Q1: fill the blanks with correct answer:

1- The gradient field valid only for -------------- .

a- Vector field b- Scalar field c- scalar & vector field d- none of them

2- The gradient of a scalar field is a ------------ field.

a) scalar b) vector c) zero d) vanish

3- When the magnitude of the arrows of the field is the same everywhere around a point, the diverge is : a) positive b) negative c) zero d) none of them

4- When curl A = 0 but div A≠ 0, then the field is :
a) Irrotational as well as solenoidal b) irrotational but not solenoidal c) rotational d)none of them

Q2: Find the directional derivative of $\varnothing = x^3 z + 3xyz$ at (x= 2, y= -2, z =1) in the direction $\vec{s} = 3i - 2j + k$.

Q3: if $V = i(x^2 z) - j(y^2 x) + k(yz)$, which of the following points is closer to the sink of the field. $p_1 = (1,1,2)$, $p_2 = (-1, 1, 1)$ (7.5 marks)

Q4: compute the integral : $\iint_R x^2 y^2 + \cos(\pi x) + \sin(\pi y)\, dA\,,\ \ R = [-2,-1] \times [0,1]$

Q5: Define the following: 1- Divergence 2- gradient of a scalar field 3- conservative field 4- curl of a vector 5- non-conformist

Q6: (a) find the unit normal to the surface sin(x) y z , at point (π , 1 , 1)

(b) show that the divergence of the sum of two vector functions is equal to the sum of their divergence.

Q7: (a) convert the point $\left(\sqrt{6}, \frac{\pi}{4}, \sqrt{2}\right)$ from cylindrical to spherical coordinates.

(b) calculate the following integral :

$$\iint_R \frac{1}{(2x+3y)^2}\, dA\ ,\quad R = [0\,,1] \times [1\,,2]$$

Q8: Show that $\nabla.(U+V) = \nabla.U + \nabla.V$, where

$$U = iU_1 + jU_2 + kU_3$$

$$V = iV_1 + jV_2 + kV_3$$

Q9: for what values of a the following field is a gradient field?

$$\vec{F} = \hat{\imath}(4x^2 + axy) + \hat{\jmath}(3y^2 + 4x^2)$$

Q10: compute the integral : $\iint_R x e^{xy}\, dA, \quad R = [-1, 2] \times [0, 1]$

(7 marks)

Q11: Suppose the function that describes the temperature at any point in the room is given by:

$$T = x^2 - y^2 + xyz + 273$$

1. Determine the rate of change of temperature from origin towards the point $(-1\,, 2, 3)$.
2. In which direction does the temperature increase at fastest rate and how much is the change?

Q12: If $V = i(x^2z) - j(2y^2z^2) + k(xy^2z)$, which of the following points is closer to the source of the field. $P1 = (1, -1,1),\ P2 = (2, -1,5)$

Q13: Which of the following vectors is solenoidal;

A) $V = 3y^4z^2i + 4x^3z^2j + 3x^2y^2$

B) $U = (2x^2 + 8xy^2z)i + (3x^3y - 3xy)k - (4y^2z^2 + 2x^3z)k$

Q14: prove that : $\Gamma\left(\frac{1}{2}\right)\ \sqrt{\pi}$

Q15: for what values of a the following field is a gradient field?

$$\vec{F} = \hat{\imath}(4x^2 + axy) + \hat{\jmath}(3y^2 + 4x^2)$$