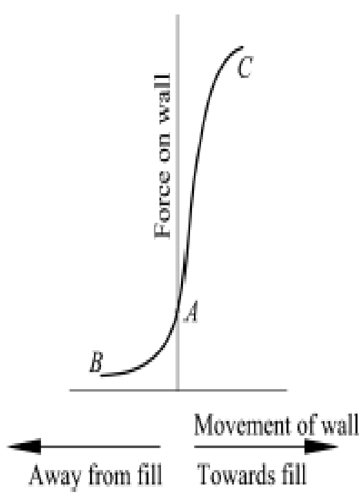
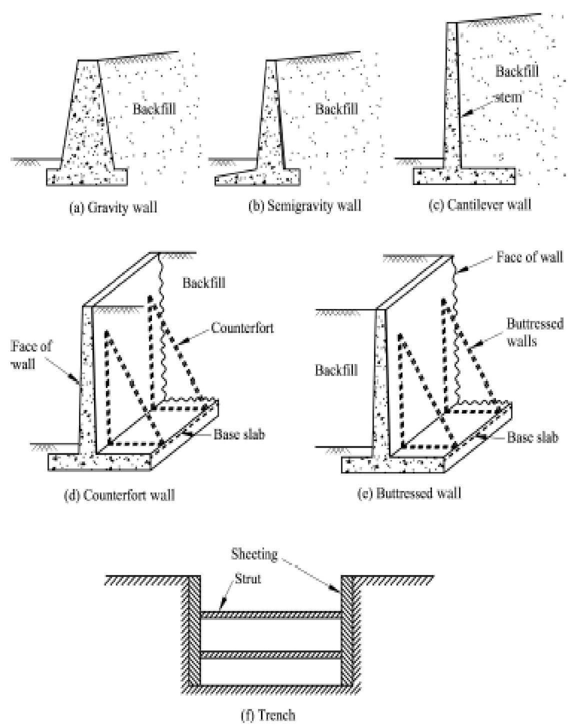
# Earth Retaining Structures

Lateral earth pressure determination is needed in the design of many types of structures and structural members, common examples are retaining walls, sheet piles, tunnels, basement walls of buildings and other walls that retain earth fills and excavation trenches. However, lateral pressures can be estimated accurately using earth pressure theories, such as Rankine's theory, Coulomb's theory and other theories.

## Relationships Between Lateral Pressure and Backfill Movement

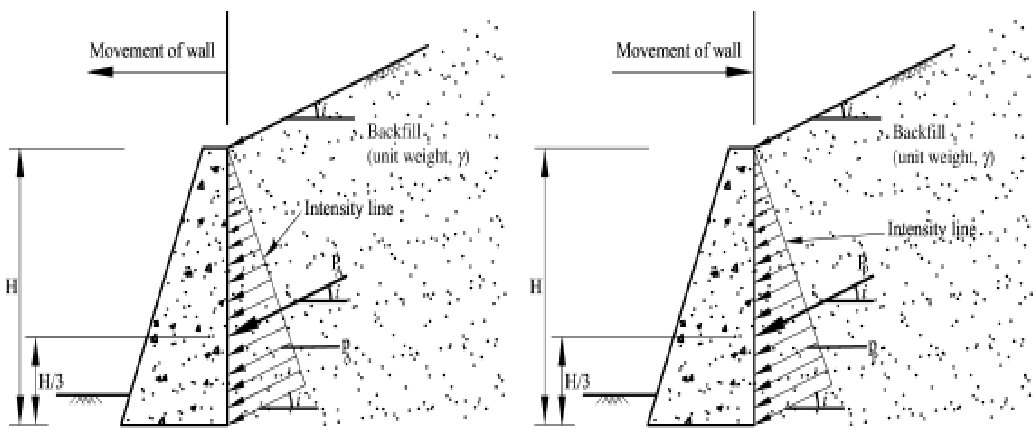
The ordinate of point A represent the force on a wall which has been held rigidly in place while a soil backfill is behind it. This is called Earth pressure at rest where there is no relative movement between the back fill and the soil. This is also called $K_o$ condition where $K_o$ is the corresponding earth pressure coefficient. $K_o$ values are around 0.4–0.5. $K_o$ is referred to as coefficient of earth pressure at rest



If the wall moves in the direction away from the backfill, the force decreases and after a small movement reaches a minimum value at point B. This is called active earth pressure. The corresponding earth pressure coefficient is $K_a$.

If the wall is forced against the backfill, the force between the wall and the fill increases, reaching a maximum value at C. This is called passive earth pressure. The corresponding earth pressure coefficient $K_p$ values are around 3–4.



(a) Active case (b) Passive case

## Rankine Theory

The minimum principal stress *OC =* $\sigma_3$ *is termed the active earth pressure and can be* computed using the following equation,

$\sin \phi = 1/2(\sigma_1 - \sigma_3) / 1/2(\sigma_1 + \sigma_3) + c \cot\phi$

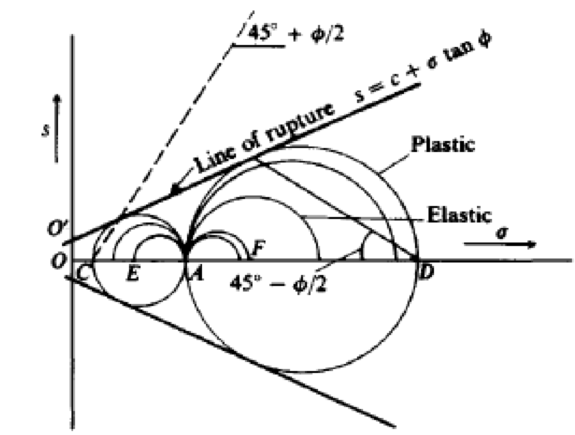
$\sigma_3(1+\sin \phi) = \sigma_1(1-\sin \phi) - 2c \cos\phi$

$\sigma_3 = \sigma_1 \tan^2(45-\phi/2) - 2c \tan(45-\phi/2)$

For horizontal back fill surface

$K_a = \tan^2(45-\phi/2) = (1-\sin \phi)/(1+\sin \phi)$

$\sigma_3 = \sigma_1 k_a - 2c k_a^{1/2}$

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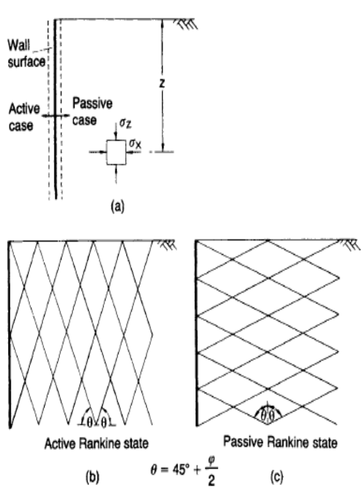
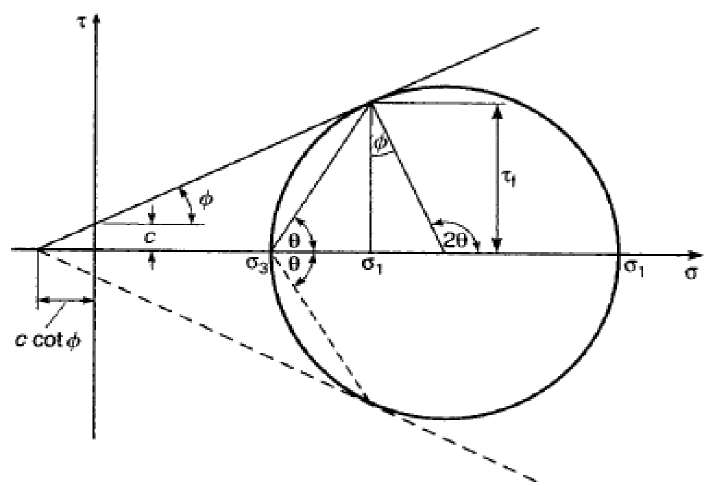


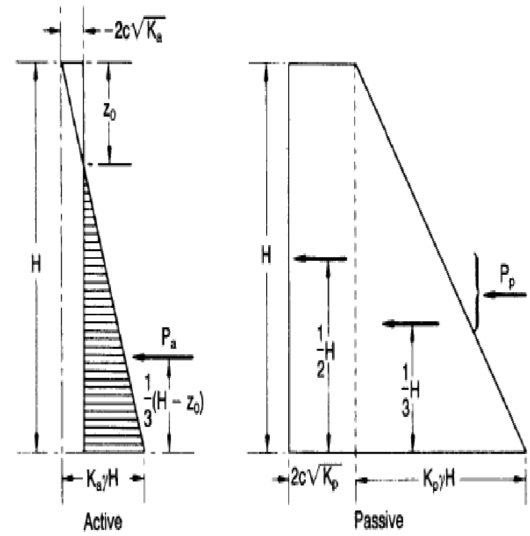
$\sigma_h = \sigma_3 \quad \sigma_v = \sigma_1 = \gamma z$

$\sigma_a = \gamma z\, k_a - 2\, c\, k_a^{1/2}$

$\sigma_a$=active pressure

$P_a = 1/2 \gamma H^2\, k_a - 2\, c\, H k_a^{1/2}$

$P_a$=Active thrust or force per meter length

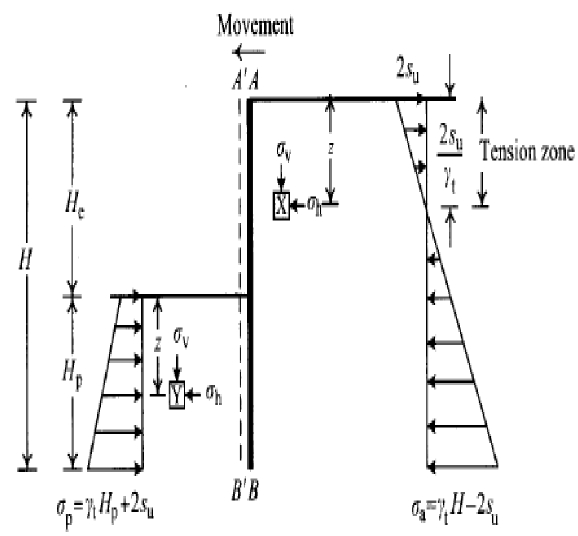



**Effect of Tension Crack**

When c is greater than zero, the value of $\sigma_a$ is zero at a particular depth $z_0$

$0 = \gamma z\, k_a - 2\, c\, k_a^{1/2}$

$z_0 = 2\, c / \gamma\, k_a^{1/2}$ **depth of tension crack**




**For c = 0**

$\sigma_a = \sigma_v\, k_a \qquad K_a = \sigma_h / \sigma_v$

$\sigma_h = \gamma z\, k_a$

**For passive condition**

$\sigma_1 = \sigma_3 \tan^2(45 + Ø/2) + 2\, c \tan(45 + Ø/2)$

$\sigma_p = \sigma_v k_p + 2\, c\, k_p^{1/2}$

$P_p = 1/2 \gamma H^2\, k_p + 2\, c\, H k_p^{1/2}$

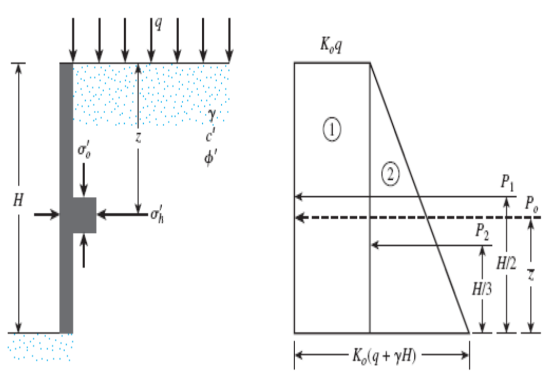
$K_p = 1/k_a = (1 + \sin Ø)/(1 - \sin Ø)$

**Effect of uniformly distributed surcharge pressure q acts on surface.**

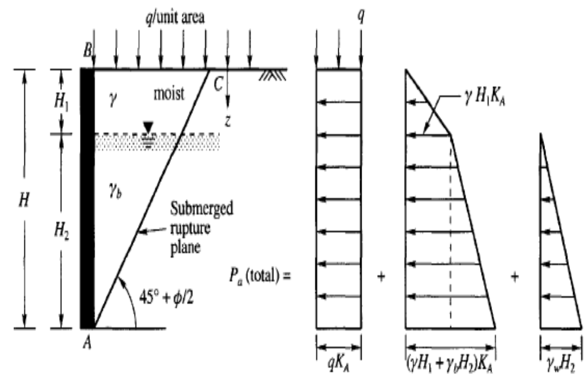
At any depth *z below the ground surface, the horizontal stress due to q*

$\sigma_a = q\,k_a + \gamma z\,k_a \quad P_a = q\,Hk_a + 1/2\gamma H^2\,k_a \quad \sigma_p = q\,k_p + \gamma z\,k_p$

$P_p = q\,Hk_p + 1/2\gamma H^2\,k_p$




(a) (b)




(a) Retaining wall (b) Pressure distribution

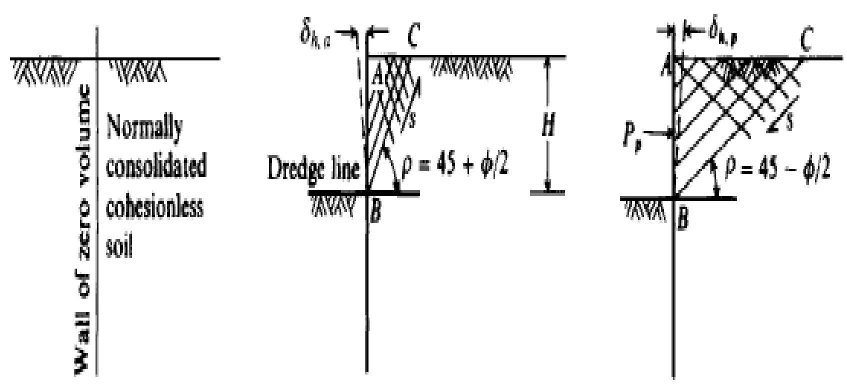
***Coefficient of lateral earth pressure at rest $k_o$***

For normally consolidated clay

$K_o = 1 - \sin \phi'$

For over consolidated clay

$K_o = 1 - \sin \phi' (OCR)^{\sin \phi'}$




(a) (b) Active case. (c) Passive case.

## Sloping soil surface

**-Active Pressure**

$\sigma_a = \gamma z$ **cosi[cosi-(cos²i-cos²Ø)^{1/2}]/[cosi+(cos²i-cos²Ø)^{1/2}]**

$$k_a = \cos i[\cos i-(\cos^2 i-\cos^2 Ø)^{1/2}]/[\cos i+(\cos^2 i-\cos^2 Ø)^{1/2}]$$

$$\sigma_a = \gamma z k_a$$

The resultant thrust on the wall

$P_a = 1/2 \gamma H^2 k_a$

where

H=height of the wall

$\gamma$=unit weight of soil

Z=depth coordinate from top of the wall

$K_a$ =active earth pressure coefficient for inclined backfill

**-Passive Pressure**

$$\sigma_p = \gamma z k_p$$

$$k_p = \cos i[\cos i+(\cos^2 i-\cos^2 Ø)^{1/2}]/[\cos i-(\cos^2 i-\cos^2 Ø)^{1/2}]$$

$k_p$= passive earth pressure coefficient for inclined backfill

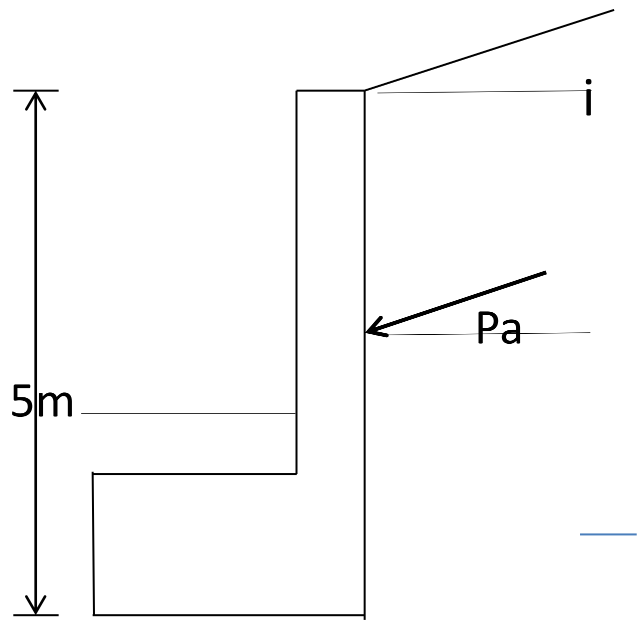
the thrust acts at a height of 1/3 H from the base of the wall.

If the backfill is level (horizontal, i=0), the above equations reduce to

$$\sigma_a = \gamma z[(1-\sin Ø)/(1+\sin Ø)] = \gamma z \tan^2(45-Ø/2)$$

$$P_a = 1/2\gamma H^2 k_a = 1/2\gamma H^2 [(1-\sin Ø)/(1+\sin Ø)] = 1/2\gamma H^2 \tan^2(45-Ø/2)$$

What is the total active earth force(thrust) per meter of wall for the retaining wall shown in the figure using the Rankin equation. The surface of the back fill inclined 10° with horizontal, Ø=20° the height of the R.W. is 5m.




$\sigma_a = \gamma z \cos i[\cos i - (\cos^2 i - \cos^2 Ø)^{1/2}]/[\cos i + (\cos^2 i - \cos^2 Ø)^{1/2}]$

$= 20 x 5 x \cos 10[\cos 10 - (\cos^2 10 - \cos^2 20)^{1/2}] / [\cos 10 + (\cos^2 10 - \cos^2 20)^{1/2}]$

$= 98 x 0.69/1.28 = 53 kN/m^2$

$P_a = 1/2 x 53 x 5 = 132.5 kN/m$

**Example**

Assume that the retaining wall shown in Figure can yield sufficiently to develop an active state. Determine the Rankine active force per unit length of the wall and the location of the resultant line of action.

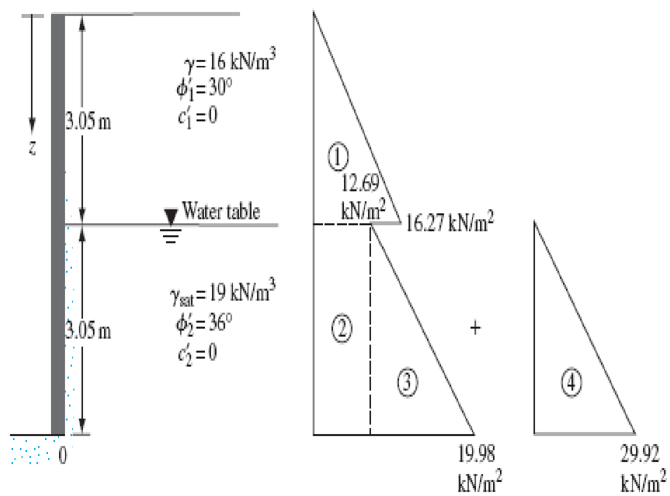
$\sigma_a = \gamma z\, k_a$

$Ka = \tan^2(45-Ø/2)$

$K_{a1} = \tan^2(45-30/2) = 1/3$

$K_{a2} = \tan^2(45-36/2) = 0.26$

The following table shows the calculation of $\sigma_a$ *and u at various depths below the* ground surface.




| Depth(m) | $\sigma_{v} = \gamma z$ (kN/m²) | $K_a$ | $\sigma_a = \gamma z\, k_a$ (kN/m²) | u (kN/m²) |
|---|---|---|---|---|
| 0 | 0 | 1/3 | 0 | 0 |
| 3.05 **(upper layer)** | 16x3.05=48.8 | 1/3 | 48.8x1/3=16.27 | 0 |
| 3.05 **(lower layer)** | 16x3.05=48.8 | 0.26 | 48.8x0.26=12.69 | 0 |
| 6.1 | 16x3.05+(19-9.81)(3.05)=76.83 | 0.26 | 19.98 | 9.81x3.05=29.92 |

The thrust force per meter length

*Pa = area 1+ area 2 + area 3 + area 4*

= 1/2(3.05)(16.27) + (12.69)(3.05) + 1/2(19.98 - 12.69)(3.05) + 1/2(29.92)(3.05)

=24.81 + 38.70 +11.12 + 45.63 = **120.26 kN/m**

The distance of the line of action of the resultant force from the bottom of the wall can be determined by taking the moments about the bottom of the wall (point O)

$$\bar{z} = \frac{(24.81)\left(3.05 + \frac{3.05}{3}\right) + (38.7)\left(\frac{3.05}{2}\right) + (11.12 + 45.63)\left(\frac{3.05}{3}\right)}{120.26} = 1.81 \text{ m}$$

## Stability of Retaining Wall

A retaining wall may fail in any of the following ways:

- It may *overturn about its toe.*

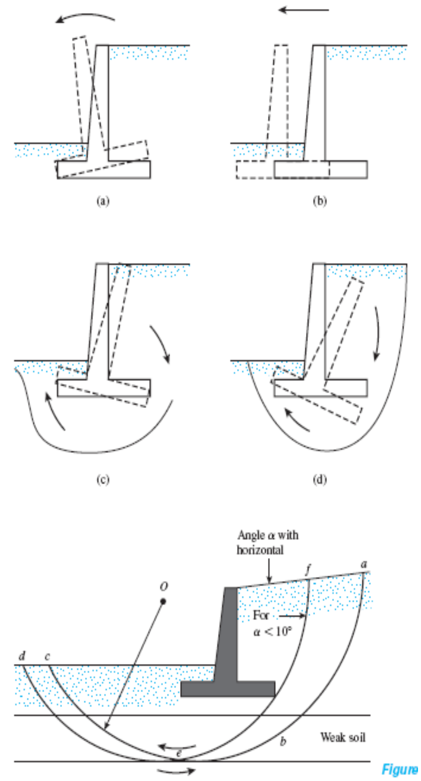
-It may *slide along its base.*

- It may fail due to the loss of *bearing capacity of the soil supporting the base.*

-It may undergo deep-seated shear failure.

- It may go through excessive settlement.

-Structural failure of any element of the wall or combined soil/structure failure




## Factor of safety against overturning

The factor of safety against overturning about the toe( about point *C) in* the figure may be expressed as

- $FS_{overturning} = \Sigma M_r / \Sigma M_o$

$\Sigma M_o$ = sum of the moments of forces tending to overturn about point *C*

$\Sigma M_r$ = sum of the moments of forces tending to resist overturning about point *C*

$$\sum M_o = P_h(H'/3)$$

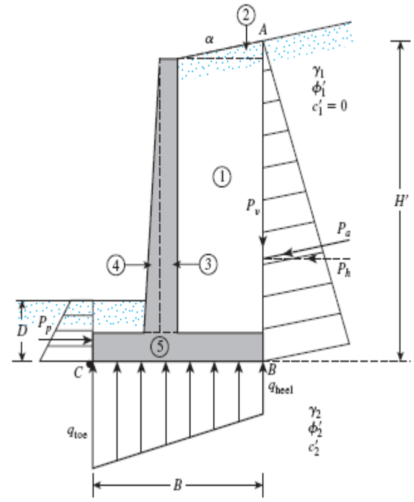
$$P_h = P_a \cos\alpha$$

$$P_v = P_a \sin\alpha$$

The moment of the force $P_v$ about *C is*

$$M_v = P_v B = P_a \sin\alpha B$$

$$F_{overturning} = (\sum M_{1,2,3,4,5 parts} + M_v)/\sum M_o = \sum Mr/\sum Mo$$




## Factor of safety against sliding

$$FS_{sliding} = \sum F_r / \sum F_d$$

$\sum F_r$=Sum of the horizontal resisting forces

$\sum F_d$=Sum of the horizontal driving forces

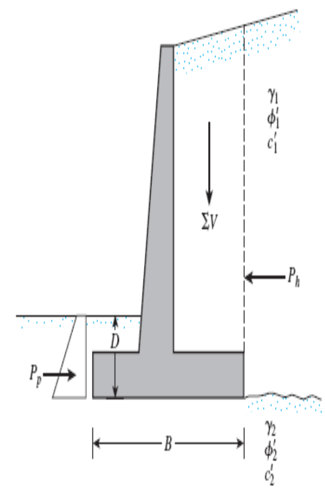
$s = \sigma \tan\delta' + c'_a B$

$\delta'$ =Angle of friction between the soil and the base slab

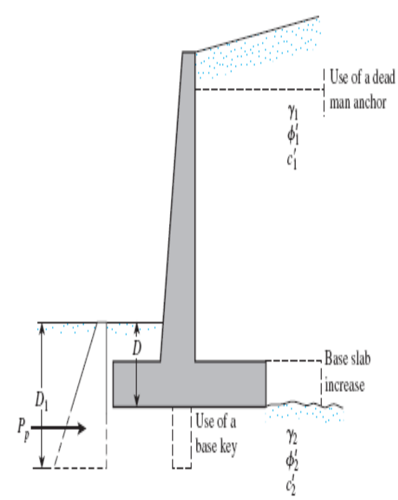
$c'_a$=adhesion between the soil and the base slab

s(area of cross section) = s(B x 1) = $B\sigma' \tan\delta' + Bc'_a$

$$\sum F_r = (\sum V)\tan\delta' + B c'_a + P_p$$




$$FS_{(sliding)} = \frac{(\sum V)\tan\delta' + Bc'_a + P_p}{P_a \cos\alpha} \geq 1.5$$




In many cases, the passive force is ignored in calculating the factor of safety with respect to sliding.

# Check for Bearing Capacity Failure

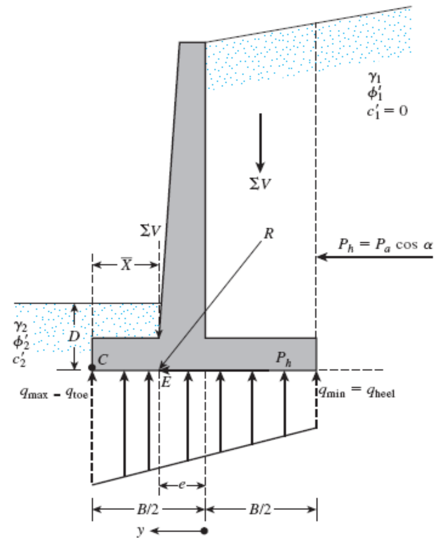
The pressure distribution under the base slab may be determined by using simple principles from the mechanics of materials.

**$q = \Sigma V/A \pm M\,y/I$**

Where

M= moment = ($\Sigma$**V**) e

**I**= Moment of inertia per unit length of the base Section= 1/12 (1) ($B^3$)

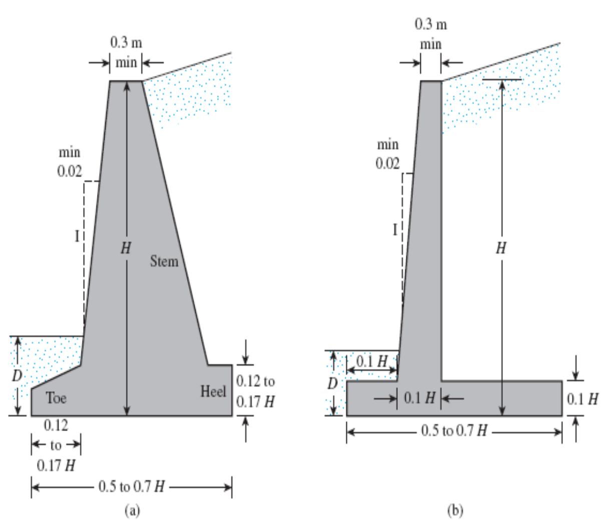



$$q_{max} = q_{toe} = \frac{\Sigma V}{(B)(1)} + \frac{e(\Sigma V)\frac{B}{2}}{\left(\frac{1}{12}\right)(B^3)} = \frac{\Sigma V}{B}\left(1 + \frac{6e}{B}\right)$$

Similarly,

$$q_{min} = q_{heel} = \frac{\Sigma V}{B}\left(1 - \frac{6e}{B}\right)$$

Note that $\Sigma$**V** includes the weight of the soil, and that when the value of the eccentricity *e becomes greater than* B/6, $q_{min}$ *becomes negative.* Thus, there will be some tensile stress at the end of the heel section. This stress is not desirable, because the tensile strength of soil is very small. If the analysis of a design shows that e>B/6 the design should be reproportioned and calculations redone.



## Example

The cross section of a cantilever retaining wall is shown in the Figure . Calculate the factors of safety with respect to overturning, sliding, and bearing capacity.

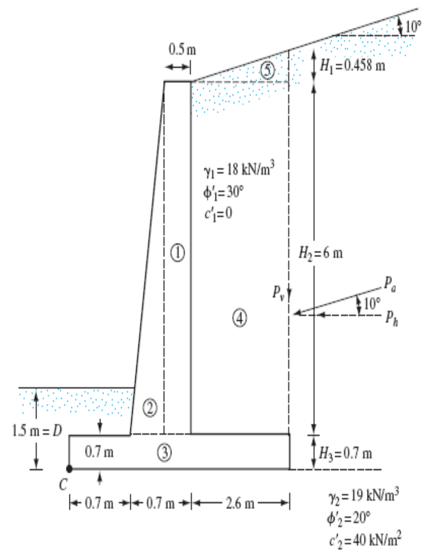
*H = H1+ H2+ H3= 2.6 tan 10° +6 + 0.7*

= 0.458 + 6+ 0.7= 7.158 m

$P_a$ = 1/2(18) (7.158)$^2$(0.3532) = 162.9 kN/m

$P_v$ = *Pa sin10° = 162.9 (sin10°)= 28.29 kN/m*

$P_h$ *=Pa cos10°= 162.9 (cos10°)= 160.43 kN/m*




**Factor of Safety against Overturning**

The following table can now be prepared for determining the resisting moment:

| Section no.ª | Area (m²) | Weight/unit length (kN/m) | Moment arm from point C (m) | Moment (kN-m/m) |
|---|---|---|---|---|
| 1 | $6 \times 0.5 = 3$ | 70.74 | 1.15 | 81.35 |
| 2 | $\frac{1}{2}(0.2)6 = 0.6$ | 14.15 | 0.833 | 11.79 |
| 3 | $4 \times 0.7 = 2.8$ | 66.02 | 2.0 | 132.04 |
| 4 | $6 \times 2.6 = 15.6$ | 280.80 | 2.7 | 758.16 |
| 5 | $\frac{1}{2}(2.6)(0.458) = 0.595$ | 10.71 | 3.13 | 33.52 |
| | | $P_v = 28.29$ | 4.0 | 113.16 |
| | | $\Sigma V = 470.71$ | | $1130.02 = \Sigma M_R$ |

$\Sigma M_o = P_h(H'/3)$

= 160.4(7.15/3)=382.79 kN.m/m

$F_{overturning} = \Sigma M_r / \Sigma M_o$

$= 1130.02/382.79 = \mathbf{2.95 > 2, OK}$

**Factor of Safety against Sliding**

$$FS_{(sliding)} = \frac{(\Sigma V)\tan\delta' + Bc'_a + P_p}{P_a \cos\alpha}$$

$P_p = 1/2\gamma H^2 k_p + 2\ c\ Hk_p^{1/2}$

$K_p = 1/k_a = (1+\sin Ø)/(1-\sin Ø) = (1+\sin 20)/1-\sin 20) = 2.04$

$P_p = 1/2 x 19 x (1.5)^2\ 2.04 + 2x\ 40 x 1.5 x (2.04)^{1/2} = 215 kN/m$

$$FS_{(sliding)} = \frac{(470.71)\tan\left(\frac{2 \times 20}{3}\right) + (4)\left(\frac{2}{3}\right)(40) + 215}{160.43}$$

$$= \frac{111.56 + 106.67 + 215}{160.43} = \mathbf{2.7 > 1.5, OK}$$

**Factor of Safety against Bearing Capacity Failure** Assume $q_{all} = 200 kPa$

$$e = \frac{B}{2} - \frac{\Sigma M_R - \Sigma M_o}{\Sigma V} = \frac{4}{2} - \frac{1130.02 - 382.79}{470.71}$$

$$= 0.411 \text{ m} < \frac{B}{6} = \frac{4}{6} = 0.666 \text{ m}$$

Again, from Eqs. (8.20) and (8.21)

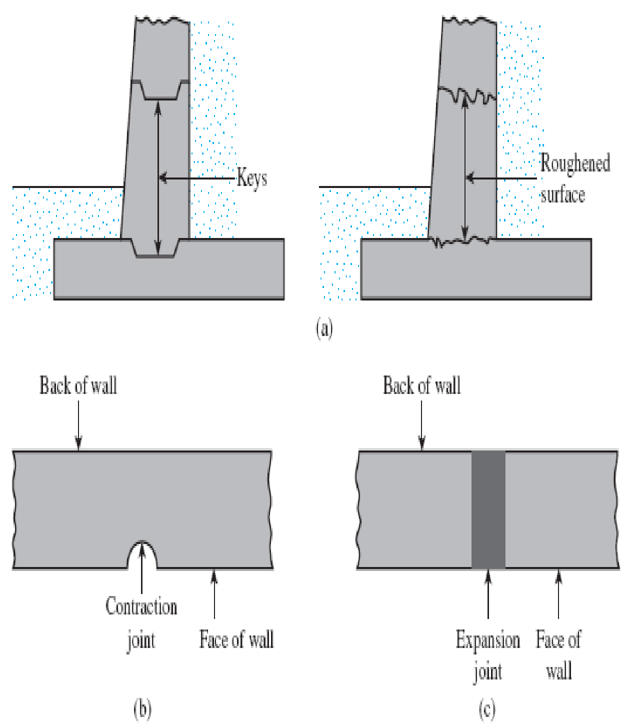
$$q_{\text{heel}}^{\text{toe}} = \frac{\Sigma V}{B}\left(1 \pm \frac{6e}{B}\right) = \frac{470.71}{4}\left(1 \pm \frac{6 \times 0.411}{4}\right) = 190.2 \text{ kN/m}^2 \text{ (toe)}$$

$$= 45.13 \text{ kN/m}^2 \text{ (heel)}$$

## Construction Joints

A retaining wall may be constructed with one or more of the following joints:

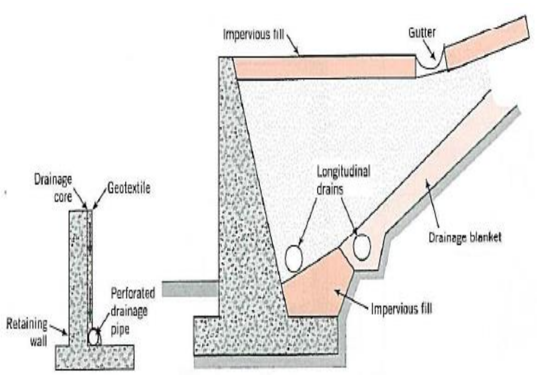
1. *Construction joints (Figure a) are vertical and horizontal joints that are* placed between two successive pours of concrete. To increase the shear at the joints, keys may be used. If keys are not used, the surface of the first pour is cleaned and roughened before the next pour of concrete.
2. *Contraction joints (Figure b) are vertical joints (grooves) placed in the face* of a wall (from the top of the base slab to the top of the wall) that allow the concrete to shrink without noticeable harm. The grooves may be about 6 to 8 mm wide and 12 to 16 mm deep.
3. *Expansion joints (Figure c) allow for the expansion of concrete caused by* temperature changes; vertical expansion joints from the base to the top of the wall may also be used. These joints may be filled with flexible joint fillers. In most cases, horizontal reinforcing steel bars running across the stem are continuous through all joints. The steel is greased to allow the concrete to expand



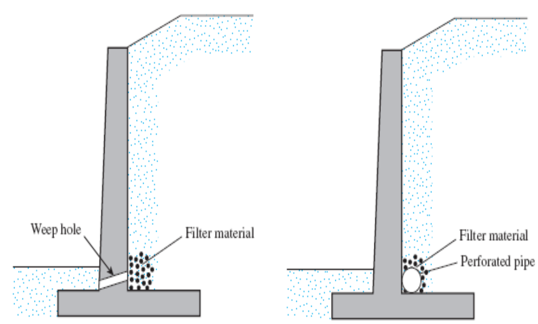
## Drainage System

Adequate drainage must be provided by means of *weep holes or perforated drainage pipes.* weep holes should have a minimum diameter of about 0.1 m and be adequately spaced. A filter material needs to be placed behind the weep holes or around the drainage pipes, geotextiles now serve that purpose.

Two main factors influence the choice of filter material: The grain-size distribution of the materials should be such that (a) the soil to be protected is not washed into the filter and (b) excessive hydrostatic pressure head is not created in the soil with a lower hydraulic conductivity (in this case, the backfill material).



(a) Simple vertical drain (b) Inclined drain (after Sibley, 1967)



(a) (b)