University of Salahaddin - Erbil
College of Administration and Economics
Department of Statistics and informatics

 **Stochastic Processes**

Fourth stage

First Course
Academic year 2022 - 2023

**Asst Prof. D. Saman Hussein Mahmood
E-mail: saman.mahmood@su.edu.krd
Number of units in a week: three units**

**Course overview:**

In [probability theory](https://en.wikipedia.org/wiki/Probability_theory), a **stochastic**  **process**, or often **random process**, is a collection of [random variables](https://en.wikipedia.org/wiki/Random_variable), representing the evolution of some [system](https://en.wikipedia.org/wiki/System) of random values over time. This is the probabilistic counterpart to a deterministic process (or [deterministic system](https://en.wikipedia.org/wiki/Deterministic_system)). Instead of describing a process which can only evolve in one way (as in the case, for example, of solutions of an [ordinary differential equation](https://en.wikipedia.org/wiki/Ordinary_differential_equation)), in a stochastic or random process there is some indeterminacy: even if the initial condition (or starting point) is known, there are several (often infinitely many) directions in which the process may evolve.

In the simple case of [discrete time](https://en.wikipedia.org/wiki/Discrete-time_stochastic_process), as opposed to [continuous time](https://en.wikipedia.org/wiki/Continuous-time_stochastic_process), a stochastic process involves a [sequence](https://en.wikipedia.org/wiki/Sequence_%28mathematics%29) of random variables and the [time series](https://en.wikipedia.org/wiki/Time_series) associated with these random variables (for example, see [Markov chain](https://en.wikipedia.org/wiki/Markov_chain), also known as discrete-time Markov chain). One approach to stochastic processes treats them as [functions](https://en.wikipedia.org/wiki/Function_%28mathematics%29) of one or several deterministic arguments (inputs; in most cases this will be the time parameter) whose values (outputs) are random variables: non-deterministic (single) quantities which have certain [probability distributions](https://en.wikipedia.org/wiki/Probability_distribution). Random variables corresponding to various times (or points, in the case of [random fields](https://en.wikipedia.org/wiki/Random_field)) may be completely different. The main requirement is that these different random quantities all take values in the same space (the [codomain](https://en.wikipedia.org/wiki/Codomain) of the function). Although the random values of a stochastic process at different times may be [independent random variables](https://en.wikipedia.org/wiki/Statistical_independence), in most commonly considered situations they exhibit complicated statistical correlations.

**Course objective:**

Course objective:

The students who succeeded in this course; • At the end of the course the students are expected to:

 1) Know the properties and usage of special probability distributions .

2) Understand the notion of stochastic process and analyze different types of stochastic processes.

4) classify states and compute probabilities for Markov Chains

5) Model different real-life situations with the help of stochastic processesa topological space instead of limited to real values representing time

**Student's obligation**

* . Closed-book policy: No use of calculators, or books will be allowed during any in-class tests/quizzes.
* Policy related to make-up exams or other work: There will be no opportunities to make up for work not submitted. However, if a student provides a legitimate excuse well in advance, scores will be prorated. Work with due date should be turned in at the beginning of class on the stated due date. Late work will not be accepted and will be deemed work not submitted.
* Policy on class attendance: Requirements for class attendance and make-up exams, assignments, and other work in this course are consistent with university policies that can be found in the online catalog at: <https://catalog.ufl.edu/ugrad/current/regulations/info/attendance.aspx>
* **Forms of teaching**
* Different forms of teaching will be used to reach the objective of the course : power point presentation for the head titles and summary of conclusion with applications by some equations, besides worksheet will be designed to let the chance for practicing on several aspects of the course in the classroom, furthermore students will contents homework . There will be classroom discussions and the lecture will give enough background to translate , solve, analyze , and evaluate problems sets.

**Assessment scheme**

**The student must be examined twice in each course. The last grade is (30).**

**Putting grades for daily activities, homework, for (10) marks.**

**The annual work of the material (40) marks.**

**The final exam out of (60) marks.**

**Student learning outcome:**

* On successful completion of the course, students should be able to:
* Explain fundamentals of probability theory, random variables and random processes.
* Understand the mathematical concepts related to probability theory and random processes.
* Understand the characterization of random processes and their properties.
* Formulate and solve the engineering problems involving random processes.
* Analyze the given probabilistic model of the problem.
* Make precise statements about random processes.
* Use computational techniques to generate simulation results.

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|  **Course Reading List and References‌:** **Stochastic Processes, Estimation, and Control** **Jason L. Speyer****Walter H. Chung**University of California, Los AngelesCopyright © 2008 by the Society for Industrial and Applied Mathematics.**Theory and Problems of Probability, Random Variables, and Random Processes****Hwei P. Hsu, Ph.D.** Professor of Electrical EngineeringFairleigh Dickinson UniversityCopyright © 1997 by The McGraw-Hill Companies.**LEVEL SETS AND EXTREMA OF RANDOM PROCESSES AND FIELDS****JEAN-MARC AZA¨IS**Universite´ de Toulouse, Institut de Mathe´matiques, Laboratoire de Statistiques et Probabilite´ s, Toulouse, France**MARIO WSCHEBOR**Universidad de la Repu´ blica, Centro de Matema´ tica, Facultad de CienciasMontevideo, UruguaA JOHN WILEY & SONS, INC., PUBLICATIONCopyright © 2009 by John Wiley & Sons, Inc. All rights reserved.**Probability and Statistics by Example: II Markov Chains: a Primer in Random Processes and their Applications**Yuri Suhov /*University of Cambridge*Mark Kelbert /*University of Wales–Swansea*© Y. Suhov and M. Kelbert 2008 |
| **17. The Topics:** | **Lecturer's name** |
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| **Week** | **Subject** |
| **Week-1** | **Capter-1 : Probability Theory** **1.1 Probability Theory as a Set of Outcomes** **1.2 Set Theory** **1.3 Probability Space and the Probability Measure**  |
| **Week-2** | **1.4 Key Concepts in Probability Theory** **Exercises**  |
| **Week-3** | **Capter-2 : Random Variables and Stochastic Processes** **2.1 Random Variables****2.2 Probabilistic Concepts Applied to Random Variables**  |
| **Week-4** | **2.3 Functions of a Random Variable** **2.4 Expectations and Moments of a Random Variable**  |
| **Week-5** | **2.5 Probability generating Function****2.6 Stochastic Processes , Extending the Concept of Random Vectors**  |
| **Week-6** | **Capter-3 : Stochastic Processes****Solved Problems About Stochastic Processes** |
| **Week-7** | **3-1 CLASSIFICATION of stochastic process**1. **1. Stochastic Processes with Discrete Parameter and State Spaces with examples.**
2. **Stochastic Processes with Continuous Parameter and Discrete State Space with examples.**
3. **Stochastic Processes with Discrete Parameter and Continuous State Space with examples.**
4. **Stochastic Processes with Continuous Parameter and State Spaces with examples.**
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| **Week-8** | **3-2 A. Stationary Processes** **B. Wide-Sense Stationary Processes****Solved Problems About Stationary Processes & Wide-Sense Stationary Processes** |
| **Week-9** | **C. Independent Processes** **D. Processes with Stationary Independent Increments****Solved Problems about Independent Processes & Processes with Stationary Independent Increments**  |
| **Week-10** |  **Capter-4 : 4-1 Markov Processes****Solved Problems about Markov Processes** |
| **Week-11** | **4-2 Discrete-Parameter Markov Chains** **A. Transition Probability Matrix** |
| **Week-12** | **Solved Problems about Discrete-Parameter Markov Chains & Transition Probability Matrix** |
| **Week-13** |  **B. Higher- Order Transition Probabilities-Chapman-Kolmogorov Equation** **C. The Probability Distribution of**  |
| **Week-14** | **Solved Problems about Higher- Order Transition Probabilities-Chapman-Kolmogorov Equation & The Probability Distribution of**  |
| **Week-15** | **D. Classification of States**  **Solved Problems about Classification of States** |

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| **18. Practical Topics (If there is any)** |  |
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| **19. Examinations:**1. **In New England, 84% of the houses have a garage and 65% of the houses have a garage and a back yard. What is the probability that a house has a back yard given that it has a garage?**
2. **bag contains 7 yellow balls and 5 red balls. One ball is taken from the bag at random and is not replaced. A second ball is then taken from the bag. Determine the probability that at least one ball is yellow?**
3. **Find the mean and variance for binomial distribution by p.g.f**
4. **define 1-Sample Space 2-Stochastic Processes 3-Stationary independent increment 2-Random variable 3-Number of success**
5. **Difference between (stochastic process and probability) with example**

1. **/Prove that 1/**

1. **Classification Stochastic Process**
* **8. Find the transition Matrix**  .
* **What is the probability that he has lost all his money at the end of 2 plays**
* **How many communicating state, absorbing state , Irreducible Markov chain with transition diagrams**

 **12- If you have the transition Matrix? Find M and F** **P=****9-A psychologist makes the following assumptions concerning the behavior of mice subjected to a particular feeding schedule. For any particular trial 80% of the mice that went right in the previous(**ثيَشوو) **experiment will go right in this trial, and 60% of those mice that went left in the previous experiment will go right in this trial. If 50% went right in the first trial, what would he predict for** 1. **The second trial? (c) The thousandth trial**

**10- If you have the transition Matrix?** * **Find the transition diagrams**  .
* **How many communicating state, absorbing state , Irreducible Markov chain and Transient and Recurrent States**

**11-** **for the experiment of tossing a coin (a) once and (b) twice.** **Explain each of them (trial, outcome, random experimental, sample space, events)? and what difference between sample space and Events****12- Find the mean and variance for Geometric distribution by p.g.f** **13-prove that** $p(T-n=t/T>n)=$**P(T=t)****14 The probability that a patient recovers(ضاك بوون لة نةخؤشي) from blood disease(نةخوشي خوين ) is 0.4 . (and probability female patient recovers is 0.2)** 1. **Find the probability that among 10 blood disease at least 2 survive.**

 **Survive: نةخوش لة ذيان بميَنيَ** 1. **probability that a 6 survive will be from 20th blood disease**
2. **probability that a 4 survive will be from 15th blood disease she is female**
3. **Find the mean and variance survive a from 18th blood disease**

15- **If you have the transition Diagram?****0.5****0.2****1****0.2****0.6****0.25****0.25** **1** **4****1****2****3*** **Find the transition Matrix**  .
* **How many communicating state, absorbing state , Irreducible Markov chain and Transient and Recurrent States**

**Find M and F****17-**Consider a family of exactly two children. We will find the probabilities:1. **both are girls**
2. **both are girls given that the elder(یه‌كه‌م مندال) child is a girl**

18- If you have the transition Matrix? * **Find the transition diagrams**  .
* **How many communicating state, absorbing state,**

 **Irreducible Markov chain, and Transient States** * **Find M**

**19- Find the mean and variance for Exponential distribution by p.g.f****20- define 1-Markov Process 2-Mutually Exclusive Events****21- You get email according to a Poisson process at a rate of A = 0.2 messages per hour.** **1-You check your email every hour. What is the probability of finding 0 and 1 new message?****22-Suppose that you have not checked your email for a whole day. What is the probability of finding 5 new messages?****23- A salesman's territory consists of three cities A, B and C. He never sells in the same city on successive days. If he sells in city A, then the next day he sells in city B. However, If he sells in either A or B, then the next day he is twice as likely (probable) to sell in city A as in other city.*** **Find the transition Matrix**
* **In the long run, how often does he sell in each of the cities?**

**24- In the diagram mouse is randomly moving from room to room*** **Find the transition Matrix**

**2-By Matrix diagram, how many communicating state, absorbing state, and Recurrent States** **25-In tennis club there are 5 boys and 3 girls in training squad. Two are chosen at random. Determine the probability that at least one boy?** **26- A player has 4$. At each play of game, he loses 1$ with probability 0.75 but wins 1$ with probability 0.25. He stops playing if he loses 4$ or win 3$ .*** **Find the transition Matrix**  .
* **What is the probability that he has lost all his money at the end of 2 plays**
* **How many communicating state, absorbing state , Irreducible Markov chain with transition diagrams**

**27- If you have the transition Matrix? Find M and F?** **28-A bag contains 7 yellow balls and 5 red balls. One ball is taken from the bag at random and is not replaced. A second ball is then taken from the bag. Determine the probability that at least one ball is yellow?** **29- Plays tennis players (A and B) five match, if any player has won three match is the winner** **1-What Probability win player A in the fourth match** **2-Expected the winning player in the third match** **3-What Probability win the player A in the fifth match  or before that** **47-In the diagram mouse is randomly moving from room to room*** **Find the transition Matrix**
* **By Matrix diagram, how many communicating state, absorbing state, and Recurrent States**

**30-Types of Queuing characteristics**9m2q1.gif**51-** 1**4**1. **Draw a picture corresponding to this transition matrix**
2. **How many communicating state, absorbing state , Irreducible Markov chain with transition diagrams**
3. **Find M**

**31- Your neighbor has 2 children. You learn that he has a son, Joe. What is the probability that Joe’s sibling is a brother?** **32-**Suppose that ﬁve good fuses and two defective ones have been mixed up. To ﬁnd the defective fuses, we test them one-by-one, at random and without replacement. What is the probability that we are lucky and ﬁnd both of the defective fuses in the ﬁrst two tests?**33- Around 1% of men are blue-green color-blind and 1 in 5 men is left-handed. Assuming these characteristics are inherited independently, calculate the probability that a man chosen at random will:**1. **Be both color-blind and left-handed**
2. **Be color-blind or left-handed**
3. **Be neither color-blind nor left-handed.**

**34- In a shipment of 20 computers, 3 are defective. Three computers are randomly selected and tested. What is the probability that all three are defective if the first and second ones are not replaced after being tested?**35-**At a middle school, 18% of all students play football and basketball and 32% of all students play football. What is the probability that a student plays basketball given that the student plays football?****60-****http://ceee.rice.edu/Books/LA/markov/img27.gif****B/****36- 1-Draw a picture corresponding to this transition matrix**  **2- How many communicating state, absorbing state , Irreducible Markov chain** 37- **A petrol station owner is considering the effect on his business (Dana) of a new petrol station (Sardar) which has opened just down the road. Currently (of the total market shared between Dana and Sardar) Dana has 80% of the market and Sardar 20%.** **P=**$\left(\begin{matrix}0.7&0.3\\0.5&0.5\end{matrix}\right) $1. **What will be the probability market share for *Dana* and *Sardar* after another two weeks ?**
2. **What would be the long-run prediction for the probability market share for Dana and Sardar?**

**62- If you have the transition Matrix? Find M and F** **P=** $\left(\begin{matrix}0&0.2&\begin{matrix}0.2&0.6\end{matrix}\\0&1&\begin{matrix}0&0\end{matrix}\\\begin{matrix}0.5\\0\end{matrix}&\begin{matrix}0\\0\end{matrix}&\begin{matrix}\begin{matrix}0\\0\end{matrix}&\begin{matrix}0.5\\1\end{matrix}\end{matrix}\end{matrix}\right)$38- **Classification of Markov Processes 2- Irreducible Markov chain** 39- **A computer is inspected at the end of every hour. It is found to be either working (up) or failed (down). If the computer is found to be up, the probability of its remaining up for the next hour is 0.90. It it is down, the computer is repaired, which may require more than one hour. Whenever, the computer is down (regardlewss of how long it has been down), the probability of its still being down 1 hour later is 0.35. ( 20 M)**1. **What is the probability to work for two hours**
2. **What would be the long-run prediction for the probability for failed**

**40-A mouse is put into the maze of the following figure. Each time period it chooses at random one of the doors in the room it is in and moves to another room. From room1 the mouse can escape to the outside (state 5) but in room 3 is a mouse trap** |
| **20. Extra notes:** |
| **21. Peer review**  |