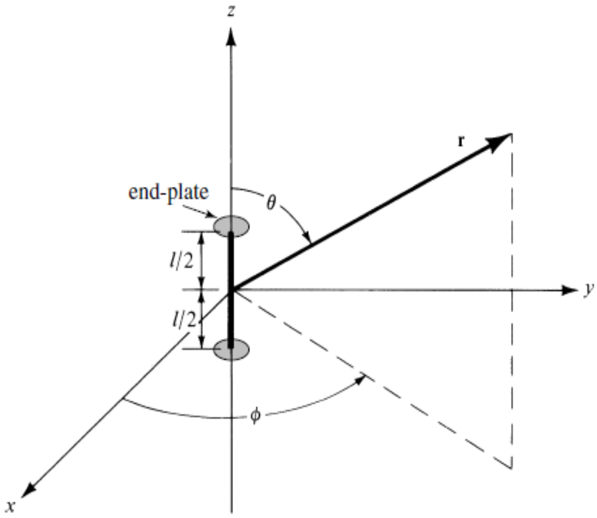
LINEAR WIRE ANTENNAS

Linear wire antennas are some of the oldest, simplest, cheapest, and in many cases the most versatile for many applications.

1. **Infinitesimal Dipole** **(L ≤ λ /50)**
2. **Small Dipole** **(λ /50 < L ≤ λ /10)**
3. **Finite Length Dipole** **(L > λ /10)**
4. **Half-Wave Dipole** **(L= λ /2)**

1. Infinitesimal Dipole

An infinitesimal linear dipole is positioned symmetrically at the origin along the *z*-axis, as shown in figure below. In addition to being very small *(l << λ)*, is very thin *(a << λ)*. The current distribution is assumed to be constant:



$$I(z) = I_0 \boldsymbol{a_z}$$

$$\vec{A} = \frac{\mu}{4\pi}\int_{-\frac{l}{2}}^{\frac{l}{2}} I(z)\frac{e^{-j\beta r}}{r}dl \quad , \quad dl = dz$$

$$\vec{A} = \frac{\mu I_0}{4\pi r}e^{-j\beta r}\boldsymbol{a_z}\int_{-l/2}^{l/2} dz = \frac{\mu I_0 l}{4\pi r}e^{-j\beta r}\boldsymbol{a_z}$$

$$A_z = \frac{\mu I_0 l}{4\pi r}e^{-j\beta r} \qquad A_x = A_y = 0$$

The transformation between rectangular and spherical coordinates is given in matrix form:

$$\begin{bmatrix} A_r \\ A_\theta \\ A_\phi \end{bmatrix} = \begin{bmatrix} \sin\theta\cos\phi & \sin\theta\sin\phi & \cos\theta \\ \cos\theta\cos\phi & \cos\theta\sin\phi & -\sin\theta \\ -\sin\phi & \cos\phi & 0 \end{bmatrix}\begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix}$$

$$A_r \ = \ A_z\cos\theta \ = \frac{\mu I_0 l}{4\pi r}\cos\theta e^{-j\beta r}$$

$$A_\theta \ = \ -A_z\sin\theta \ = -\frac{\mu I_0 l}{4\pi r}\sin\theta e^{-j\beta r}$$

$$A_\phi = 0$$

Since no variation in ϕ direction is assumed, so that $\frac{\partial}{\partial\phi} = 0$

The magnetic field $"\boldsymbol{H}"$ can be obtained by:

$$\text{curl A} = \nabla \times A = B = \mu H \rightarrow H = \frac{1}{\mu}(\nabla \times A) = \frac{1}{\mu}\begin{bmatrix} \frac{a_r}{r^2 sin\theta} & \frac{a_\theta}{r sin\theta} & \frac{a_\phi}{r} \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial\theta} & \frac{\partial}{\partial\phi} \\ Ar & A\theta & A\phi \end{bmatrix}$$

$$H_\phi = \frac{1}{\mu r}\left[\frac{\partial}{\partial r}(rA_\theta) - \frac{\partial}{\partial\theta}(A_r)\right]$$

$$H_\phi = \frac{1}{\mu r}\left[\frac{\partial}{\partial r}\left(-r\frac{\mu I_0 l}{4\pi r}\sin\theta\, e^{-j\beta r}\right) - \frac{\partial}{\partial\theta}\left(\frac{\mu I_0 l}{4\pi r}\cos\theta\, e^{-j\beta r}\right)\right]$$

$$H_\phi = \frac{1}{\mu r}\left[j\beta\frac{\mu I_0 l}{4\pi}\sin\theta\, e^{-j\beta r} + \frac{\mu I_0 l}{4\pi r}\sin\theta\, e^{-j\beta r}\right] = j\beta\frac{I_0 l}{4\pi r}\sin\theta\left(1+\frac{1}{j\beta r}\right) e^{-j\beta r}$$

$$H_\phi = j\beta\frac{I_0 l}{4\pi r}\sin\theta\left(1+\frac{1}{j\beta r}\right) e^{-j\beta r}$$

$$H_r = H_\theta = 0$$

The electric field $\boldsymbol{E}$ can be found using:

$$E = \frac{1}{j\omega\epsilon}(\nabla \times H)$$

$$E_r = \eta\frac{I_0 l}{2\pi r^2}\cos\theta\left(1+\frac{1}{j\beta r}\right) e^{-j\beta r}$$

$$E_\theta = j\eta\frac{\beta I_0 l}{4\pi r}\sin\theta\left(1+\frac{1}{j\beta r}-\frac{1}{(\beta r)^2}\right) e^{-j\beta r}$$

$$E_\phi = 0$$

In far-field region βr >>1, the electric and magnetic fields are approximated as:

$$E_\theta \cong j\eta\frac{\beta I_0 l}{4\pi r}\sin\theta\, e^{-j\beta r}$$

$$H_\phi \cong j\frac{\beta I_0 l}{4\pi r}\sin\theta\, e^{-j\beta r}$$

$$E_r = E_\phi = H_r = H_\theta = 0$$

The following formula can be used to find E_θ in the far field:

$$E_\theta = j\eta\frac{\beta}{4\pi r}\sin\theta\, e^{-j\beta r}\int_{-l/2}^{l/2} I(x,y,z)\, e^{-j\beta z\cos\theta} dz$$

The average power density can be found using:

$$W_{av} = W_r\boldsymbol{a_r} = \frac{1}{2}Re\left[E_\theta \times H_\phi^*\right] = \frac{1}{2}\left(j\eta\frac{\beta I_0 l}{4\pi r}\sin\theta\, e^{-j\beta r}\right) * \left(-j\frac{\beta I_0 l}{4\pi r}\sin\theta\, e^{j\beta r}\right)\boldsymbol{a_r}$$

$$W_{av} = W_r\boldsymbol{a_r} = \eta\frac{\beta^2 I_0^2 l^2}{32\pi^2 r^2} sin^2\theta\,\boldsymbol{a_r} = \eta\frac{\left(\frac{2\pi}{\lambda}\right)^2 I_0^2 l^2}{32\pi^2 r^2} sin^2\theta\,\boldsymbol{a_r}$$

$$W_{av} = \frac{\eta}{8} I_0^2\left(\frac{l}{\lambda}\right)^2\frac{sin^2\theta}{r^2}\,\boldsymbol{a_r}$$

The radiation power is:

$$P_{rad} = \int_0^{2\pi}\int_0^{\pi} W_r\boldsymbol{a_r}\,.\,r^2 sin\theta d\theta d\phi\,\boldsymbol{a_r} = \int_0^{2\pi}\int_0^{\pi}\frac{\eta}{8}I_0^2\left(\frac{l}{\lambda}\right)^2\frac{sin^2\theta}{r^2}\,.\,r^2 sin\theta d\theta d\phi$$

$$P_{rad} = \frac{\eta}{8}I_0^2\left(\frac{l}{\lambda}\right)^2 2\pi\int_0^{\pi} sin^3\theta d\theta = \frac{\eta}{8}I_0^2\left(\frac{l}{\lambda}\right)^2 2\pi\frac{4}{3} = \frac{I_0^2}{2}\eta\frac{2\pi}{3}\left(\frac{l}{\lambda}\right)^2$$

$$P_{rad} = \eta \frac{\pi I_0^2}{3}\left(\frac{l}{\lambda}\right)^2 = 120\pi \frac{\pi I_0^2}{3}\left(\frac{l}{\lambda}\right)^2 = 40\pi^2\left(\frac{l}{\lambda}\right)^2 I_0^2$$

$$P_{rad} = \frac{I_0^2}{2} R_{rad} \rightarrow R_{rad} = \frac{2P_{rad}}{I_0^2}$$

And the radiation resistance is:

$$R_{rad} = \frac{2 * 40\pi^2 \left(\frac{l}{\lambda}\right)^2 I_0^2}{I_0^2}$$

$$R_{rad} = 80\pi^2\left(\frac{l}{\lambda}\right)^2 \quad (\Omega)$$

$$R_{in} = \frac{R_{rad}}{\sin^2\left(\frac{\beta l}{2}\right)} \quad (\Omega)$$

For infinitesimal dipole $\frac{l}{\lambda} = \frac{1}{50}$ then $R_{rad} = 80\pi^2\left(\frac{1}{50}\right)^2 = 0.316\ \Omega$

The radiation resistance of an infinitesimal dipole is very small, so a very large mismatch occurs when connected to practical transmission lines have characteristic impedances of 50 or 75 **Ω**, and the radiation efficiency will be very small.

The **radiation intensity** which is given by:

$$U = r^2 W_{av} = r^2 * \frac{\eta}{8} I_0^2 \left(\frac{l}{\lambda}\right)^2 \frac{sin^2\theta}{r^2}$$

$$U = \frac{\eta}{8} I_0^2 \left(\frac{l}{\lambda}\right)^2 sin^2\theta \qquad U_n(\theta) = sin^2\theta \qquad U_{max} = \frac{\eta}{8} I_0^2 \left(\frac{l}{\lambda}\right)^2$$

The maximum directivity and maximum effective aperture are:

$$D_{max} = \frac{4\pi U_{max}}{P_{rad}} = 4\pi \frac{\frac{\eta}{8} I_0^2 \left(\frac{l}{\lambda}\right)^2}{\frac{I_0^2}{2}\eta \frac{2\pi}{3}\left(\frac{l}{\lambda}\right)^2}$$

$$D_{max} = \frac{3}{2}$$

$$Aem = \frac{\lambda^2}{4\pi} D_0 = \frac{\lambda^2}{4\pi} * \frac{3}{2}$$

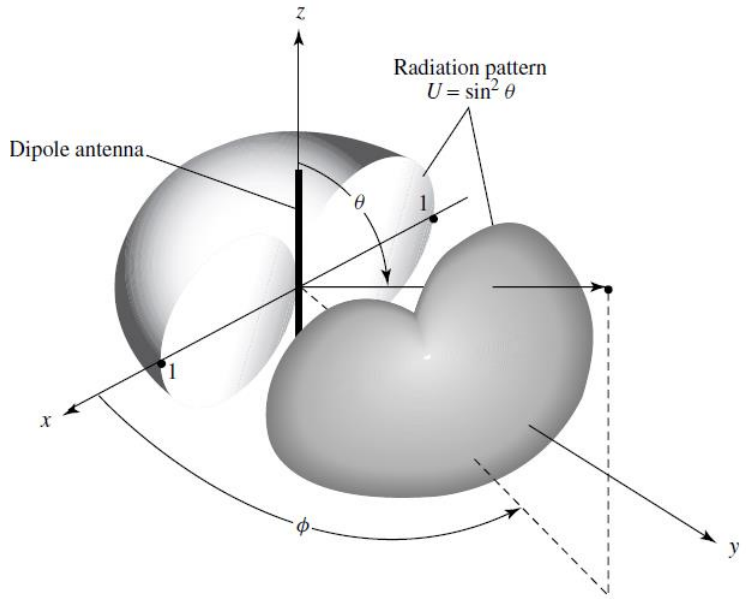
$$Aem = 0.119\lambda^2 \quad (m^2)$$

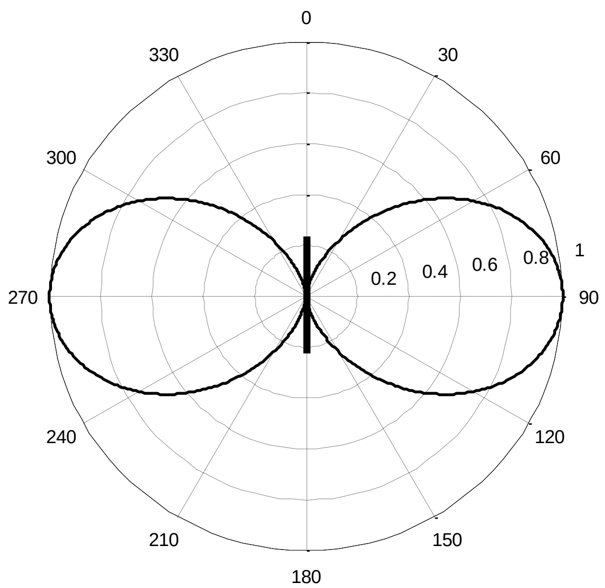
To find the HPBW: $\theta_m = 90^o$

$U_n(\theta)|_{\theta=\theta_h} = 0.5 \rightarrow sin^2\theta_h = 0.5 \rightarrow sin\theta_h = 0.707 \rightarrow \theta_h = 45^o, 135^o$

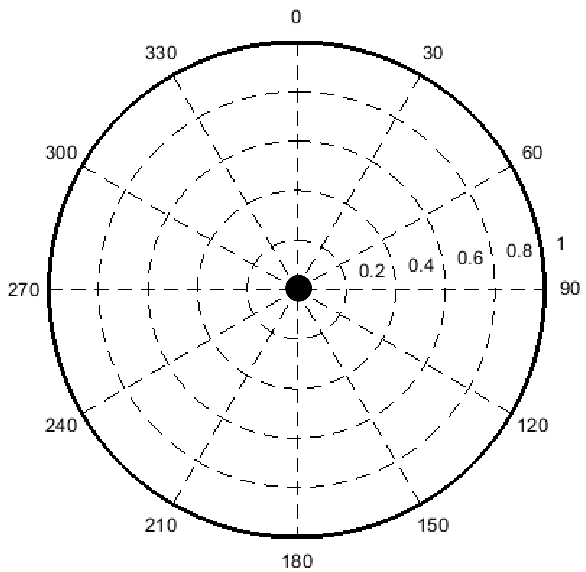
$HPBW = 2|\theta_m - \theta_h| = 2|90 - 45| = 90^o \ \ OR\ \ HPBW = 135^o - 45^o = 90^o$

The figure below shows the two and three-dimensional radiation pattern of infinitesimal dipole.





Radiation Pattern in E-plane (x-z plane)



Radiation Pattern in H-plane (x-y plane)

Example:

Find the radiation resistance of an infinitesimal dipole whose overall length is *a) l =* $\lambda/50$. *b) l =* $\lambda/75$

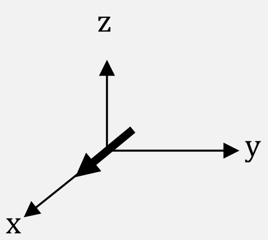
Solution:

$$a)\ R_{rad} = 80\pi^2\left(\frac{l}{\lambda}\right)^2 = 80\pi^2\left(\frac{\lambda/50}{\lambda}\right)^2 = 80\pi^2\left(\frac{1}{50}\right)^2 = 0.316\ \Omega$$

$$b)\ R_{rad} = 80\pi^2\left(\frac{l}{\lambda}\right)^2 = 80\pi^2\left(\frac{\lambda/75}{\lambda}\right)^2 = 80\pi^2\left(\frac{1}{75}\right)^2 = 0.14\ \Omega$$

Example:

Derive an expression for the electric fields, magnetic fields, and radiation intensity in far region if an infinitesimal dipole is placed symmetrically along the x-axis.



Solution:

In general:

$$E \cong j\eta \frac{\beta I_0 l}{4\pi r} \sin\gamma \, e^{-j\beta r} \qquad sin\gamma = \sqrt{1 - cos^2\gamma}$$

$$cos\,\gamma = \vec{\boldsymbol{a_x}} \cdot \vec{\boldsymbol{a_r}} \qquad \vec{\boldsymbol{a_r}} = \vec{\boldsymbol{a_x}} \sin\theta \cos\phi + \vec{\boldsymbol{a_y}} \sin\theta \sin\phi + \vec{\boldsymbol{a_z}} \cos\theta$$

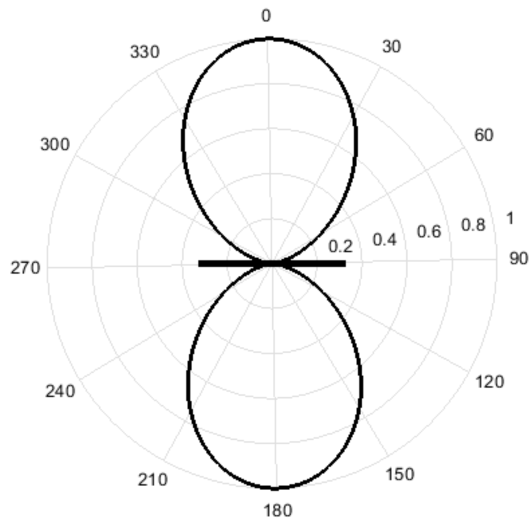
$$cos\,\gamma = \vec{\boldsymbol{a_x}} \cdot \left(\vec{\boldsymbol{a_x}} \sin\theta \cos\phi + \vec{\boldsymbol{a_y}} \sin\theta \sin\phi + \vec{\boldsymbol{a_z}} \cos\theta\right)$$

$$cos\,\gamma = sin\,\theta\, cos\,\phi$$

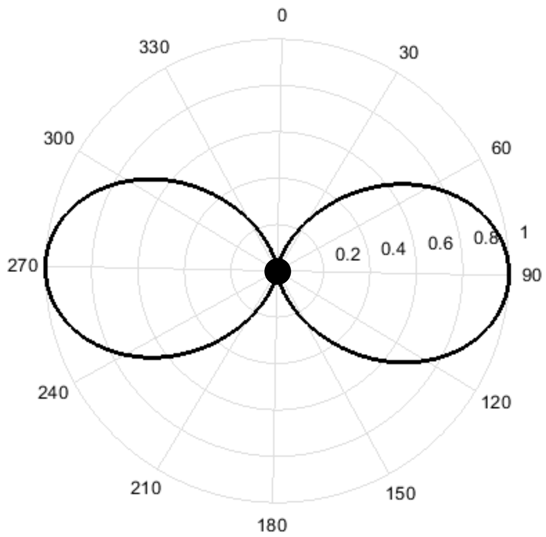
$$E \cong j\eta \frac{\beta I_0 l}{4\pi r} \sqrt{1 - (sin\,\theta \cos\phi)^2}\, e^{-j\beta r}$$

$$and \quad H \cong j \frac{\beta I_0 l}{4\pi r} \sqrt{1 - (sin\,\theta\, cos\,\phi)^2} e^{-j\beta r}$$

$$U = W_{av} r^2 = \frac{1}{2} Re[E \text{ x } H^*].\, r^2 = \eta \frac{\boldsymbol{\beta^2} I_0^2 l^2}{32\pi^{\boldsymbol{2}}} [1 - (sin\,\theta\, cos\,\phi)^2]$$



Radiation Pattern in E-plane (x-z plane)



Radiation Pattern in H-plane (x-y plane)

H.W Repeat the previous example for an infinitesimal dipole is:

1. placed symmetrically along the y-axis.

2. placed symmetrically in the x-y plane along the line y=x.

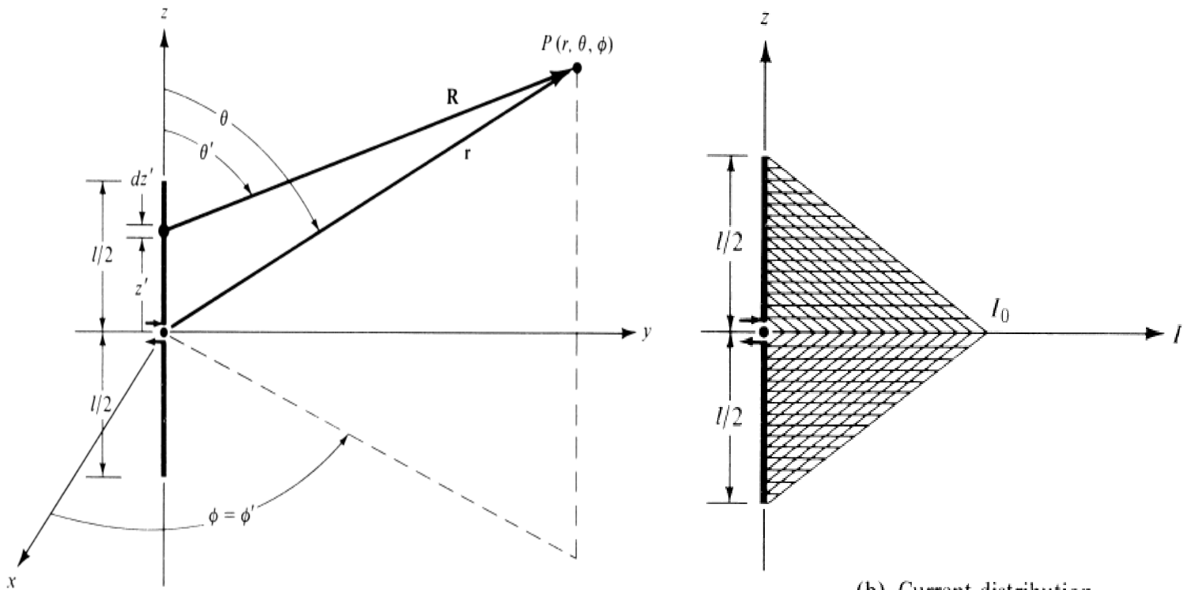
2. Small Dipole

The current distribution of small dipole ($\lambda/50 < l \leq \lambda/10$), is the triangular variation.

$$I(z) = \begin{cases} I_0\left(1+\frac{2}{l}z\right)\boldsymbol{a_z} & -l/2 < z < 0 \\ I_0\left(1-\frac{2}{l}z\right)\boldsymbol{a_z} & 0 < z < l/2 \end{cases}$$

$$\vec{A} = \frac{\mu}{4\pi}\left[\int_{-l/2}^{0} I_0\left(1+\frac{2}{l}z\right)\boldsymbol{a_z}\frac{e^{-j\beta r}}{r}dz + \int_{0}^{l/2} I_0\left(1-\frac{2}{l}z\right)\boldsymbol{a_z}\frac{e^{-j\beta r}}{r}dz\right]$$

$$\vec{A} = A_z\boldsymbol{a_z} = \frac{\mu I_0 l}{8\pi r}e^{-j\beta r}\boldsymbol{a_z}$$



(a) Dipole and geometry (b) Current distribution

Since the magnetic potential vector $\overrightarrow{\boldsymbol{A}}$ for the triangular distribution is one-half of the corresponding one for the constant (uniform) current distribution, the corresponding fields are also one-half. Thus, we can write the **E** and **H**-fields radiated by a small dipole in far-field region ($\beta r >> 1$) as:

$$E_\theta \cong j\eta\frac{\beta I_0 l}{8\pi r}\sin\theta\, e^{-j\beta r} \quad and \quad H_\phi \cong j\frac{\beta I_0 l}{8\pi r}\sin\theta\, e^{-j\beta r}$$

$$E_r = E_\phi = H_r = H_\theta = 0$$

The radiation resistance is one-fourth of that for infinitesimal dipole:

$$R_{rad} = 20\pi^2\left(\frac{l}{\lambda}\right)^2 \quad (\Omega)$$

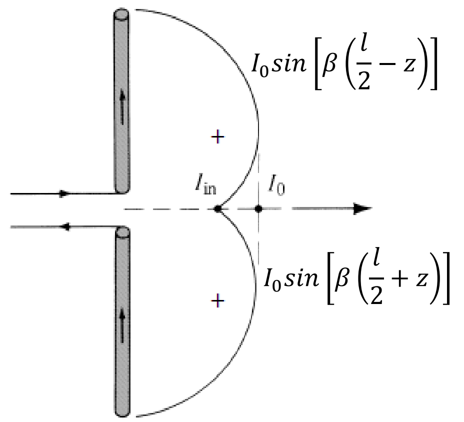
Directivity, effective aperture, the radiation pattern, and HPBW are the same of the infinitesimal dipole:

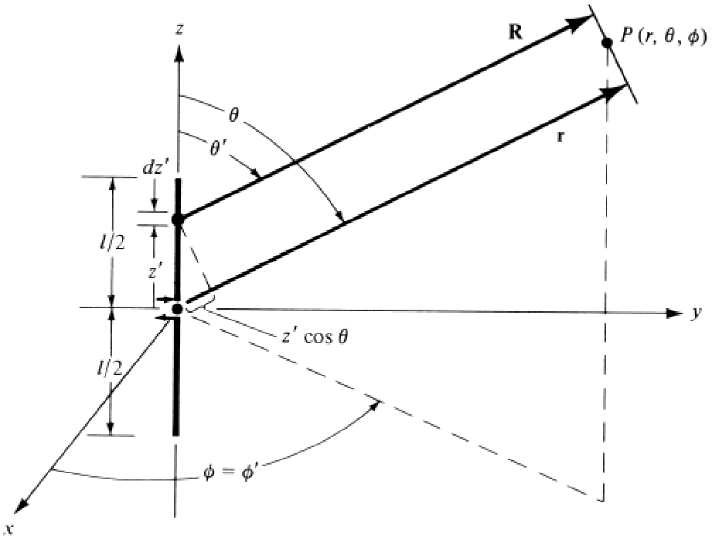
$$D_o = \frac{3}{2} \quad , \quad Aem = 0.119\lambda^2 \quad (m^2) \quad , \quad HPBW = 90^o$$

3. Finite Length Dipole

For a very thin dipole of length > λ/10 placed along the z-axis, it has a sinusoidal current distribution as shown.

$$I(z) = \begin{cases} I_0 sin\left[\beta\left(\frac{l}{2}+z\right)\right]\boldsymbol{a_z} & -l/2 < z < 0 \\ I_0 sin\left[\beta\left(\frac{l}{2}-z\right)\right]\boldsymbol{a_z} & 0 < z < l/2 \end{cases}$$





$$H_\phi \cong j\frac{I_0 e^{-j\beta r}}{2\pi r}\left[\frac{\cos\left(\frac{\beta l}{2}cos\theta\right)-\cos\left(\frac{\beta l}{2}\right)}{sin\theta}\right]$$

$$E_\theta = \eta H_\phi = j\eta\frac{I_0 e^{-j\beta r}}{2\pi r}\left[\frac{\cos\left(\frac{\beta l}{2}cos\theta\right)-\cos\left(\frac{\beta l}{2}\right)}{sin\theta}\right]$$

For the dipole, the average power density can be written as:

$$W_{av} = W_r\boldsymbol{a_r} = \frac{1}{2}\left[E_\theta\boldsymbol{a_\theta} \times H_\phi^*\boldsymbol{a_\phi}\right]$$

$$W_{av} = \eta\frac{I_0^2}{8\pi^2 r^2}\left[\frac{\cos\left(\frac{\beta l}{2}cos\theta\right)-\cos\left(\frac{\beta l}{2}\right)}{sin\theta}\right]^2\boldsymbol{a_r}$$

And the total power radiated by the dipole is:

$$P_{rad} = \int_0^{2\pi}\int_0^{\pi} W_r\boldsymbol{a_r}\,.\,r^2 sin\theta d\theta d\phi\,\boldsymbol{a_r} = \int_0^{2\pi}\int_0^{\pi}\eta\frac{I_0^2}{8\pi^2 r^2}\left[\frac{\cos\left(\frac{\beta l}{2}cos\theta\right)-\cos\left(\frac{\beta l}{2}\right)}{sin\theta}\right]^2 r^2 sin\theta d\theta d\phi$$

$$P_{rad} = \eta\frac{I_0^2}{4\pi}\int_0^{\pi}\frac{\left[\cos\left(\frac{\beta l}{2}cos\theta\right)-\cos\left(\frac{\beta l}{2}\right)\right]^2}{sin\theta}\,d\theta = 30\,I_0^2\int_0^{\pi}\frac{\left[\cos\left(\frac{\beta l}{2}cos\theta\right)-\cos\left(\frac{\beta l}{2}\right)\right]^2}{sin\theta}\,d\theta$$

This integral is so difficult to be solved and cannot be solved numerically. Table (1) below shows the solution of the above integral for dipole length $0.1\lambda \le l \le 3\lambda$.

Table (1) The value of the integral: $I = \int_0^{\pi} \frac{\left[\cos\left(\frac{\beta l}{2} cos\theta\right) - \cos\left(\frac{\beta l}{2}\right)\right]^2}{sin\theta}\, d\theta$ for dipole length $0.1\lambda \le l \le 3\lambda$

Dipole Length(l)		0.1 λ	0.15 λ	0.2 λ	0.25 λ	0.3 λ	0.35 λ	0.4 λ	0.45 λ	0.5 λ
Value of I		0.0032	0.0157	0.0480	0.1120	0.2197	0.3811	0.6021	0.8833	1.2188
Dipole Length(l)	0.55 λ	0.6 λ	0.65 λ	0.7 λ	0.75 λ	0.8 λ	0.85 λ	0.9 λ	0.95 λ	λ
Value of I	1.5962	1.9970	2.3980	2.7733	3.0968	3.3446	3.4979	3.5448	3.4827	3.3181
Dipole Length(l)	1.05 λ	1.1 λ	1.15 λ	1.2 λ	1.25 λ	1.3 λ	1.35 λ	1.4 λ	1.45 λ	1.5 λ
Value of I	3.0673	2.7550	2.4123	2.0740	1.7756	1.5496	1.4225	1.4122	1.5256	1.7582
Dipole Length(l)	1.55 λ	1.6 λ	1.65 λ	1.7 λ	1.75 λ	1.8 λ	1.85 λ	1.9 λ	1.95 λ	2 λ
Value of I	2.0938	2.5057	2.9596	3.4162	3.8349	4.1781	4.4146	4.5226	4.4924	4.3272
Dipole Length(l)	2.05 λ	2.1 λ	2.15 λ	2.2 λ	2.25 λ	2.3 λ	2.35 λ	2.4 λ	2.45 λ	2.5 λ
Value of I	4.0433	3.6686	3.2399	2.7999	2.3931	2.0609	1.8385	1.7506	1.8093	2.0128
Dipole Length(l)	2.55 λ	2.6 λ	2.65 λ	2.7 λ	2.75 λ	2.8 λ	2.85 λ	2.9 λ	2.95 λ	3 λ
Value of I	2.3449	2.7770	3.2704	3.7800	4.2584	4.6610	4.9499	5.0977	5.0905	4.9291

The radiation intensity is:

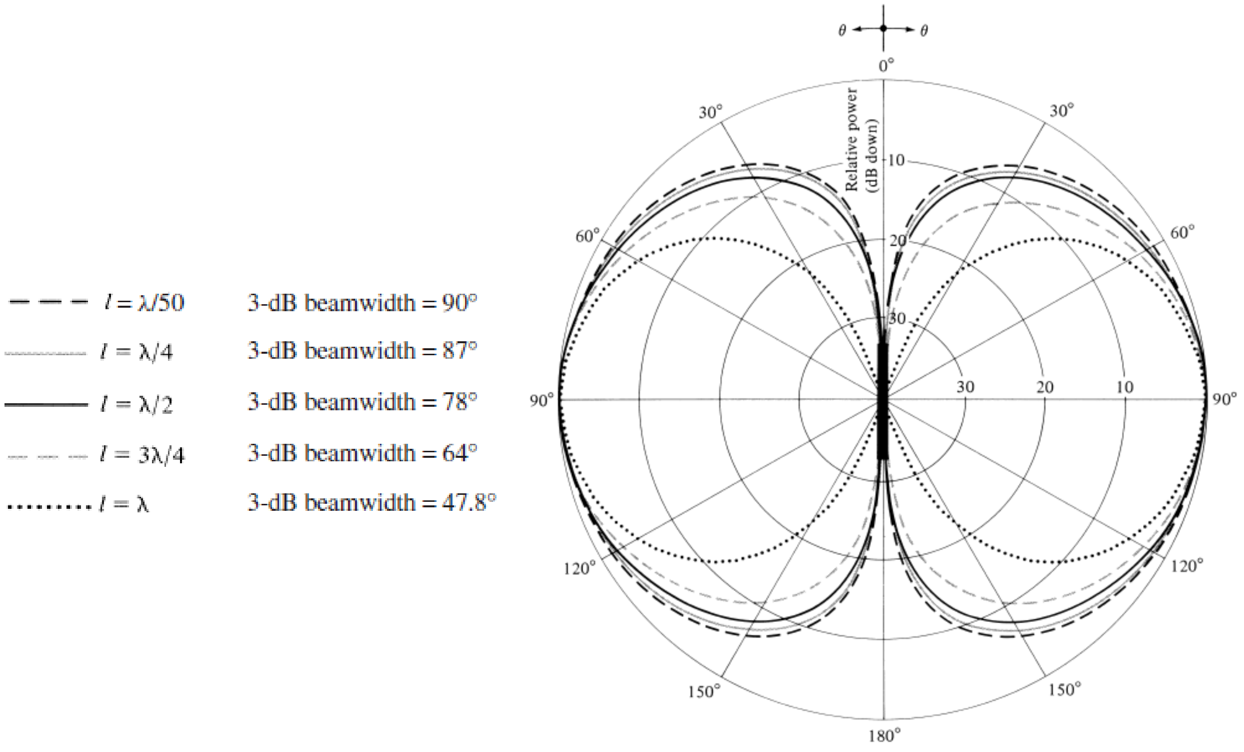
$$U = \eta \frac{I_0^2}{8\pi^2}\left[\frac{\cos\left(\frac{\beta l}{2} cos\theta\right) - \cos\left(\frac{\beta l}{2}\right)}{sin\theta}\right]^2 \qquad U_{max} = \eta \frac{I_0^2}{8\pi^2} U_m = \frac{15 I_0^2}{\pi} U_m$$

The value of Um can be obtained from table (2).

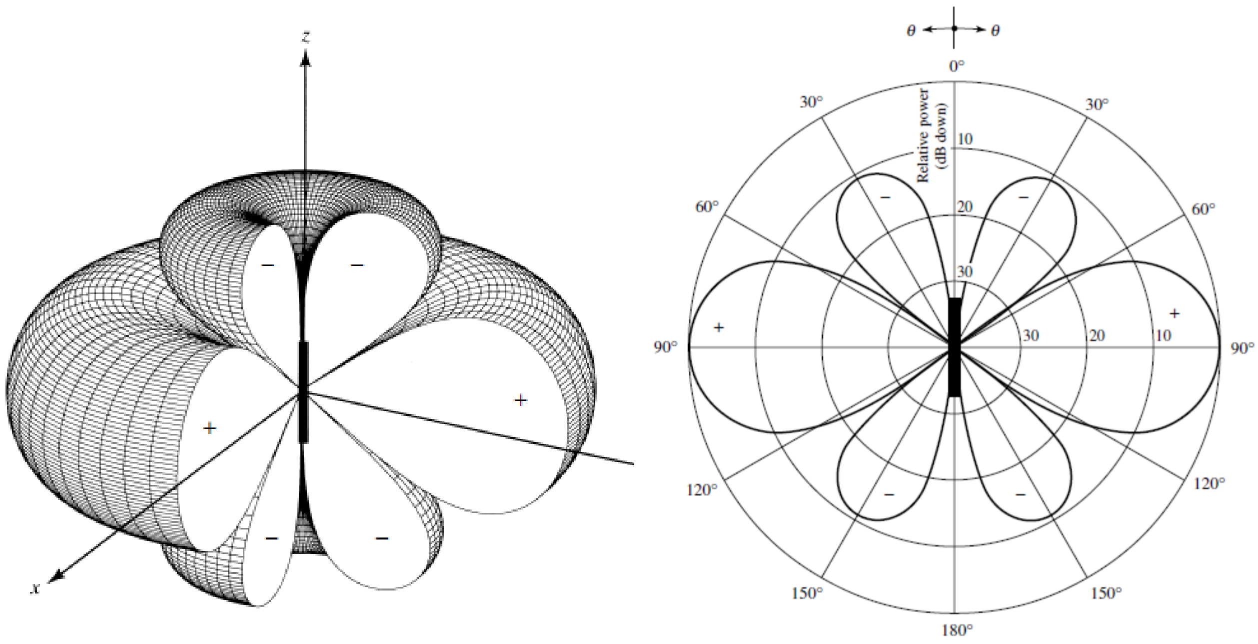
Table (2) The maximum value of $U(\theta) = \left[\frac{\cos\left(\frac{\beta l}{2} cos\theta\right) - \cos\left(\frac{\beta l}{2}\right)}{sin\theta}\right]^2$ at $0 \le \theta \le 2\pi$ for dipole length $0.1\lambda \le l \le 3\lambda$

Dipole Length(l)		0.1 λ	0.15 λ	0.2 λ	0.25 λ	0.3 λ	0.35 λ	0.4 λ	0.45 λ	0.5 λ
Um		0.0024	0.0119	0.0365	0.0858	0.1699	0.2981	0.4775	0.7116	1.0000
Dipole Length(l)	0.55 λ	0.6 λ	0.65 λ	0.7 λ	0.75 λ	0.8 λ	0.85 λ	0.9 λ	0.95 λ	λ
Um	1.3373	1.7135	2.1141	2.5211	2.9142	3.2725	3.5759	3.8066	3.9509	4
Dipole Length(l)	1.05 λ	1.1 λ	1.15 λ	1.2 λ	1.25 λ	1.3 λ	1.35 λ	1.4 λ	1.45 λ	1.5 λ
Um	3.9509	3.8066	3.5759	3.2725	2.9142	2.5211	2.1141	1.7135	1.4875	1.9572
Dipole Length(l)	1.55 λ	1.6 λ	1.65 λ	1.7 λ	1.75 λ	1.8 λ	1.85 λ	1.9 λ	1.95 λ	2 λ
Um	2.4718	3.0119	3.5543	4.0740	4.5454	4.9445	5.2499	5.4455	5.5204	5.4708
Dipole Length(l)	2.05 λ	2.1 λ	2.15 λ	2.2 λ	2.25 λ	2.3 λ	2.35 λ	2.4 λ	2.45 λ	2.5 λ
Um	5.2995	5.0161	4.6362	4.1803	3.6724	3.1375	2.6011	2.0862	2.3823	3.0781
Dipole Length(l)	2.55 λ	2.6 λ	2.65 λ	2.7 λ	2.75 λ	2.8 λ	2.85 λ	2.9 λ	2.95 λ	3 λ
Um	3.8206	4.5788	5.3189	6.0054	6.6046	7.0864	7.4264	7.6072	7.6205	7.4663

Elevation Plane Amplitude Patterns for a Thin Dipole with Sinusoidal Current Distribution



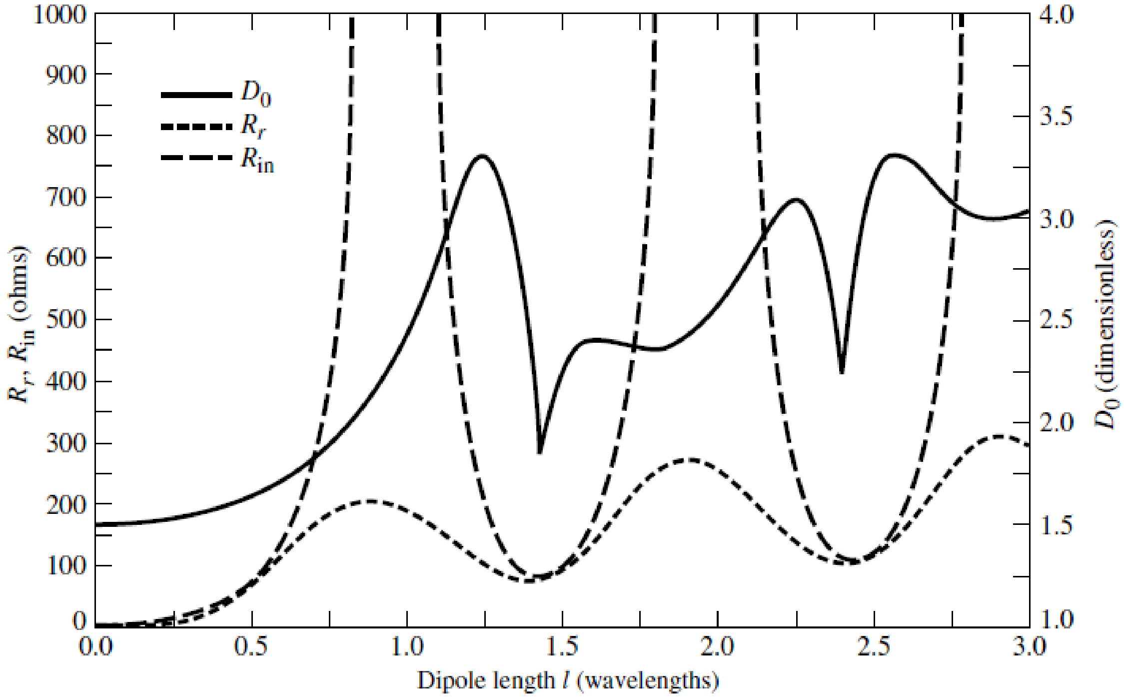
As the length of the dipole increases beyond one wavelength ($l > \lambda$), the number of lobes begin to increase. The radiation pattern for a dipole with $l = 1.25\lambda$ is shown in Figure below:



(a) Three-dimensional

(b) Two-dimensional

Radiation Resistance, Input Resistance and Directivity of a Thin Dipole with Sinusoidal Current Distribution



4. Half-Wave Dipole (λ/2)

One of the most commonly used antennas is the half-wavelength *(l = λ/2)* dipole. Because its radiation resistance is 73Ω, which is very near the 75Ω characteristic impedances of some transmission lines.

The electric and magnetic field components of a half-wavelength dipole can be obtained by letting $l = \lambda/2$, thus:

$$\frac{\beta l}{2} = \frac{2\pi}{2\lambda} * \frac{\lambda}{2} = \frac{\pi}{2}$$

$$H_\phi \cong j\frac{I_0 e^{-j\beta r}}{2\pi r}\left[\frac{\cos\left(\frac{\pi}{2}cos\theta\right)}{sin\theta}\right]$$

$$E_\theta \cong j\eta\frac{I_0 e^{-j\beta r}}{2\pi r}\left[\frac{\cos\left(\frac{\pi}{2}cos\theta\right)}{sin\theta}\right]$$

The average power density and radiation intensity can be written, respectively, as:

$$W_{av} = \eta\frac{I_0^2}{8\pi^2 r^2}\left[\frac{\cos\left(\frac{\pi}{2}cos\theta\right)}{sin\theta}\right]^2$$

$$U = W_{av}r^2 = \eta\frac{I_0^2}{8\pi^2}\left[\frac{\cos\left(\frac{\pi}{2}cos\theta\right)}{sin\theta}\right]^2 \qquad and \qquad U_{\max(\theta=90)} = \eta\frac{I_0^2}{8\pi^2}$$

$$U_n(\theta) = \left[\frac{\cos\left(\frac{\pi}{2}cos\theta\right)}{sin\theta}\right]^2$$

$$P_{rad} = \eta\frac{I_0^2}{4\pi}\int_0^{\pi}\frac{\left[\cos\left(\frac{\pi}{2}cos\theta\right)\right]^2}{sin\theta}\,d\theta$$

$$= \eta\frac{I_0^2}{4\pi}(1.2188) = 36.564I_0^2 \quad (W) \qquad the\ value\ 1.2188\ is\ from\ table\ (1)$$

and the ***radiation resistance*** is:

$$R_{rad} = 2 * 36.564 \cong 73\ \Omega$$

$$D_o = \frac{4\pi U_{max}}{P_{rad}} = 4\pi\frac{\eta\frac{I_0^2}{8\pi^2}}{\eta\frac{I_0^2}{4\pi}(1.2188)}$$

$$D_o = 1.643$$

$$Aem = \frac{\lambda^2}{4\pi}D_0 = \frac{\lambda^2}{4\pi} * 1.643 = 0.13\lambda^2 \quad (m^2)$$

To find the HPBW:

$\theta_m = 90^o\ ,\ 270^o$

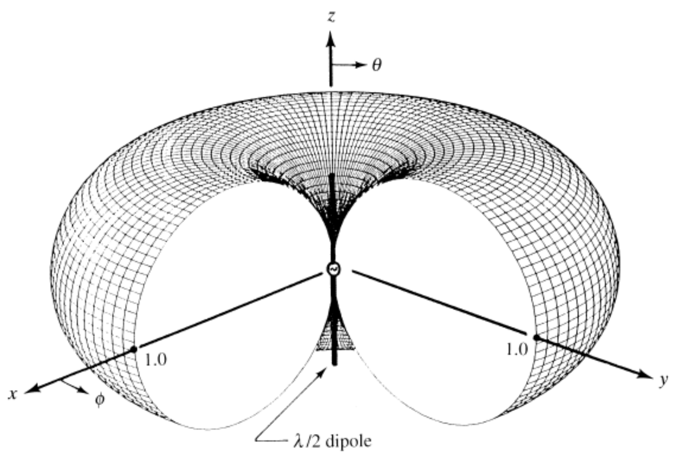
$$U_n(\theta)|_{\theta=\theta h} = 0.5 \rightarrow \left[\frac{\cos\left(\frac{\pi}{2}cos\theta_h\right)}{sin\theta_h}\right]^2 = 0.5 \rightarrow \frac{\cos\left(\frac{\pi}{2}cos\theta_h\right)}{sin\theta_h} = 0.707$$

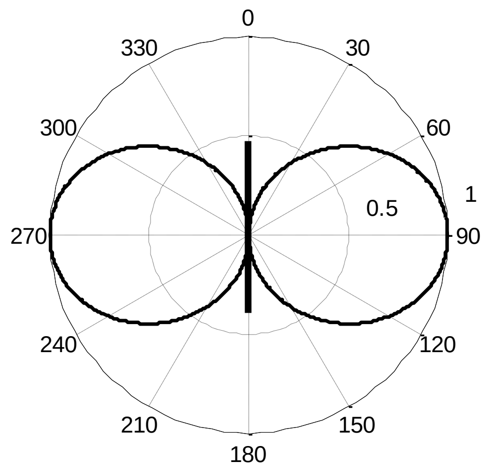
$$sin\theta_h = \frac{1}{0.707}\cos\left(\frac{\pi}{2}cos\theta_h\right) \rightarrow \theta_h = \sin^{-1}\left(\frac{1}{0.707}\cos\left(\frac{\pi}{2}cos\theta_h\right)\right)$$

By using trial and error, we can find the value of $\boldsymbol{\theta_h \cong 51^o}$

$$HPBW = 2|\theta_m - \theta_h| = 2|90 - 51| = 78^o$$

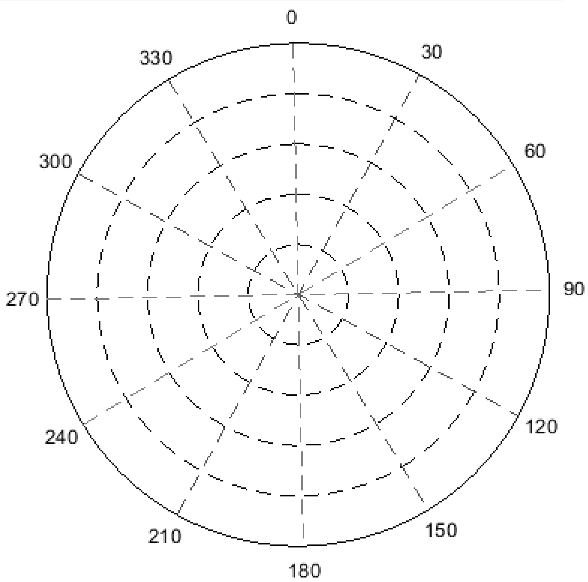
Figure below shows the two and three-dimensional radiation pattern for a λ/2 dipole.





Example: For a $\lambda/4$ dipole, find the radiation resistance, maximum directivity and maximum aperture, HPBW & FNBW and draw the radiation pattern.

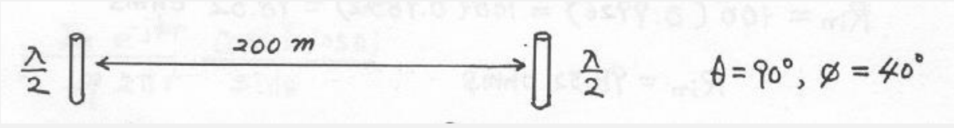
Solution:



H.W. For a $3\lambda/4$ dipole, find the radiation resistance, maximum directivity and maximum aperture, HPBW & FNBW and draw the radiation pattern.

Example: A $\boldsymbol{\lambda/2}$ dipole radiates a time-averaged power of **600 W** at a frequency of 300 MHz. A second $\boldsymbol{\lambda/2}$ dipole is placed at a point P(r, θ, φ), where ***r* = 200 m**, **θ = 90**°,**φ= 40**°. It is oriented so that its axis is parallel to that of the transmitting antenna. What is the available power at the terminals of the second (receiving) dipole?

Solution:



$f = 300\,MHz \qquad \lambda = \frac{c}{f} = \frac{3\times10^8}{300\times10^6} = 1\,m$

$\frac{2D^2}{\lambda} = \frac{2\left(\frac{\lambda}{2}\right)^2}{\lambda} = 0.5\,m$

$r = 200 \gg 0.5 \qquad so\ far\ field\ region$

$P_r = P_t G_t G_r \left(\frac{\lambda}{4\pi R}\right)^2$

$for\ lossless\ antenna\ G_t = D_t\ ,G_r = D_r$

$D(for\boldsymbol{\lambda/2}) = 1.643$

$P_r = 600 * 1.643 * 1.643 * \left(\frac{1}{4\pi * 200}\right)^2 = 0.25\,mW$

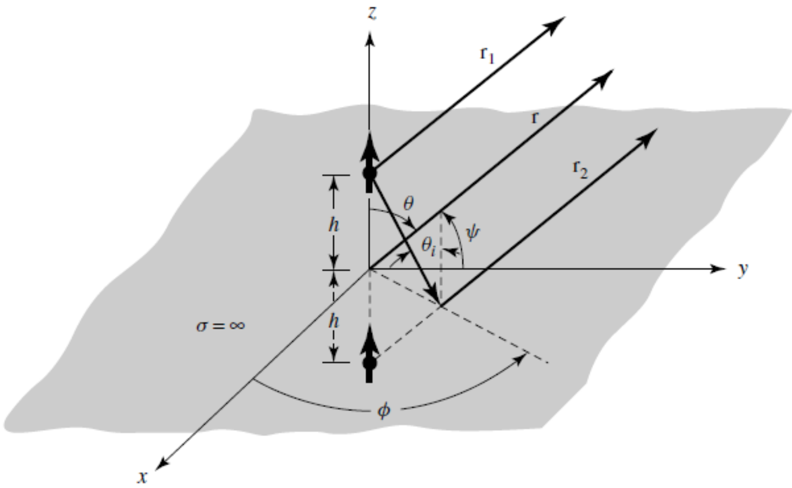
H.W: Find P_r at ***r* = 200 m**, $\boldsymbol{\theta}$ **= 50°**, $\boldsymbol{\varphi}$ **= 20?**

Example: A $3\lambda/4$ dipole is radiating into free-space. Input power to the dipole is 100 W. Assuming an overall efficiency of 50%, find the power density (in W/m^2) at (r = 500 m, θ = 60°, φ = 0)?

Solution:

5. Linear Elements Near or On Infinite Perfect Conductors

Any energy from the radiating element directed toward the ground undergoes a reflection. The amount of reflected energy and its direction are controlled by the geometry and constitutive parameters of the ground the analysis procedure for hhe Vertical electric dipole above infinite perfect electric conductor shown in figure below.



Vertical electric dipole above infinite perfect electric conductor

The electric field of the ***infinitesimal*** dipole of length ***l*** *is given by:*

$$E_\theta \cong j\eta \frac{\beta I_0 l}{4\pi r_1}\sin\theta_1\, e^{-j\beta r_1}$$

The reflected component can be written as:

$$E_\theta \cong j\eta \frac{\beta I_0 l}{4\pi r_2}\sin\theta_2\, e^{-j\beta r_2}$$

For far-field observations:

$\theta_1 \cong \theta_2 \cong \theta$

$\{r_1 \cong r - hcos\theta$ and $r_2 \cong r + hcos\theta\}$ for phase variation

$r_1 \cong r_2 \cong r$ for amplitude variation

The total electric field is equal to:

$$E_\theta \cong j\eta \frac{\beta I_0 l}{4\pi r_1}\sin\theta_1\, e^{-j\beta r_1} + j\eta \frac{\beta I_0 l}{4\pi r_2}\sin\theta_2\, e^{-j\beta r_2} \cong j\eta \frac{\beta I_0 l}{4\pi r}\sin\theta\left[\, e^{-j\beta(r-hcos\theta)} + e^{-j\beta(r+hcos\theta)}\right]$$

$$E_\theta \cong j\eta \frac{\beta I_0 l}{4\pi r}\, e^{-j\beta r}\, \sin\theta\, [2\cos(\beta h\ \cos\theta)] \qquad z \geq 0$$

The total radiated power over the upper hemisphere

$$P_{rad} = \pi\eta \left(\frac{I_0 l}{\lambda}\right)^2 \left[\frac{1}{3} - \frac{\cos(2\beta h\,)}{(2\beta h\,)^2} + \frac{\sin(2\beta h\,)}{(2\beta h\,)^3}\right]$$

The radiation intensity can be written as:

$$U = \frac{\eta}{2}\left(\frac{I_0 l}{\lambda}\right)^2 \ \sin^2\theta \ \cos^2(\beta h \ \cos\theta)$$

The maximum value of U occurs at $\theta \ = \ \pi/2$ and is given by:

$$U_{max} = U_{\theta=\pi/2} = \frac{\eta}{2}\left(\frac{I_0 l}{\lambda}\right)^2$$

which is ***four times greater than that of an isolated element***

The directivity can be written as:

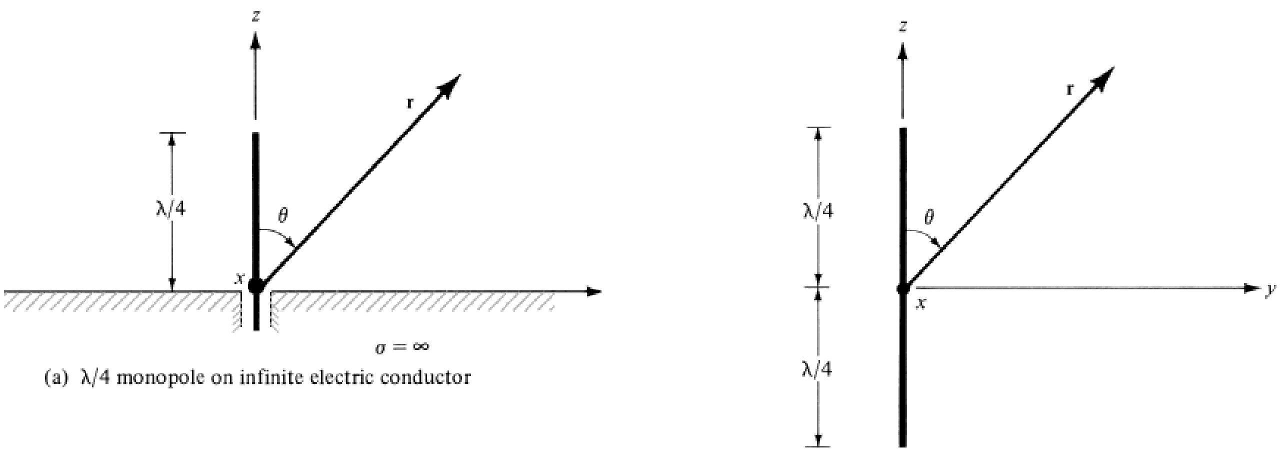
$$D = \frac{2}{\left[\frac{1}{3} - \frac{\cos(2\beta h)}{(2\beta h)^2} + \frac{\sin(2\beta h)}{(2\beta h)^3}\right]}$$

for βh = 0 then D = 3. The maximum value occurs when βh = 2.881 *(h* =0.4585*)*, and it is equal to $D_{max} = 6.566$ ***which is greater than four times that of an isolated element:***

$$R_{rad} = 2\pi\eta\left(\frac{l}{\lambda}\right)^2\left[\frac{1}{3} - \frac{\cos(2\beta h)}{(2\beta h)^2} + \frac{\sin(2\beta h)}{(2\beta h)^3}\right]$$

whose value for $\boldsymbol{\beta h} \rightarrow \infty$ **is the same** and for $\boldsymbol{\beta h} = \ 0$ ***is twice that of the isolated element***

In practice, a wide use has been made of a quarter-wavelength ***monopole*** (l = λ/4) ***mounted*** above a ground plane, and fed by a coaxial line, as shown in Figure below. For analysis purposes, a λ /4 image is introduced and it forms the λ /2 equivalent.



(a) λ/4 monopole on infinite electric conductor

(b) Equivalent of λ/4 monopole on infinite electric conductor

Quarter-wavelength monopole on an infinite perfect electric conductor

The input impedance of a λ/4 monopole above a ground plane is equal to one-half that of an isolated λ/2 dipole

$$Z_{in}(monopole) = \frac{1}{2} Z_{in}(dipole)$$

$Z_{in}(\lambda/4 monopole) = \frac{1}{2} Z_{in}(\lambda/2 \ dipole) = \frac{1}{2}(73 + j42.5) = 36.5 + j21.25$

Example: Determine the smallest ***height*** that an ***infinitesimal*** vertical electric dipole of $\boldsymbol{l} = \boldsymbol{\lambda/50}$ must be placed above an ***electric ground plane*** so that its pattern has only one null occurs at **30**° from the ***vertical***. For that height, find the directivity and radiation resistance?

Solution:

$$E_\theta = j\eta \frac{\beta I_0 l}{4\pi r} e^{-j\beta r} \sin\theta \, [2\cos(\beta h \cos\theta)]$$

$$E_\theta|_{\theta=30} = 0 \Rightarrow j\eta \frac{\beta I_0 l}{4\pi r} e^{-j\beta r} \sin\theta \, [2\cos(\beta h \cos\theta)] = 0$$

$$\cos(\beta h \cos 30) = 0 \qquad \Rightarrow \beta h \cos 30 = \frac{\pi}{2} \qquad \Rightarrow \frac{2\pi}{\lambda} h(0.867) = \frac{\pi}{2}$$

$$h = 0.288\lambda$$

$$2\beta h = 2\frac{2\pi}{\lambda}(0.288\lambda) = 3.632$$

$$D = \frac{2}{\left[\frac{1}{3} - \frac{\cos(2\beta h)}{(2\beta h)^2} + \frac{\sin(2\beta h)}{(2\beta h)^3}\right]} = \frac{2}{\left[\frac{1}{3} - \frac{\cos(3.632)}{(3.632)^2} + \frac{\sin(3.632)}{(3.632)^3}\right]} = 5.12 = 7.1\, dB$$

$$R_{rad} = 2\pi\eta \left(\frac{l}{\lambda}\right)^2 \left[\frac{1}{3} - \frac{\cos(2\beta h)}{(2\beta h)^2} + \frac{\sin(2\beta h)}{(2\beta h)^3}\right]$$

$$= 2\pi\eta \left(\frac{\lambda/50}{\lambda}\right)^2 \left[\frac{1}{3} - \frac{\cos(3.632)}{(3.632)^2} + \frac{\sin(3.632)}{(3.632)^3}\right] = 0.37\, \Omega$$

H.W

A very short ($\boldsymbol{l} \leq \boldsymbol{\lambda/50}$) vertical electric dipole is mounted on a pole a height h above the ground perfectly conducting, and of infinite extent. The dipole is used as a transmitting antenna in a VHF ($f = 50\, MHz$) ground-to-air communication system. In order for the communication system transmitting antenna signal not to interfere with a nearby radio station, it is necessary to place a null in the vertical dipole system pattern at an angle of 80◦ from the vertical. What should the shortest height (h) of the dipole be to achieve the desired specifications?