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Sent: Sunday, January 29, 2023 11:48 PM

To: sanhan.khasraw@su.edu.krd

Subject: zbMATH - Review receipt for DE076033327

Dear Sanhan Muhammad Salih Khasraw,

Thank you for your contribution to zbMATH Open. Your help is greatly appreciated.

Your review has been submitted as follows:

Document:

DE076033327

Li, Jing Jian; Ma, Jicheng; Zhu, Wenying

On 7-valent symmetric Cayley graphs of finite simple groups

Classification:

05C25 20D05 20D60

Keywords:

Cayley graph ; symmetric graph ; simple groups

Review Text:

Suppose G is a group and S is a subset of G for which $S=\{g^{-1} \; |\; g \in S\}$ and S does

not contain the identity of G. A Cayley graph on G, denoted by $Cay(G, S)$, with respect to S is

the graph with vertex set G and the edge set $\{\{g, sg\} \; |\; g \in G, s \in S\}$. The valency of a

Cayley graph $\Gamma = Cay(G, S)$ is $|S|$, and it is connected if and only if $G=\langle S \rangle$.

There are plenty of papers deal with the classification of the symmetric Cayley graphs of non-abelian

simple groups, and in the most cases the classifications are conditional.

In this paper, the authors try to investigate 7-valent symmetric Cayley graphs of finite simple groups.

The following theorem is the main result of the paper.

\begin{theorem} Let G be a finite non-abelian simple group and let$\Gamma = Cay(G, S)$ be a

symmetric Cayley graph on G with valency 7. For $\alpha \in V \Gamma$, we have either Γ

is a normal Cayley graph or one of the following holds:

\begin{center} \begin{tabular}{ c c c }

& $Aut \Gamma$ & G \\

\hline 1 & $[2^{18}].$P$\Omega^+(12, 2)$ & $\Omega(11, 2)$ \\

2 & $(\mathds{Z}_7 \rtimes \mathds{Z}_3) \times A_8$ & PSL(3, 2) \\

3 & $((\mathds{Z}_7 \rtimes \mathds{Z}_3) \times A_8) \rtimes \mathds{Z}_2$ & PSL(3, 2) \\

4 & $\mathds{Z}_2.$PSU(6, 2) & PSU(5, 2) \\

5 & $\mathds{Z}_2^2.$PSU(6, 2) & PSU(5, 2) \\ & $\mathds{Z}_4 \times$PSU(6, 2) & \\

6 & $\mathds{Z}_2^4 \times$Sp(6, 4) & PSU(4, 4) \\

7 & $(\mathds{Z}_7 \rtimes \mathds{Z}_3) \times$ AGL(3, 2) & PSL(2, 7) \\ \hline \end{tabular}

\end{center} \begin{enumerate} \item [(1)] $Aut \Gamma$ acts quasiprimitively on $V \Gamma$ with

$L = soc(Aut \Gamma)$ being a non-abelian simple group. And $(L, G) \cong P\Omega^+(12, 2),

\Omega(11, 2)), (Sp(6, 4), PSU(4, 4)) or (A7,A6).$ \item [(2)] There exists a maximal intransitive normal

subgroup K of $A=Aut\Gamma$ such that the socle of A/K, say $\bar{L}$, is a simple group

containing $\bar{G} = GK/K \cong G$ properly. Moreover, one of the following holds.

\item [(i)] $\bar{L}$ is classical simple group. Then $(Aut \Gamma, G)$ is one of the seven cases in the

table above.

\item [(ii)] Otherwise, up to isomorphism, we have either $(Aut \Gamma, G) \cong (\mathds{Z}_7 \times

M_{24}, M_{23}),$ or $(\bar{L}, \bar{G}) \cong (A_n, A_{n-1})$ with $\bar{L}_{\bar{\alpha}}$ has a

subgroup of index n for $\bar{\alpha} \in V\Gamma_K$.

\end{enumerate}

\end{theorem}

Remarks to Subject Editor:

Best regards,

Your zbMATH staff

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