PhD Research Proposal	پرۆپۆزەلی توێژینەوە بۆ بەدەستهێنانی بروانامەی دکتۆرا
1. Title of PhD research proposal	ناونیشانی پرۆپۆزەلی توێژینەوەی پێشنیازکراو

Energy of some graphs associated with finite groups

2. General information		زانیاری گشتی
Name and surname of the supervisor 1	Dr. Sanhan Muhammad Salih Khasraw	ناوی سیانی سەرپەرشتیار ١
Scientific title	Assistant Professor	پلەی زانستی سەرپەرشتیار ١
E-mail	sanhan.khasraw@su.edu.krd	ئیمێلی سەرپەرشتیار ١
Mobile	07504566814	ژمارەی مۆبایل
Place of the work	Salahaddin University-Erbil	شوێنی بنەرەتی کارکردنی سەرپەرشتیار
Department	Mathematics	ناوی بەشی زانستی
College / faculty	Education	کۆلیژ / فاکەلتی/سکول
university's name	Salahaddin University-Erbil	ناوی زانکۆی میلاکی سەرپەرشتیار
Name and surname of the supervisor 2 (If it is available)	Dr. Rashad Rashid Haji	ناوی سیانی سەرپەرشتیار ٢ (ئەگەر هەیە)
Scientific title	Assistant Professor	پلەی زانستی سەرپەرشتیاری ٢
E-mail	rashad.haji@su.edu.krd	ئیمێلی سەرپەرشتیار ٢
Mobile	07504909486	ژمارەی مۆبایل
Department	Mathematics	ناوی بەشی زانستی
College / faculty	Education	کۆلیژ/ فاکەلتی/سکول
university's name	Salahaddin University-Erbil	ناوی زانکۆ

3. Summary (Abstract) of PhD research proposal
This should be not more than 200 words and not less than 75 words.

The study will focus on the energy of some graphs associated to finite groups. For this reason, we generalize the concept of coprime graph of subgroups of a group into two different ways. Also, we try to study some unsolved problems related to energy of graphs.

4. Introduction پێشەکی
To be completed by the primary supervisor: an overview of the proposed research project, focusing on the background of the project and rationale for the research.

The study of graph theory has actually begun many years ago. There are many real-life problems that can be related to graph theory. One of the problems is the electric network which was discovered by Kirchoff in 1847. From the electric network's problem, Kirchoff developed the basic concepts and theorem related to trees in graphs. To solve the system of simultaneous linear equation which gives the current in each branch and around each circuit of an electric network, Kirchoff replaced each electrical network by its underlying graph. Marcus stated that a graph consists of points which are called vertices, and connections which are called edges, which are indicated by line segments or curves joining certain pairs of vertices. In 2021, Ivan Gutman introduced the Sombor index and studied some of its mathematical properties. Also, many mathematicians studied properties of Sombor index. In addition, there are many researches which relate both the group and graph theory.

For a graph Γ on n vertices, the energy $E(\Gamma)$ of Γ is defined to be the sum of the absolute values of the eigenvalues of the adjacency matrix, $A(\Gamma)$, of Γ. In this project, we study and generalize the concept of the **coprime graph of subgroups of a group**, introduced in [5], into two different ways which help to determine the scope of the study. We study some connections between algebraic properties of a group and graph theoretic properties of its **generalized coprime graph of subgroups of a group**. Also, the energy and Sombor energy of the **generalized coprime graph of subgroups of a group** will be studied. Furthermore, we try to discuss some unsolved problems related to energy of graphs.

5. Research objectives
Clarify the research objectives and planned methodology to meet the challenges of the project. Include details of the research plan and relate to the previous work carried out by others.

The objectives of this research are to:

1. Study the graph properties of the Coprime graph of subgroups of a group introduced in [5].
2. Study the energy of the Coprime graph of subgroups of a group
3. Generalize the definition of the Coprime graph of subgroups of a group into two different ways and interplay between their properties with algebraic properties of groups.
4. Study the energy of the Generalized coprime graph of subgroups of a group introduced in part-(3).
5. determine some topological indices, especially Sombor index, of the Generalized coprime graph of subgroups of a group introduced in part-(3).
6. determine polynomials related to topological indices, especially Sombor index, of the Generalized coprime graph of subgroups of a group introduced in part-(3).
7. Study some unsolved problems related to energy of graphs.

6. Methodology and data collection
In this section the supervisor should describe the methodology of the proposed research

Some basic concepts in graph theory and group theory are included. Computer algebra system GAP (Groups, algorithms and programming) and MATLAB are required for this project.

7. Scope and limit to the research
Details of anticipated problems and proposed resolutions

The research focuses on the graph theory in groups. The groups considered in this study are finite groups, such as dihedral group and quaternion group. Other than that, this study also focuses on a class of graphs. Graph properties of such class of graphs are obtained from the graph constructed. Topological indices and their polynomials are computed. Some unsolved problems related to energy of graphs will also be discussed in this study.

8. Duration and timeline

6 months to read and understand previous studies.
6 months to make the required derivatives.
3 months to make GAP(MATLAB) programs.
6 months for computation and application.
3 months to write the theoretical part.
3 months to write the application.
3 months to write conclusions, recommendations, introduction and previous studies

9. Conclusions
The project supervisor summaries the research objectives and clarify their expected findings; include why the research has scientific value.

The properties of graphs for some groups are obtained. Topological indices of a class of graphs of some finite groups are determined, namely the coprime graph of subgroups of a group. Polynomials related to topological indices of a class of graphs of some finite groups are presented. Some unsolved problems related to energy of graphs will also be discussed in this study.

10. References   سەرچاوەکان

[1] C. Elphick, M. Farber, F. Goldberg, P. Wocjan, Conjectured bounds for the sum of squares of positive eigenvalues of a graph, Discrete Math., 339(9): 2215–2223, 2016.
[2] C. Elphick, P. Wocjan, Conjectured lower bound for the clique number of a graph, arXiv: 1804.03752, 2018.
[3] N. Ghanbari, On the Sombor characteristic polynomial and Sombor energy of a graph, Comput. Appl. Math. 41 (2022) 242, http://dx.doi.org/10.1007/s40314-022-01957-5.
[4] H.S. Ramane, D.V. Kitturmath, On the conjecture of Sombor energy of a graph, Examples and Counterexamples, 3 (2023) 100115.
[5] P. Devi, Studies on graphs associated with subgroups of groups, Ph.D Thesis, The Gandhigram Rural Institute (Deemed to be University), 2016"
[6] K.C. Das, A.S. C¸ evik, I.N. Cangul and Y. Shang, On sombor index, Symmetry 13 (2021) 140.
[7] I. Gutman, Geometric approach to degree-based topological indices: Sombor indices. MATCH Commun. Math. Comput. Chem. 86 (2021) 11–16.
[8] I. Milovanovic, E. Milovanovic and M. Matejic, On some mathematical properties of sombor indices, Bull. Int. Math. Virtual Inst. 11 (2021) 341–353.
[9] I. Gutman, Some basic properties of sombor indices, Open J. Discrete Appl. Math. 4 (2021) 1–3.
[10] J. Rada, J.M. Rodr´ıguez and J.M. Sigarreta, General properties on sombor indices, Discrete Appl.

Math. 299 (2021) 87-97.

[11] N. Ghanbari and S. Alikhani, Sombor index of certain graphs, arXiv preprint arXiv:2102.10409, 2021.

[12] V. Kulli, Sombor indices of certain graph operators, Int. J. Engin. Sci. Res. Technol. 10 (2021) 127–134.

[13] I. Red˘zepovi´c, Chemical applicability of sombor indices, J. Serbian Chem. Soc. 86(5) (2021) 445–457.

[14] M.N. Husin, R. Hasni, N.E. Arif and M. Imran, On topological indices of certain families of nanostar dendrimers. Molecul. 21 (2016) 821.

[15] D. Kumara, A. Mahasinghe and H. Erandi, On the abc and ga indices of the corona products of some graphs, Eurasian Chem. Commun. 3 (2021) 257-263.

11. General notes :   هەر زانیارییەکی گشتی دیکە کە سەرپەرشتیار بە گرنگی بزانێت

1. The student needs to group theory, graph theory, GAP and MATLAB programming.
2. The student obtains the exact specialization in algebraic graph theory.

12. بەش زانستی لیژنەی لایەن لە پرۆپۆزەل پەسەندکردنی

ژمارەی کۆنووسی کۆبوونەوە:1
ڕێکەوتی کۆبوونەوە:
بریار: پەسەند کرا ⭕   پەسەند نەکرا ⭕

ناوی سیانی و واژووی لیژنەی زانستی بەش

واژوو:
ناوی سەرۆکی لیژنەی زانستی بەش   مۆری بەش

واژوو:
ناوی سەرۆکی بەش:

13.

پەسەندکردنی پرۆپۆزەل لە لایەن ئەنجومەنی کۆلێژ/فاکەڵتی

ژمارەی کۆنوسی کۆبوونەوە:
ڕێکەوتی کۆبوونەوە:
بریار: پەسەند کرا ⭕   پەسەند نەکرا ⭕

واژوو:
ناو ڕاگری کۆلێژ: مۆری کۆلێژ

تێبینی: تکایە فۆرمەکە تەنها بە یەك زمان (زمانی توێژینەوە) پڕ بکرێتەوە.

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