Q1. Let U and W be two subspaces of a vector space V over F. Then prove that U+W is a subspace of V.

Q2. If M={(x, y, z, w): x+z+2w=0} and N={(x, y, z, w): x-y+z+w=0} are subspaces of R^4, then find dim(M+N) and M+N.

Q3. Find the transition matrix from the basis $S_1=\{A_1=(-3, 7), A_2=(-13, 19)\}$ for R^2 to the basis $S_2=\{`A_1^*=(-7, 5), A_2^*=(2, 1)\}$.

Q4. Find a basis for a subspace M={p(x): p(-x) = - p(x)} of $P_2(R)$.

Q5. Let A=(1, 1, 0, 0), B=(-3, 0, 1, 0), C=(2, -1, 0, 1) be vectors in R^4. Test whether the vector D=(1, -1, 1, 0) is a linear combination of vectors A, B, C or not.

Q6. Let $V=P_1(R)$, the set of all polynomials of degree at most one over the field R. Define the operations of addition and scalar multiplication as follows: for all $(a_1+b_1x), (a_2+b_2x)\in V$ and $r\in R$,

$(a_1+b_1x)+(a_2+b_2x)=(a_1+a_2)+(b_1+b_2)x$ and

$r(a_1+b_1x)=ra_1+(r^2b_1)x$. Test whether V is a vector space or not.

Q7. Define transition matrix. If the transition matrix from the basis $S_1=\{A_1=(-5, 4), A_2=(-9, 10)\}$ for R^2 to the basis $S_2=\{A_1^*, A_2^*\}$ is $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$. Then find A_1^*, A_2^*.

Q8. Find basis for the subspace M={p(x): p(-x) = p(x)} of $P_2(R)$.

Q9. If M={(x, y, z, w): x+3z+w=0} and N={(x, y, z, w): x+y+z-w=0} are subspaces of R^4, then find dim(M+N) and M+N.

Q10. Prove that in any n-dimensional vector space V over a field F, every set of n linearly independent vectors is a basis.

Q11. Let {(1,-1),(2,0)} be a basis for R^2. Find a LT, $T:R^2 \rightarrow P_2(R)$ such that T(1,-1)=1-2x and $T(2,0)=1+x+x^2$.

Q12. Let $T \in L(P_1(R),R^2)$ defined by T(a+bx)=(a-2b, 3a+b). Find left and right inverses to T if exists.

Q13. Let $T \in L(R^3, R^2)$ defined by T(x,y,z)=(x-2y,y+z). Find basis for R^3 and basis for R^2 for which the matrix for T relative to these bases is of normal form.

Q14. Let $T:M_n(R) \rightarrow M_n(R)$ be a function defined by $T(A)=(A-A^t)/2$. Show that T is a linear transformation and find its kernel.

Q15. Let {(2,-1),(1,1)} be a basis for R^2. Find a LT, $T:R^2 \rightarrow P_2(R)$ such that T(2,-1)=1+3x and $T(1,1)=1-x^2$.

Q16. Let $T \in L(R^2, P_1(R))$ defined by T(a,b)=(a-2b) - (3a)x. Find left and right inverses to T if exists.

Q17. Let $T \in L(R^3, R^2)$ defined by T(x,y,z)=(x+y,y-2z). Find basis for R^3 and basis for R^2 for which the matrix for T relative to these bases is of normal form.

Q18. Let $T:M_n(R) \rightarrow M_n(R)$ be a function defined by $T(A)=(A+A^t)/2$. Show that T is a linear transformation and find its kernel.

Q19. In R^4, let M={(x, y, z, w) : x+z-w=0} and N={(x, y, z, w) : x=0}. Find $M \cap N$ and M+N.

Q20. Let $S = \{2,\ 1-x,\ 3x^2\}$ be a subset of $P_2(R)$. (i) Show that S is a basis for $P_2(R)$.

(ii) Find coordinate vector to $u = 3 - x^2$ relative to the basis S.

Q21. Let {(1, 0, 0), (1, 1, 0), (1, 1, 1)} be a basis for R^3. Find a linear transformation $T:R^3 \rightarrow R^2$ such that T(1, 0, 0)=(1, 1), T(1, 1, 0)=(1, 2) and T(1, 1, 1)=(1, 1).

Q22. If $T \in L(M_{2x2}(R), R^4)$ defined by $T\begin{bmatrix} a & b \\ c & d \end{bmatrix} = (2a, -c, 0, 3b)$, then find dimKerT and dimImT.

Q23. Let $T \in L(R^2, R^2)$ defined by $T(x, y)=(x+y, x+y)$. Test whether T is diagonalizable or not.

Q24. Let U be a vector space over F and $T \in L(U, U)$. Let $u_1, \ldots, u_m$ be eigenvectors of T associate with distinct eigenvalues $\lambda_1, \ldots, \lambda_m$. Then show that $\{u_1, \ldots, u_m\}$ is linearly independent.

Q25. Let $U=R^2$ and $u=(x_1, y_1)$, $v=(x_2, y_2) \in R^2$. Define $< , >: R^2 \times R^2 \to R$ by $<u, v>= x_1x_2-x_1y_2-x_2y_1+3y_1y_2$. Test whether $< , >$ is an inner product on R^2 or not.

Q26. Let $<U, <, >>$ be an inner vector space over R. Suppose that the vectors $\{u_1, \ldots, u_8\}$ are orthonormal. Show that the vectors $\{w_1, w_2\}$ are also orthonormal, where
$w_1=(u_1+ u_2+ u_3+ u_4)/2$ and $w_2=(u_5+ u_6+ u_7+ u_8)/2$.

Q27. Let $M = \left\{ \begin{bmatrix} a & b \\ c & d \end{bmatrix}, a+b+d=0 \right\}$ and $N = \left\{ \begin{bmatrix} a & b \\ c & d \end{bmatrix}, c=0 \right\}$ be two subspaces of $M_{2x2}(R)$. Find $M \cap N$ and M+N.

Q28. Let $P=\begin{bmatrix} \frac{1}{\sqrt{5}} & \frac{2}{\sqrt{5}} \\ -\frac{2}{\sqrt{5}} & \frac{1}{\sqrt{5}} \end{bmatrix}$ be a transition matrix from S={(2, 1),(0, 3)} to $S^*=\{u_1^*, u_2^*\}$. Then find u_1^* and u_2^*.

Q29. Let {(1, 1, 0), (1, 0, 1), (0, 1, 1)} be a basis for R^3. Find a linear transformation $T:R^3 \to P_2(R)$ such that $T(1, 1, 0)=1-x$, $T(1, 0, 1)=x^2$ and $T(0, 1, 1)=1+2x^2$.

Q30. If $T \in L(R^3, R^2)$ defined by $T(x, y, z)=(x+y+z, 0)$, then find dimKerT and dimImT.

Q31. Let $V=R^2$ and $T \in L(R^2, R^2)$ defined by $T(x, y)=(y, x)$. Find a basis for R^2 which diagonalizes T.

Q32. Let U be n-dimensional vector space over F, $T \in L(U, U)$ and λ is an eigenvalue for T. Then show that $GM(\lambda) \leq AM(\lambda)$.

Q33. Let <U, <, >> be an inner product space over R and let $x, y \in U$. Prove that $<x, y>=0$ if and only if $\|x - y\| = \|x + y\|$.

Q34. Let S={(1, -2, 0, 1), (-1, 0, 0, -1), (0, 0, 0, 1)} be a basis for a subspace M of R^4. Find an orthogonal basis for M with usual inner product on R^4.

Q35. a) If $M = \left\{ \begin{bmatrix} a & b \\ c & d \end{bmatrix}, b = a + d, c = 0 \right\}$ and $N = \left\{ \begin{bmatrix} a & b \\ c & d \end{bmatrix}, a = 2d, b = 0 \right\}$ are subspaces of $M_{2x2}(R)$, then find $M \cap N$ and M+N.

b) If the transition matrix from the basis S={ A_1=(1, -1), A_2=(1, 3)} to the basis $S^* = \{ A_1^*, A_2^* \}$ is $P = \begin{bmatrix} -1 & 1 \\ 3 & 1 \end{bmatrix}$, then find A_1^* and A_2^*.

Q36. a) Let $T: C^{(1)}(0,1) \rightarrow C(0,1)$ defined by $T(f(x))=f'(x)e^x$. Show that T is a LT and find kerT.

b) Let U and V be vector spaces over F and $T \in L(U, V)$. Then prove that T is one-one if and only if KerT={O}.

Q37. a) Let $T \in L(P_1(R), R^3)$ defined by T(a+bx)=(a, -b, a-2b). Find left inverse of T if exists.

b) Let $T \in L(R^3, R^3)$ defined by T(x, y, z)=(-3x, x+y, x+y-3z). Test whether T is diagonalizable or not.

Q38. a) Find an orthonormal basis for a subspace $M=\{a+bx+cx^2 \in P_2(R): a=b-c\}$ of $P_2(R)$ with usual inner product.

b) Let $U=R^2$ and $u=(x_1, x_2)$, $v=(y_1, y_2) \in R^2$, define $<,>: R^2 \times R^2 \rightarrow R$ by

$<u, v> = x_1y_1 - x_2y_1 - x_1y_2 + x_2y_2$. Test whether $<,>$ is an inner product on R^2 or not.

Q39. a) In R^4, let $M=\{(x, y, z, w) \in R^4 \mid x+y+w = 0\}$. Show that M is a subspace.

b) Find a basis for $P_3(R)$ containing the vectors $1+x$ and $2x^2$.

Q40. a) Let $\{(1,0),(2,1)\}$ be a basis for R^2. Find a LT, $T:R^2 \rightarrow P_2(R)$ such that $T(1,0)=1+x$ and $T(2,1)=x-x^2$.

b) Let $T \in L(R^3, R^2)$ defined by $T(x, y, z)=(x-y, 2x+z)$. Find right inverse to T if exists.

Q41. a) Let U be n-dimensional vector space over F, $T \in L(U,U)$ and $\Delta(t)$ be a characteristic polynomial for T. Show that λ is an eigenvalue for T if and only if λ is a root of $\Delta(t)$ $[\Delta(\lambda)=0]$.

b) Test whether $T:R^3 \rightarrow R^3$ defined by $T(x,y,z)=(2x+y, y-z, 2y+4z)$ is diagonalizable or not.

Q42. a) Let u and v be any two vectors in an Euclidean vector space $<U, <,>>$. Then $4 <u, v> = \|u+v\|^2 - \|u-v\|^2$.

b) Find an orthogonal basis for the subspace $M=\{(x, y, z, w): x - y - 2z = 0\}$ of an Euclidean space R^4 with usual inner product.

Q43. Let $M = \left\{ \begin{bmatrix} a & b \\ c & d \end{bmatrix}, c = a + d, b = 0 \right\}$ and $N = \left\{ \begin{bmatrix} a & b \\ c & d \end{bmatrix}, a = 3d, c = 0 \right\}$ be two subspaces of $M_{2x2}(R)$. Find $M \cap N$ and M+N.

Q44. If the transition matrix from the basis $S=\{ u_1=(-23, 14), u_2=(-26, 21)\}$ of R^2 to the basis $S^* = \{u_1^*, u_2^*\}$ is $P = \begin{bmatrix} 3 & -1 \\ 4 & 1 \end{bmatrix}$, then find u_1^* and u_2^*.

Q45. Let $\{(1, -1), (0, 2)\}$ be a basis for R^2. Find a linear transformation $T:R^2 \rightarrow R^3$ such that $T(1, -1)=(2, 1, 1)$ and $T(0, 2)=(0, 1, -1)$.

Q46. If $T \in L(R^3, P_2(R))$ defined by $T(a, b, c)=b+(a+b)x+2cx^2$, then find left inverse to T if exists.

Q47. Let $T \in L(R^3, R^3)$ defined by $T(x, y, z)=(3x-z, y-x+2z, 4z)$. Test whether T is diagonalizable or not.

Q48. Let U be a vector space over F and $T \in L(U,U)$. Let $u_1, \ldots, u_m$ be eigenvectors of T associate with distinct eigenvalues $\lambda_1, \ldots, \lambda_m$. Then prove that $\{u_1, \ldots, u_m\}$ is linearly independent.

Q49. Let $U=R^2$ and $u=(x_1,x_2)$, $v=(y_1,y_2) \in R^2$. Define $< , >: R^2 \times R^2 \rightarrow R$ by $<u, v>=x_1y_1-x_2y_1-x_1y_2+3x_2y_2$. Test whether $< , >$ is an inner product on R^2 or not.

Q50. Let $S=\{(1,-2,0,1),(-1,0,0,-1),(0,0,0,1)\}$ be a basis for a subspace M of R^4. Find an orthogonal basis for M with usual inner product.

Q51. If $M=\{(x, y, z, w) : x+2z-w=0\}$ and $N=\{(x, y, z, w) : x+y+z+w=0\}$ are two subspaces of R^4, then find $M \cap N$ and $M+N$.

Q52. Find a basis for a vector space $P_3(R)$ containing the vectors $x+1$ and $2x^2$.

Q53. Let $\{(1, -5), (0, 2)\}$ be a basis for R^2. Find a linear transformation $T:R^2 \rightarrow R^3$ such that $T(1, -5)=(2, 4, 0)$ and $T(0, 2)=(1, 0, -1)$.

Q54. If $T \in L(M_{2x2}(R), R^4)$ defined by $T\begin{bmatrix} a & b \\ c & d \end{bmatrix} = (-b, a, 5d, 2c)$, then find left inverse to T if exists.

Q55. Let $T \in L(R^3, R^3)$ defined by $T(x, y, z)=(2x+z, 3y-x+2z, 4z)$. Test whether T is diagonalizable or not.

Q56. Let U be n-dimensional vector space over F, $T \in L(U,U)$ and $\Delta(t)$ be a characteristic polynomial for T. Prove that λ is an eigenvalue for T if and only if λ is a root of $\Delta(t)$.

Q57. Let $U=R^2$ and $u=(x_1,x_2)$, $v=(y_1,y_2)\in R^2$. Define $< , >: R^2 \times R^2 \to R$ by

$<u, v>=x_1y_1-x_2y_1-x_1y_2+2x_2y_2$. Test whether $< , >$ is an inner product on R^2 or not.

Q58. Let S={(1, 1, 2, 2), (-1, 1, 2, 2), (0, 1, 3, 5)} be a basis for a subspace M of R^4. Find an orthogonal basis for M with usual inner product.

Q59. In $P_2(R)$, let $M=\{a+bx+cx^2: a+2b-c=0\}$ and $N=\{a+bx+cx^2: a+b+2c=0\}$.

Find $M \cap N$ and M+N.

Q60. Let $S = \{(1,1,0),(1,0,1),(0,1,1)\}$ be a subset of R^3.

1. Show that S is a basis for R^3.
2. Find coordinate vector to $u = (1,-2,3)$ relative to the basis S.

Q61. Let {(1, -1, 0), (1, 0, 2), (0, 1, 1)} be a basis for R^3. Find a linear transformation $T:R^3\to R^2$ such that T(1, -1, 0)=(2, 1), T(1, 0, 2)=(1, 0) and T(0, 1, 1)=(0, 0).

Q62. If $T\in L(R^3, P_2(R))$ defined by $T(a, b, c)=c+(a-b)x+2ax^2$, then find dimKerT and dimImT.

Q63. Let $T\in L(R^3, R^3)$ defined by T(x, y, z)=(2x+y, y-z, 2y+4z). Test whether T is diagonalizable or not.

Q64. Let U be n-dimensional vector space over F and $T\in L(U, U)$. If T has n distinct eigenvalues, then show that T is diagonalizable.

Q65. Let $U=R^2$ and $u=(x_1,y_1)$, $v=(x_2,y_2)\in R^2$. Define $< , >: R^2 \times R^2 \to R$ by

$<u, v>=2x_1x_2+x_1y_2+x_2y_1+3y_1y_2$. Test whether $< , >$ is an inner product on R^2 or not.

Q66. Let S={(1, 1, 0, 0),(-1, 0, 2, 2),(0, 1, 3, 5)} be a basis for a subspace M of R^4. Find an orthogonal basis for M with usual inner product on R^4.

Q67. If M={(x, y, z) : x+y+z=0} and N={(x, y, z) : x=z} are two subspaces of R^3, then find

M+N.

Q68. Let $S=\{3,-1+x, x^2\}$ be a subset of $P_2(R)$.

1. Show that S is a basis for $P_2(R)$.
2. Find coordinate vector to $u = 3-x^2$ relative to the basis S.

Q69. Let {(1, 0, -1), (0, 1, 2), (-2, -1, 0)} be a basis for R^3. Find a linear transformation $T:R^3 \rightarrow R^2$ such that T(1, 0, -1)=(1, -1), T(0, 1, 2)=(0, 1) and T(-2, -1, 0)=(0, 0).

Q70. If $T \in L(P_2(R), R^3)$ defined by $T(a+bx+cx^2)=(a-b, 0, c)$, then find dimKerT and dimImT.

Q71. Let $T \in L(R^3, R^3)$ defined by T(x, y, z)=(x-2z, 0, -2x+4z). Test whether T is diagonalizable or not.

Q72. Let U be n-dimensional vector space over F, $T \in L(U, U)$ and λ is an eigenvalue for T. Then show that $GM(\lambda) \leq AM(\lambda)$.

Q73. Let $U=R^2$ and $u=(x_1,y_1)$, $v=(x_2,y_2) \in R^2$. Define $< , >: R^2 \times R^2 \rightarrow R$ by $<u, v>=3x_1x_2+x_1y_2+x_2y_1+4y_1y_2$. Test whether $< , >$ is an inner product on R^2 or not.

Q74. Let S={(1, 0, -1, 3), (-1, 2, 0, 5), (0, -2, 1, 3)} be a basis for a subspace M of R^4. Find an orthogonal basis for M with usual inner product on R^4.