CHAPTER ONE

Coulomb's Law

Introduction

Electricity is the name given to a wide range of electrical phenomena that, in one form or another, just about everything around us, like the lightning in the sky the glowing of a lamp, and the nerve impulses that travel along our nervous system. The control of electricity is evident in many devices, from microwave ovens to computers. In this technological age, it is important to understand the basics of electricity and of how these basic ideas are used to enhance our current comfort, safety, and prosperity.

The modern word **electric** comes from the Greek word for **amber**. The ancient Greeks as early as 700 B.C. noticed that a fossil material called amber, when rubbed with wool attracts small objects such as pieces of paper or feathers. When the amber behaves this way, it is said to have become **electrified** or **electrically charged**.

Electrostatics is the study of electrical charges **at rest** or that electricity exists in matter without applying any external force or potential. **Electrodynamics** is the study of electrical charges **in motion** or that electricity exists in matter by applying external force or potential.

The structure of the Atom

Matter is anything that has mass and occupies space. Matter is composed of very small particles called **atoms**. Everything around us is made of atoms. There are more kinds of atoms. Atoms can join together in many different ways. That’s why we have different substances, like air, apples, and toys. Everything in the world is made up of atoms. A very highly simplified model of an atom has most of the mass in a small, dense center called the **nucleus**. The nucleus has positively charged **protons** and neutral **neutrons**. Negatively charged **electrons** move around the nucleus at much greater distance. The whole atom **does not have a charge** either. That's because the protons and the electrons balance each other out. The number of protons or electron in neutral atom of an element is called the **atomic number (Z)** of the element. The number of protons and neutron within the nucleus of any particular atom specifies the **mass number** of the atom **(A)**. Atoms of different element differ from one another in the number of electrons and protons. The elements ${}_{Z}^{A}X$ which are equal in atomic number but different in mass number called **Isotopes**.

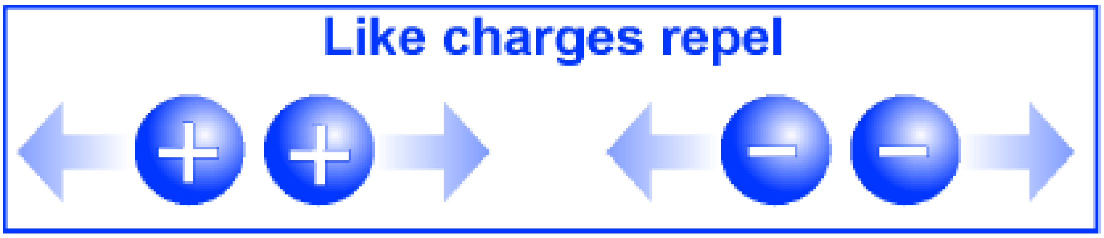
Properties of electric charge

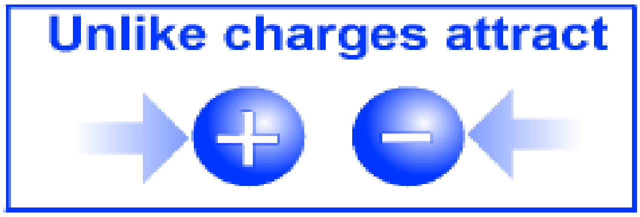
Electric charge is a property of tiny particles in atoms. An intrinsic property of protons and electrons, which make up all matter, is electric charge. Protons are about **1800 times** more **massive** than electrons, but they carry an **amount** of positive charge **equal to** the negative charge of electrons. Neutrons have slightly more mass than protons and have no net charge.

	Mass (kg)	Charge (coulombs)
Electron	9.109×10^{-31}	-1.602×10^{-19}
Proton	1.673×10^{-27}	$+1.602 \times 10^{-19}$
Neutron	1.675×10^{-27}	0

A number of simple experiments demonstrate the existence of **electric forces** and **charges**. For example, after running a comb through your hair on a dry day, you will find that the comb attracts bits of paper. The attractive force is often strong enough to suspend the paper. The same effect occurs when materials such as glass or rubber are rubbed with silk or fur. Another simple experiment is to rub an inflated balloon with wool. The balloon then adheres to a wall, often for hours. When materials behave in this way, they are said to be **electrified** or to have become **electrically charged**.

In a series of simple experiments, it is found that there are two kinds of **electric charges**, which were given the names **positive** and **negative** by **Benjamin Franklin** (1706–1790). To verify that this is true, consider a hard rubber rod that has been rubbed with fur. When a glass rod that has been rubbed with silk is brought near the rubber rod, the two **attract** each other. On the other hand, if two charged rubber rods (or two charged glass rods) are brought near each other, the two **repel** each other. This observation shows that the rubber and glass are in two different states of electrification. On the basis of these observations, we conclude that **like charges repel one another and unlike charges attract one another**.





Using the convention suggested by **Franklin**, the electric charge on the glass rod is called **positive** and that on the rubber rod is called **negative**.

Another important aspect of **Franklin's** model of electricity is the implication **that electric charge is always conserved**. That is, when one object is rubbed against another, **charge is not created** in the process. The electrified state is due to a **transfer of charge** from one object to the other. One object gains some amount of negative charge while the other gains an equal amount of positive charge. For example, when a glass rod is rubbed with silk, the silk obtains a negative charge that is equal in magnitude to the positive charge on the glass rod. We now know from our understanding of atomic structure that negatively charged electrons are transferred from the glass to the silk in the rubbing process. Similarly, when rubber is rubbed with fur, electrons are transferred from the fur to the rubber, giving the rubber a net negative charge and the fur a net positive charge. This process is consistent with the fact that **neutral uncharged matter** contains as many **positive charges** (protons within atomic nuclei) **as negative charges** (electrons).

The **fundamental charge** is the electrical charge on an **electron** and has a magnitude of **1.6×10^{-19} coulombs**. The SI unit for measuring the magnitude of electric charge is the **coulombs (C).**

In 1909, **Robert Millikan** (1868–1953) discovered that **electric charge** always occurs as **some integral(N)** multiple of **a fundamental amount of charge (e).** In modern terms, the electric charge is said to be **quantized,** where (q) is the standard symbol used for the charge. This means that only the integral (1, 2, …., n) number of electrons can be transferred from one body to the other. Charges are not transferred in fractions. Hence, a body possesses total charge only in integral multiples of electric charge. That is, electric charge exists as discrete packets, and we can write q= N e

Example: How many electrons are there in one coulomb of negative charge?

Solution:

$\mathbf{q = N\,e}$

The number of electrons (N) is $\mathbf{N=q/e}$

$\mathbf{N} = 1/\ 1.602\times10^{-19} \quad = 6.24 \times 10^{18}$

We conclude that electric charge has the following important properties:

- **Two kinds of changes occur in nature, with the property that unlike charges attract one another and like charges repel one another.**
- **Charge is conserved.**
- **Charge is quantized**.

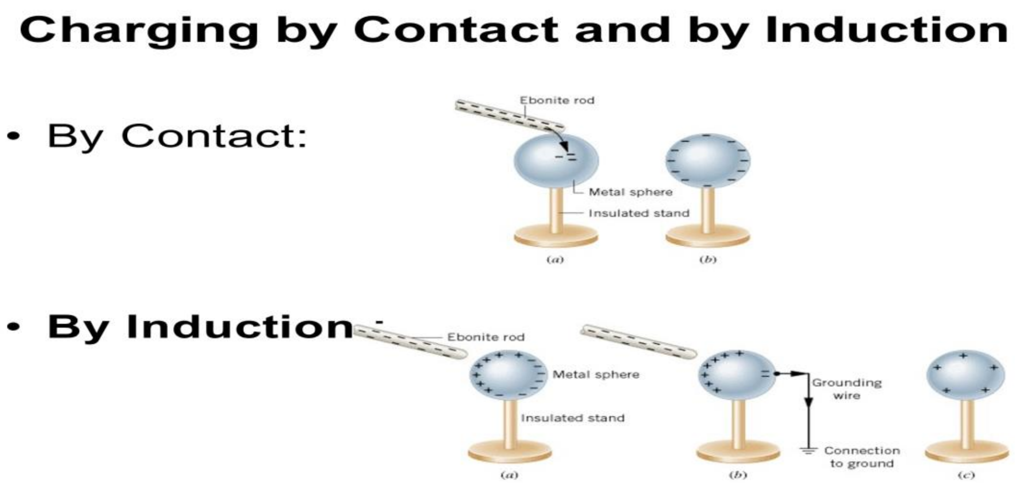
Charging implies either adding electrons to an object, removing electrons from an object, or separating out positive and negative charges within an object. This can be accomplished in three different ways:

Friction: Rubbing two materials together can rub electrons off of one and onto the other.

Conduction: Touching an object to a charged object could lead to a flow of charge between them (same charge)

Induction: If a charged object is brought near (but not touching) a second object, the charged object could attract or repel electrons (depending on its charge) in the second object. This yields a separation charge in the second object, an induced charge separation. Most important is the use of a **grounding wire**. A grounding wire is simply a conductor that connects the object to the ground. Think of the earth as a huge reservoir of charge, it can either **gain** or **donate electrons** as needed. Depending on what the situation is, either electron will travel up the **grounding wire** to the object being charged, or travel down to the ground. (opposite charge)

An **electroscope** is a simple device that is used to detect the presence of electric charges. The charged electroscope can then be used to determine the sign of an unknown charge.



Conductors and Insulators

Not only can electric charge exist on an object, but it can also move through an object. It is convenient to classify materials in terms of the ability of electrons to move through the material:

Electrical conductors are materials in which some of the electrons are free electrons that are not bound to atoms and can move relatively freely through the material. materials such as **copper, aluminum,** and **silver** are good electrical conductors.

Electrical insulators are materials in which all electrons are bound to atoms and cannot move freely through the material. Materials such as **glass, rubber,** and **wood** fall into the category of electrical insulators.

Semiconductors are the third class of materials, and their electrical properties are somewhere between those of insulators and those of conductors. **Silicon** and **germanium.**

Coulomb’s Law:

Objects become charged when the number of electrons and protons becomes unbalanced. A tiny imbalance in either positive or negative charge on an object is the cause of static electricity. Electrons can be added or removed from atoms easily. Remove electrons substance becomes positive. Add electrons substance becomes negative. In general, negative or positive charges that are exerting an electric force. Electric forces are created between all electric charges. Because there are two kinds of charge (positive and negative) the electrical force between charges can attract or repel.

Charles Coulomb (1736–1806) measured the **magnitudes of the electric forces** between charged objects using the **torsion balance**, which he invented. From experimental observations, Coulomb found that the force between **two point charges** (a particle of zero sizes that carries an electric charge) $\mathbf{q_1}$ and $\mathbf{q_2}$ exert on each otherwise **proportional** to **each charge** and **proportional** to the **inverse square of their separation distance** between them. Thus Coulomb established what we now call **Coulomb's law**, which states that "**The magnitude of the electric force between two point charges is directly proportional to the product of the charges and inversely proportional to the square of the distance between them**". We can express Coulomb’s law as an equation giving the magnitude of the electric force between two point charges:

$$\boxed{F_e = k_e \frac{|q_1||q_2|}{r^2}}$$ -------------- (1-1) **(Coulomb's law)**

Where $\mathbf{k_e}$ is a constant called the **Coulomb constant**. The value of the Coulomb constant depends on the choice of units. The SI unit of charge is the coulomb (C). The Coulomb constant $\mathbf{k_e}$ in SI units has the value ($\mathbf{k_e = 8.987\,5\times10^9\ N.m^2/\ C^2} = \mathbf{9\times10^9\ N.m^2/\ C^2}$). This constant is also written in the form

$$\boxed{k_e = \frac{1}{4\pi\varepsilon_0}}$$ --------------- (1-2) **(Coulomb constant)**

Where the constant $\boldsymbol{\varepsilon_0}$ (epsilon -nought) is known as the **permittivity of free space** and has the value ($\boldsymbol{\varepsilon_0} = \mathbf{8.8542\times10^{-12} C^2/\ N.m^2}$).

1coulombs = $\mathbf{2.9\times10^9}$ **stat coulombs.**

$\mathbf{k_e = 1}$ **dyne.cm^2/ stat coulombs^2**

Coulomb's law equation provides an accurate description of the force between two objects whenever the objects act as point charges.

A charged conducting sphere interacts with other charged objects as though all of its charges were located at its center. While the charge is uniformly spread across the surface of the sphere, the center of charge can be considered to be the center of the sphere. The sphere acts as a point charge with its excess charge located at its center. Since Coulomb's law applies to point charges, the distance (r) in the equation is the distance between the centers of charge for both objects (not the distance between their nearest surfaces). When dealing with Coulomb’s law, you must remember that force is a **vector quantity** and must be treated accordingly. The law expressed in vector form for the electric force exerted by a charge $\mathbf{q_1}$ on a second charge $\mathbf{q_2}$, written $\mathbf{F_{12}}$, is

$$\boxed{\mathbf{F}_{12} = k_e \frac{q_1 q_2}{r^2}\hat{\mathbf{r}}}$$ -------- (1-3) **(Vector form of Coulomb's law)**

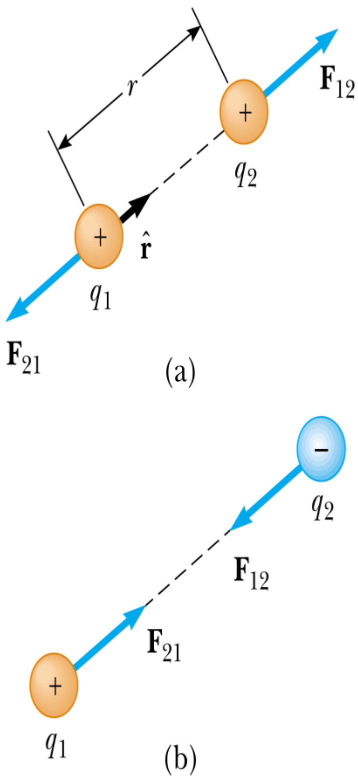
Where is $\hat{\mathbf{r}}$ **unit vector**, directed from $\mathbf{q_1}$ toward $\mathbf{q_2}$, as shown in figure (a).

From equation (1-3), we see that if $\mathbf{q_1}$ and $\mathbf{q_2}$ have the **same sign**, as in figure (a), the product $\mathbf{q_1 q_2}$ is **positive**.

If $\mathbf{q_1}$ and $\mathbf{q_2}$ are of **opposite signs**, as shown in figure (b), the product $\mathbf{q_1 q_2}$ is **negative**.

These **signs** describe the **relative direction** of the force. A **negative product** indicates an **attractive force**, so that the charges each experience a **force toward the other** thus, the force on one charge is in a direction relative to the other.

A **positive product** indicates a **repulsive force** such that each charge experiences a force away from the other.



When **more than two charges** are present, the force between **any pair** of them is given by equation (1-3). Therefore, the resultant force on any one of them equals the vector sum of the forces exerted by the various individual charges. For example, if four charges are present, then the resultant force exerted **by** particles 2, 3, and 4 **on** particle 1 is:

$\mathbf{F}_1 = \mathbf{F}_{21} + \mathbf{F}_{31} + \mathbf{F}_{41}$

Example1: Two-point charges

Suppose that two point charges, each with a charge of +1.00 Coulomb are separated by a distance of 1.00 meter. Determine the magnitude of the electrical force of repulsion between them.

Solution:

$$\boxed{F_e = k_e \frac{|q_1||q_2|}{r^2}}$$

$F_{elect} = (9.0 \times 10^9\ N \cdot m^2/C^2) \cdot |1.00\ C| \cdot |1.00\ C| / (1.00\ m)^2$

$F_{elect} = 9.0 \times 10^9$ N

Newton's universal law of gravitation

Every object in the universe attracts every other object with a force that is directly proportional to the mass of each body and that is inversely proportional to the square of the distance between them.

$\mathbf{F_g = G\ m1\ m2/ r^2}$

Where $\mathbf{G = 6.67 \times 10^2\ \ N.\ m^2\ / kgm^2}$

Example2: The Hydrogen Atom

The electron and proton of a hydrogen atom are separated (on average) by a distance of approximately 5.3×10^{-11}m. Find the magnitudes of the electric force and the gravitational force between the two particles.

Solution

From Coulomb's law, we find that the magnitude of the electric force is

$$F_e = k_e \frac{|q_1||q_2|}{r^2} = 9\times10^9 \frac{(1.6\times10^{-19})^2}{(5.3\times10^{-11})^2}$$

$F_e = 8.2\times10^{-8}$ N

From Newton's law of universal gravitation, the magnitude of the gravitational force is

$$F_g = G\frac{m_e m_p}{r^2} = 6.67\times10^{-11}\frac{(9.1\times10^{-31})(1.67\times10^{-27})}{(5.3\times10^{-11})^2}$$

$F_g = 3.6\times10^{-47}$ N

$F_e / F_g = 2.2\times10^{39}$

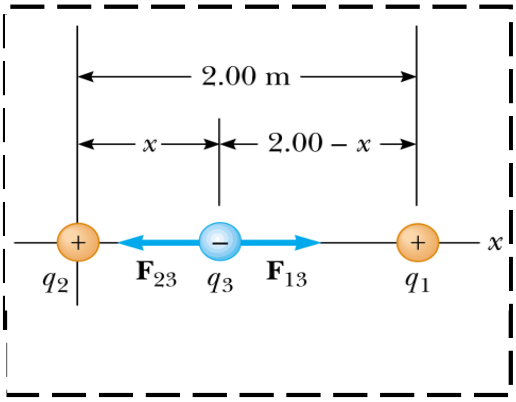
The **electric force** is about (**10^{39}**) **time stronger** than the **gravitational force**, thus the gravitational force between charged atomic particles is **negligible** when compared with the electric force.

Example3: The Resultant Force Zero

Three point charges lie along the x-axis as shown in the figure. The positive charge q_1=15μC is at x =2m, the positive charge q_2 = 6μC is at the origin, and the resultant force acting on q_3 is zero. What is the x coordinating of q_3?

Solution

Because q_3 is negative and q_1 and q_2 are positive, the forces $\mathbf{F}_{13}$ and $\mathbf{F}_{23}$ are both attractive, as indicated in the figure. From Coulomb's law, $\mathbf{F}_{13}$ and $\mathbf{F}_{23}$ have magnitudes



$$F_{13} = k_e\frac{|q_1||q_3|}{(2.00 - x)^2} \qquad F_{23} = k_e\frac{|q_2||q_3|}{x^2}$$

For the resultant force on q_3 to be zero, $\mathbf{F}_{23}$ must be equal in magnitude and opposite in direction to $\mathbf{F}_{13}$. Setting the magnitudes of the two forces equal, we have $k_e \frac{|q_2||q_3|}{x^2} = k_e \frac{|q_1||q_3|}{(2.00 - x)^2}$

Noting that k_e and $|q_3|$ are common to both sides and so can be dropped; we solve for x and find that

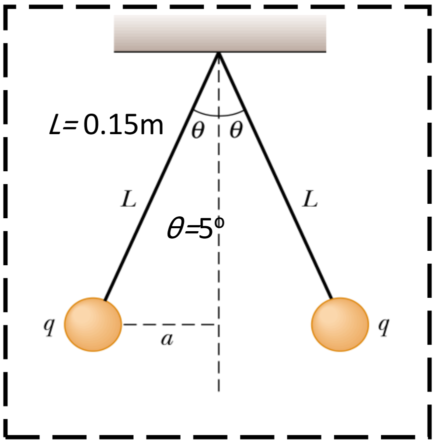
$(2 - x)^2 |q_2| = x^2 |q_1|$

$(2 - x)^2 (6 \times 10^{-6}) = x^2 (15 \times 10^{-6})$

$x = (\quad) \text{ m}$ the coordinate of q_3 from q_2.

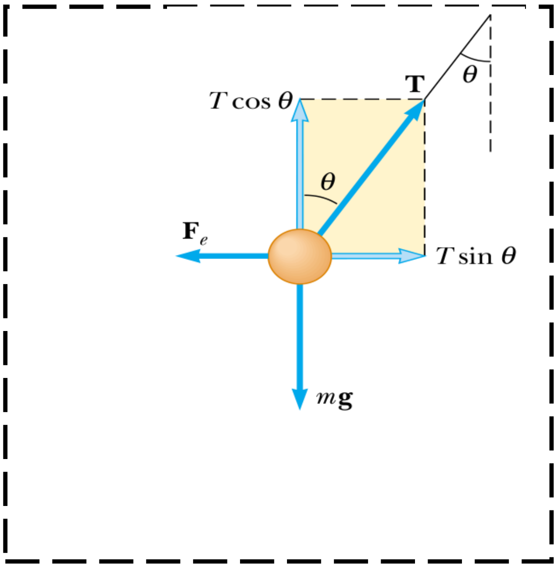
Example4: The Charge on the Spheres

Two identical small charged spheres, each having a mass of 3×10^{-2}kg, hang in equilibrium as shown in figure. The length of each string is 0.15m, and the angle θ is 5°. Find the magnitude of the charge on each sphere.



Solution

By drawing the **free-body diagram** for the left-hand sphere, as in this figure ,the sphere is in **equilibrium** under the effect of the forces, the **tension force T** from the string, the **electric force** Fe from the other sphere, and the **gravitational force mg**.



Because the sphere is in equilibrium, the forces in the horizontal and vertical directions must separately add up to zero:

$$\sum F_x = T\sin\theta - F_e = 0 \quad \text{------- (1)}$$

$$\sum F_y = T\cos\theta - mg = 0 \quad \text{------ (2)}$$

From these results, by divide Eq.(1) on Eq.(2) we can obtain the value of the magnitude of the electric force F_e as:

$F_e = mg \tan\theta = (3 \times 10^{-2})$ (9.8) tan5° $= 2.6 \times 10^{-2}$ N

From the figure, we see that **sin θ = a/L** .Therefore

a = L sin θ = 0.16 sin5° = 0.013m

The separation of the spheres is 2a =0.026m. From Coulomb's law, the magnitude of the electric force is $F_e = k_e \dfrac{|q|^2}{r^2}$

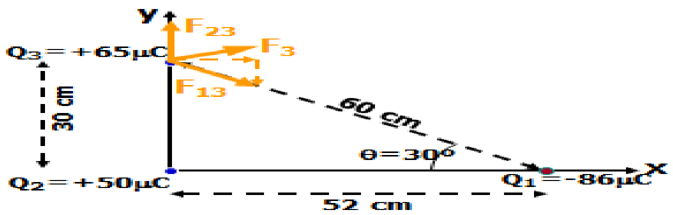
Where **r =2a = 0.026m** and $|q|$ is the magnitude of the charge on each sphere. This equation can be solved for $|q|^2$ to give

$$|q|^2 = \frac{F_e r^2}{k_e} = \frac{(2.6\times10^{-2})(0.026)^2}{9\times10^9} = 1.96\times10^{-15}\,\mathbf{C}^2$$

$$|q| = 4.4\times10^{-8}\,\mathbf{C}$$

Example 5: The Resultant Force

Calculate the net electrostatic force on charge Q_3 due to the charges Q_1 and Q_2 as shown in the figure.



Solution

The magnitude of F_{23} is

$$F_{23} = k_e \frac{|q_2||q_3|}{r^2}$$

$$= 9\times10^9 \frac{(50\times10^{-6})(65\times10^{-6})}{(0.3)^2} = 330N$$

In the coordinate system shown in figure, the **repulsive force F**$_{23}$ is to the **up** (in the positive y direction).

$F_{23,x}$ = 0 N , and $F_{32,y}$ = 330 N.

The magnitude of the force **F**$_{13}$ exerted by q_1 on q_3 is

$$F_{13} = k_e \frac{|q_1||q_3|}{r^2}$$

$$= 9\times10^9 \frac{(86\times10^{-6})(65\times10^{-6})}{(0.6)^2} = 139.75N$$

The **attractive force F**$_{13}$ makes an angle of **30°** with the **x-axis**. Therefore, the **x** and **y** components of **F**$_{13}$ with magnitude given by

$F_{13x} = F_{13} \cos30^o = 120$ N

$F_{13y} = F_{13} \sin30^o = -70$ N

F_{23x}= 120 N, and F_{23y} = -70 N

Combining **F_{13}** with **F_{23}** by the **rules of vector addition**, we arrive at the **x** and **y components** of the resultant force acting on **q_3**:

$F_{3x} = F_{13x} + F_{23x} = 120 + 0 = 120$ N

$F_{3y} = F_{13y} + F_{23y} = (-70) +33\ 0 = 260$ N

The resultant force is $F^2{}_3 = F^2{}_{3x} + F^2{}_{3y}$

$F^2{}_3 = 120)^2 + (260)^2$

$F_3 = (\quad)$ N

tan $\theta = F_{3y} / F_{3x} = 260 / 120 = 2.16$

$\Theta=(\ )\ °$

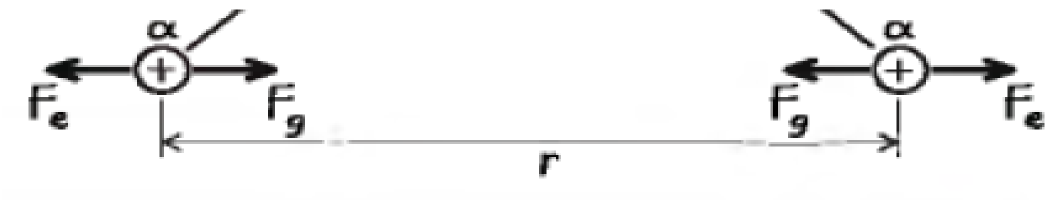
The resultant force acting on q_3 in unit vector form as, **F_3** =(120 I +260J) N

H.W

H.W1: Two balloons are charged with an identical quantity and type of charge (-6.25 nC). They are held apart at a separation distance of (61.7 cm). Determine the magnitude of the electrical force of repulsion between them.

H.W2: Two balloons with charges of +3.37 µC and -8.21 µC attract each other with a force of 0.0626 Newton. Determine the separation distance between the two balloons.

H.W3: Compute the electrostatic force between two alpha particles (nucleus of He) by a distance of approximately 10^{-11}cm, as shown in the figure, and compare with the force of gravitational attraction between them.



H.W4: Consider three point charges located at the corners of a right triangle as shown in the figure, where q_1 = q_3 = 5µC, q_2 =-2µC, and a = 0.1m. Find the resultant force exerted on q_3.