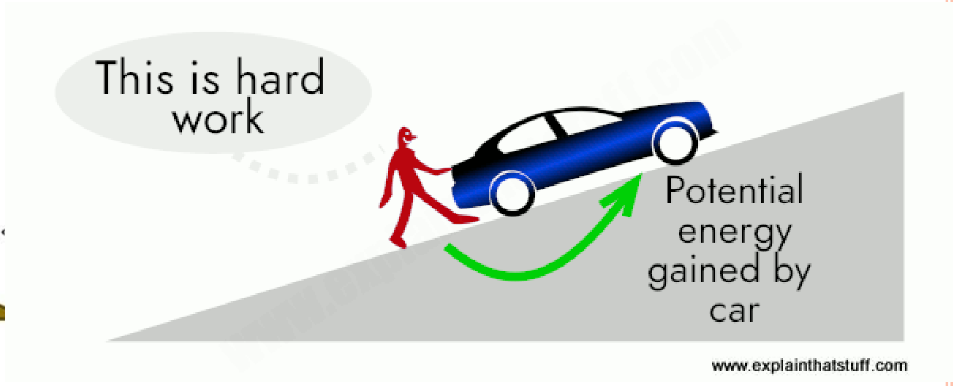




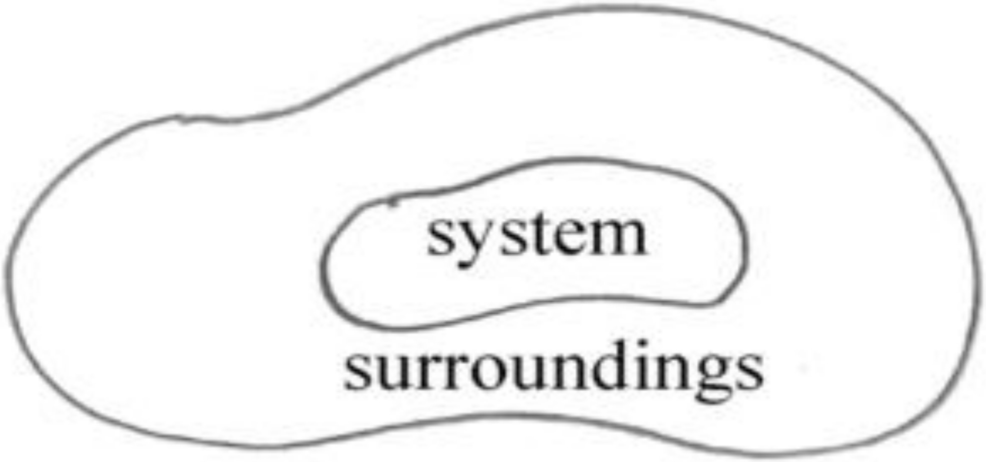
Introduction

- If the quantity of a subject does not change with time, it means that the quantity is "conserved". The best way to explain that quantity is the energy. If the energy is conserved in a system, then it is known that the total amount of this energy remains constant even its shape changes.

Conservation of Energy

- When a system and its surroundings undergo a transition from an initial state to a final state, the total change in energy is zero,

$$\Delta E^{\text{total}} = \Delta E_{\text{system}} + \Delta E_{\text{surroundings}} = 0\,.$$



$$\Delta E^{\text{closed}}_{\text{system}} = 0\,.$$

Conservative and non-conservative forces

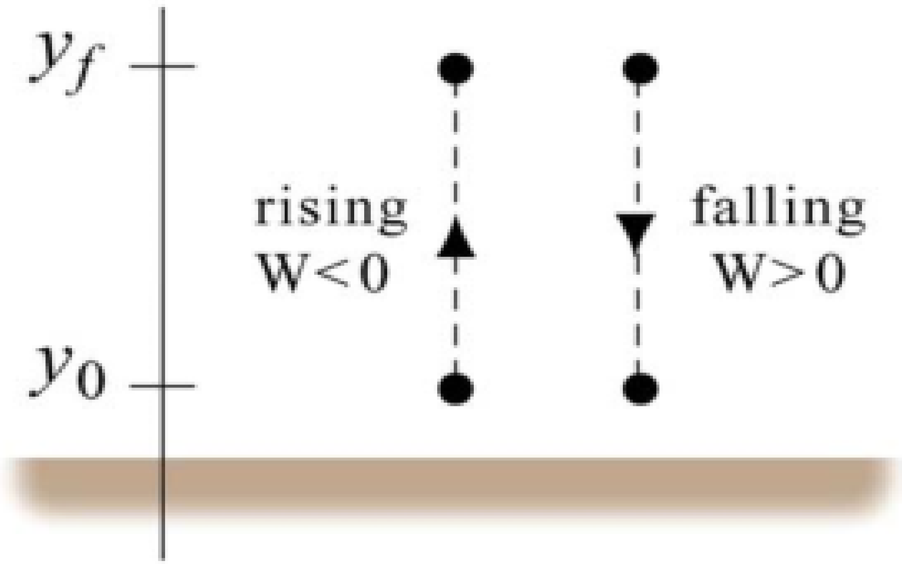
We defined the work done by a force $\vec{\mathbf{F}}$, on an object which moves along a path from an initial point $\vec{\mathbf{r}}_0$ to a final point $\vec{\mathbf{r}}_f$, as the integral of the component of the force tangent to the path with respect to the displacement of the point of contact of the force and the object,

$$W = \int_{\text{path}} \vec{\mathbf{F}} \cdot d\vec{\mathbf{r}} \,.$$

Does the work done on the object by the force depend on the path taken by the object?

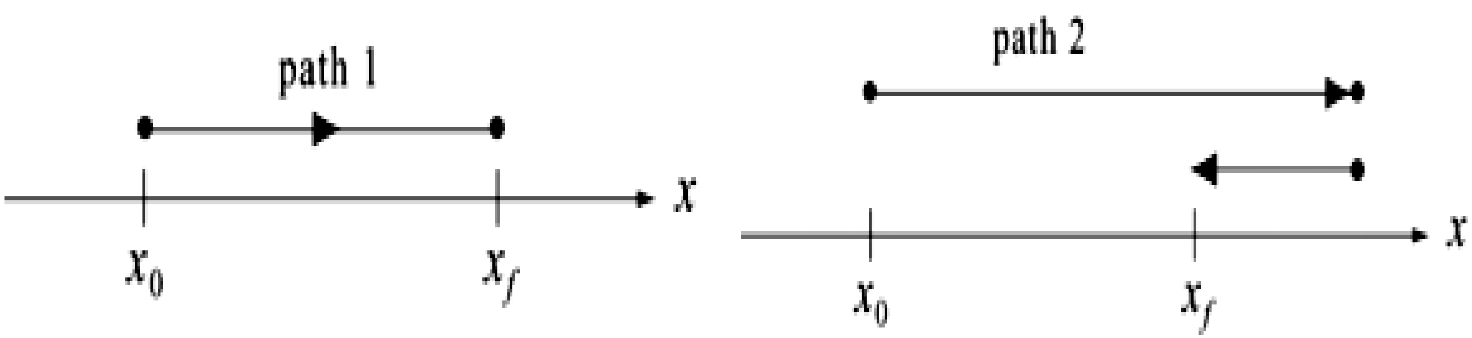
- motion of an object under the influence of a gravitational force near the surface of the earth

$$W_{grav} = \int_{path} \vec{\mathbf{F}} \cdot d\vec{\mathbf{r}} = \int_{y_0}^{y_f} F_{grav,y}\, dy = \int_{y_0}^{y_f} -mg\, dy = -mg(y_f - y_0)$$

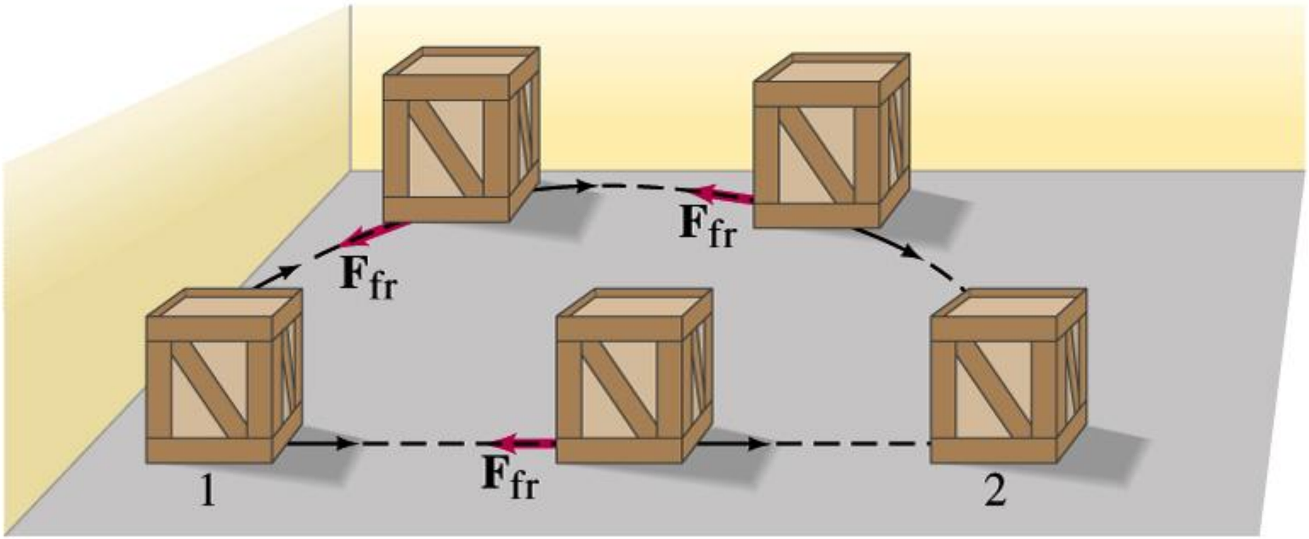


Now consider the motion of an object on a surface with a kinetic frictional force between the object and the surface and denote the coefficient of kinetic friction by μ_k. Let's compare two paths from an initial point x_0 to a final point x_f. The first path is a straight-line path. Along this path the work done is just

$$W_{\text{friction}} = \int_{\text{path 1}} \vec{\mathbf{F}} \cdot d\vec{\mathbf{r}} = \int_{\text{path 1}} F_x \, dx = -\mu_k N s_1 = -\mu_k N \Delta x < 0$$



A nonconservative force

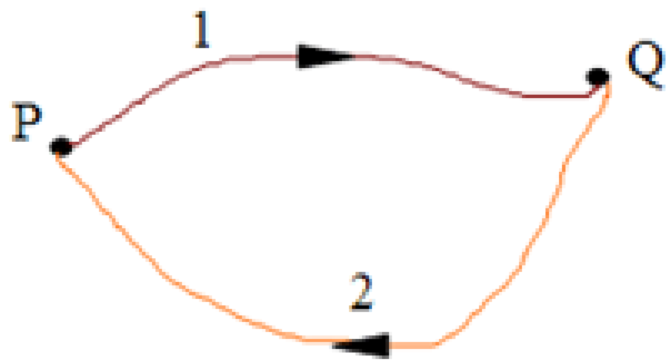


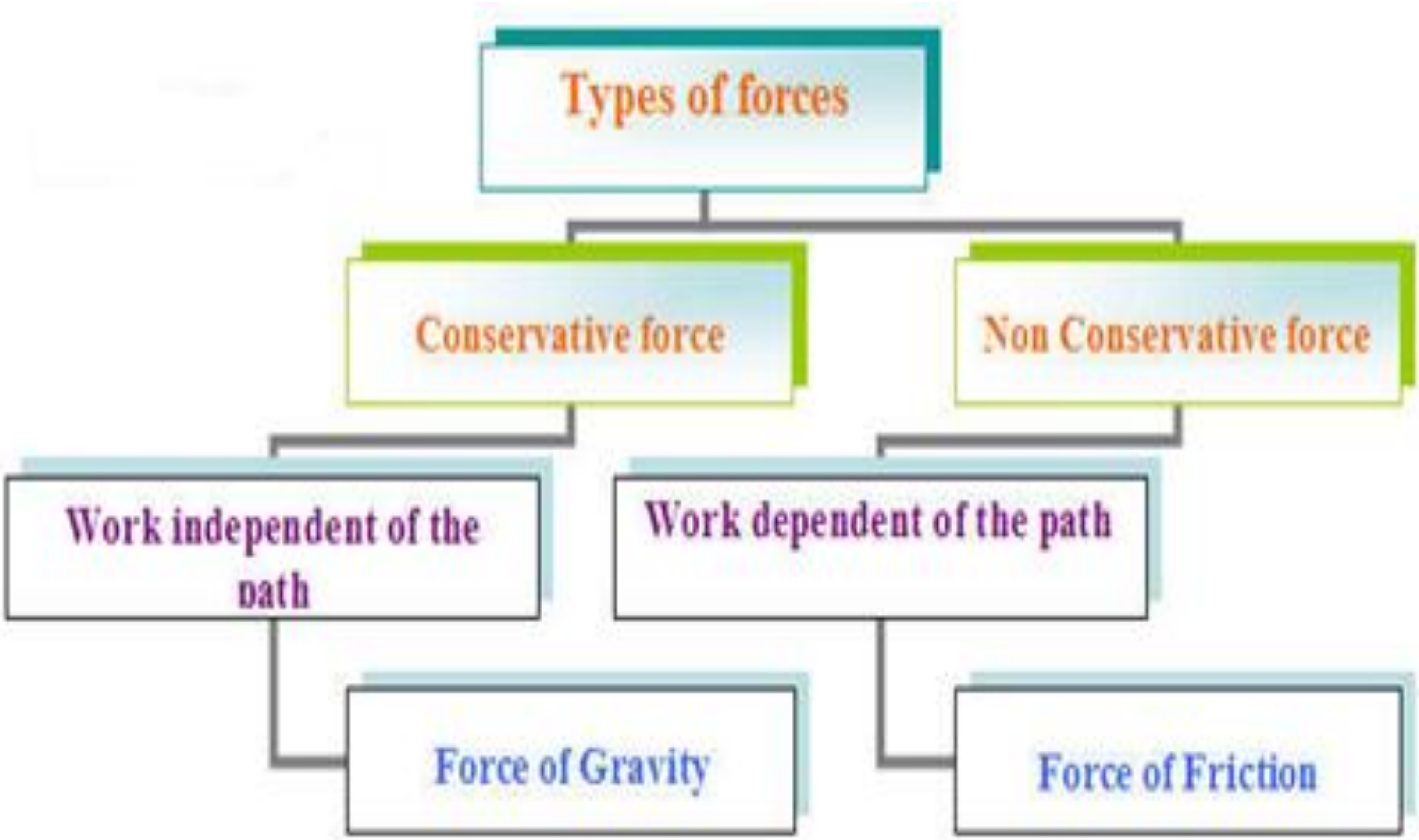
Friction is a nonconservative force!

Conservative forces

- A force is conservative when the *work* done by that *force* acting on a particle moving between two points is *independents* of the path the particle takes between the points.

$$W_{PQ}(\text{along } 1) = -W_{PQ}(\text{along } 2)$$




Types of forces
Conservative force
Non Conservative force
Work independent of the path
Work dependent of the path
Force of Gravity
Force of Friction

LAW OF CONSERVATION MECHANICAL ENERGY

- **The law of conservation of mechanical energy states that the total mechanical energy of a system remains constant for conservative force only. This means that when the kinetic energy increased the potential energy decrease.**

Potential Energy

- the amount of lost kinetic energy that can be recovered during an interaction. When only internal conservative forces act on the system, the sum of the changes of the kinetic and potential energies of the system is zero.
- The change in potential energy associated with a particular conservative force is the negative of the work done by that force.

The amount of change in the potential energy is given by:

$$U = -\int_{x_i}^{x_f} F_x(x)dx = -W = U_f - U_i$$

$$\Delta U = U_2 - U_1 = -W$$

Conservation of Mechanical Energy

$$W = \Delta K$$

Work-Energy Principle

$$-W = \Delta U$$

Definition of Potential Energy

$$0 = \Delta K + \Delta U$$

$$K_1 + U_1 = K_2 + U_2$$

$$E_1 = E_2$$

$$\Delta U_{system} = -\Delta K_{system}$$

Gravitational potential energy

The work done by the gravitational force is

$$U_f - U_i = mg(y_f - y_i)$$

If $y_i = 0 \quad \Rightarrow \quad U_i = 0, \quad U_f = +mgy_f$

If $y_f = 0 \quad \Rightarrow \quad U_f = 0, \quad U_i = -mgy_i$

There is no difference for the sign of the potential energy for the object. You can take whether $y_f = 0$ or $y_i = 0$. The difference between two points will be always equal to each other.

Potential energy for a spring

The Hooke law states that

$$F(x) = -k\,x$$

and the work done by the spring is given by

$$W = -\frac{1}{2}k(x_f^2 - x_i^2).$$

Since the potential energy is

$$\Delta U = -W \quad \Rightarrow \quad U_f - U_i = \frac{1}{2}k(x_f^2 - x_i^2)$$

If $x_i = 0 \quad \Rightarrow \quad U = \frac{1}{2}kx^2$

As seen in the last Equation, the potential energy for a spring can not be "negative".

Non-conservative forces

We know that the mechanical energy is written as

$$K_f - K_i = W_{net}$$

Now, we know that the work done by an object may not be conservative all time. Then we write the net work done by the force as

$$W_{net} = W_{cons} + W_{noncons}$$

and we know $W_{cons} = -(U_f - U_i)$**, so**

$$W_{net} = -(U_f - U_i) + W_{noncons}$$

and we said

$$K_f - K_i = -(U_f - U_i) + W_{noncons}$$

$$K_f + U_f = K_i + U_i + W_{noncons}$$

$$E_f = E_i + W_{noncons}$$

Potential Energy Summary

- Potential energy is only associated with conservative forces. It is the negative of the work done by the conservative force.
- The zero point of potential energy is arbitrary and should be chosen where it is most convenient.
- Potential energy is not something a body has by itself, but rather is associated with the interaction of two or more objects.

The Center of Mass of Two Particles

We will use the center of mass to calculate the kinematics and dynamics of the system as a whole, regardless of the motion of the individual particles.

Center of Mass for Two Particles in One Dimension

If a particle with mass m_1 has a position of x_1 and a particle with mass m_2 has a position of x_2, then the position of the center of mass of the two particles is given by:

$$x_{cm} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2} \qquad v_{cm} = \frac{m_1 v_1 + m_2 v_2}{m_1 + m_2} \qquad a_{cm} = \frac{m_1 a_1 + m_2 a_2}{m_1 + m_2}$$

By applying Newton's Second Law we can state that

$$F_{12} = m_1 a_1 \text{ and } F_{21} = m_2 a_2.$$

We can now substitute this into our expression for the acceleration of the center of mass:

$$a_{cm} = \frac{F_{12} + F_{21}}{m_1 + m_2}$$

Let M be the total mass of the system. Thus $M = m_1 + m_2$ and:

$$\sum \mathbf{F}_{\text{ext}} = M\mathbf{a}_{\text{cm}}$$

$$x_{cm} = \frac{1}{M} \sum m_n x_n$$

$$v_{cm} = \frac{1}{M} \sum m_n v_n$$

$$a_{cm} = \frac{1}{M} \sum m_n a_n$$

$$\sum F_{ext} = M a_{cm}$$

Impulse

- Impulse is force applied over a time period. Impulse can be defined mathematically, and is denoted by *I*:

$$I = F\Delta t$$

$$I = F\Delta t \Rightarrow I = ma\Delta t \Rightarrow I = m\frac{\Delta v}{\Delta t}\Delta t$$

$$I = m\Delta v \Rightarrow I = mv_f - mv_i$$

Momentum

Newton defined the quantity of motion or the *momentum*, $\vec{\mathbf{p}}$, to be the product of the mass and the velocity

$$\vec{\mathbf{p}} = m\,\vec{\mathbf{v}}\,.$$

Momentum is a vector quantity, with direction and magnitude. The direction of momentum is the same as the direction of the velocity. The magnitude of the momentum is the product of the mass and the instantaneous speed.

Units: In the SI system of units, momentum has units of $[\mathrm{kg\cdot m\cdot s^{-1}}]$. There is no special name for this combination of units.

$$\frac{dp}{dt} = m\frac{dv}{dt} \Rightarrow \frac{dp}{dt} = ma \Rightarrow \frac{dp}{dt} = \Sigma F$$

This change in momentum is called the *impulse*,

$$\vec{\mathbf{I}} = \vec{\mathbf{F}}_{\text{ave}}\,\Delta t = \Delta\vec{\mathbf{p}}.$$

The Impulse-Momentum Theorem

$$I = mv_f - mv_i$$

$$I = p_f - p_i \Rightarrow I = \Delta p$$

This equation is known as the Impulse-Momentum Theorem. Stated verbally, **an impulse given to a particle causes a change in momentum of that particle.**

Conservation of Momentum

$$\frac{dp}{dt} = \Sigma F_{ext}$$

- When there is no external force on the system:

$$\Sigma F_{ext} = 0$$

The conservation of linear momentum: ***When there is no net external force acting on a system of particles the total momentum of the system is conserved.***