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Second Year Students
1st Semester
2020-2021

Mathematics III

Vectors (Chap. 12)

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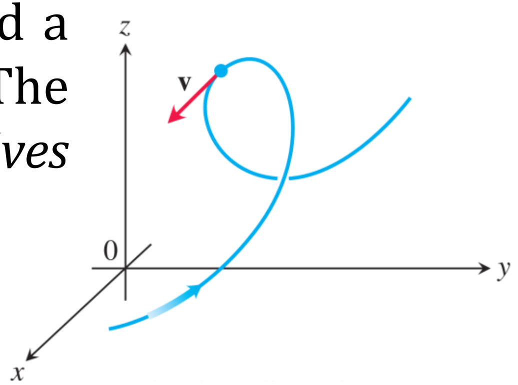
Vectors

- Some of the things we measure are determined simply by their magnitudes.
- To record **mass**, **length**, or **time**, for example, we need only *write down a number and name an appropriate unit of measure*.

- We need more information to describe a **force**, **displacement**, or **velocity** and it is called a **vector**.
- To describe a **force**, *we need to record the direction in which it acts as well as how large it is*.
- To describe a body's **displacement**, we have to *say in what direction it moved as well as how far*.
- To describe a body's **velocity**, *we have to know its direction of motion, as well as how fast it is going*

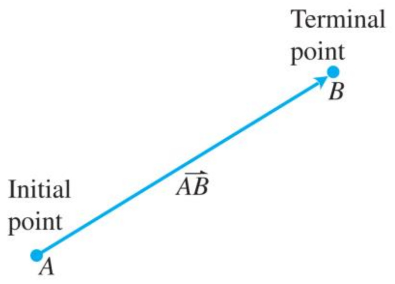
Vectors (Cont.)

A quantity such as *force, displacement*, or *velocity* is called a **vector** and is represented by a *directed line segment.* The *arrow points in the direction of the action* and its *length gives the magnitude of the action.*



DEFINITIONS

- The vector represented by the directed line segment $\overrightarrow{\boldsymbol{AB}}$ has initial point **A** and terminal point **B** and *its length is denoted* by $|\overrightarrow{\boldsymbol{AB}}|$.
- Two vectors are equal if they have the same length and direction.



Vectors (Cont.)

v is a two-dimensional vector in the plane, the component form **v** of is:

$$\mathbf{v} = (v_1, v_2)$$

v is a three-dimensional vector in the space, the component form of **v** is:

$$\mathbf{v} = (v_1, v_2, v_3)$$

v_1 = horizontal component, v_2 = Vertical Component, and v_3 = perpendicular component.

If **v** is two-dimensional with $P(x_1, y_1)$ and $Q(x_2, y_2)$ as points in the plane, then the standard position vector **v** is:

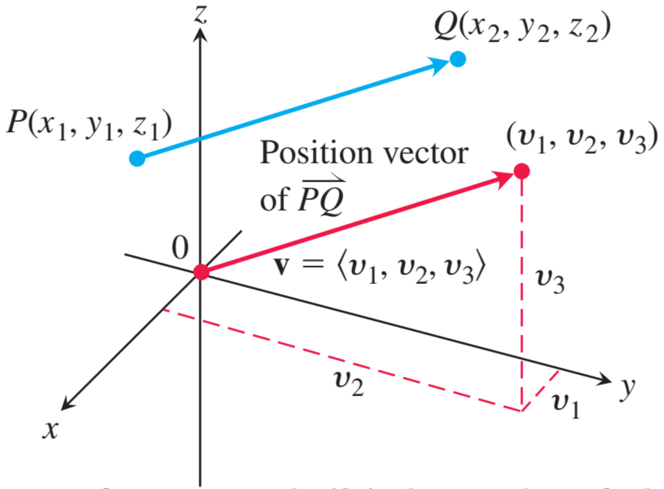
$$\mathbf{v} = (x_2 - x_1, y_2 - y_1)$$

Two vectors are equal if and only if their standard position vectors are identical.

Vectors Length

The **magnitude** or **length** of the vector $\mathbf{v} = \overrightarrow{PQ}$ is the nonnegative number.

$$|\mathbf{v}| = \sqrt{v_1^2 + v_2^2 + v_3^2} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$



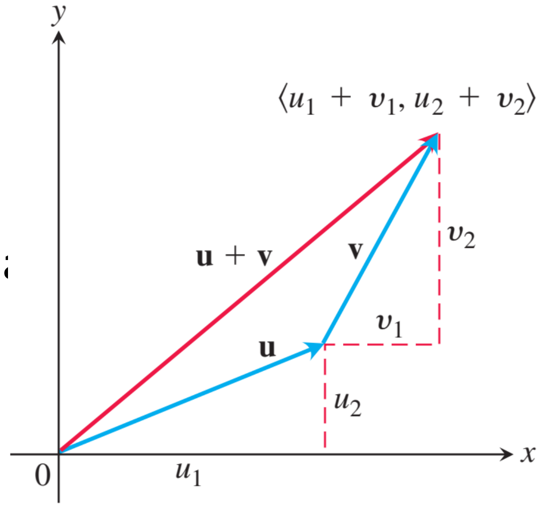
EXAMPLE 1

Find the (**a**) component form and (**b**) length of the vector with initial point P(-3, 4, 1) and terminal point Q(-5, 2, 2).

Vector Algebra Operations

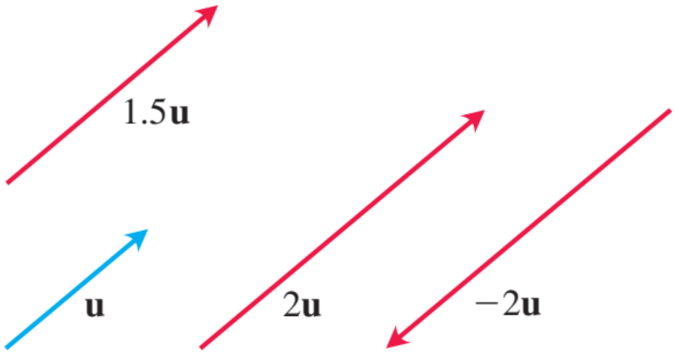
DEFINITIONS

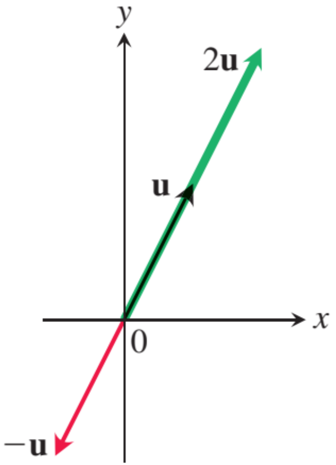
Let $\mathbf{u} = (u_1, u_2, u_3)$ and $\mathbf{v} = (v_1, v_2, v_3)$ be vectors with k a scalar.

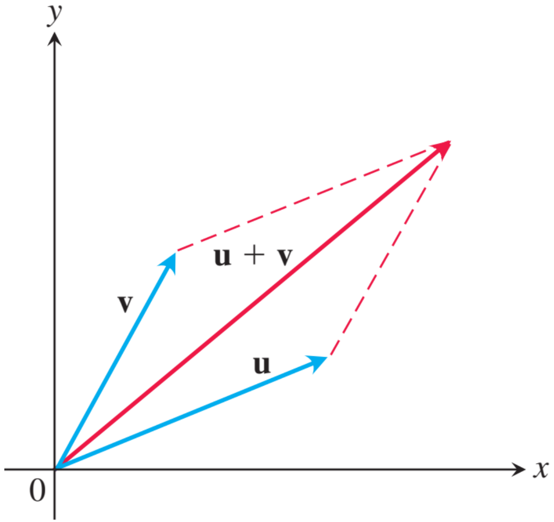


<ins>*Addition*</ins>: $\mathbf{u} + \mathbf{v} = (u_1 + v_1, u_2 + v_2, u_3 + v_3)$

<ins>*scalar multiplication*</ins>: $k\mathbf{u} = (ku_1, ku_2, ku_3)$







Vector Algebra Operations (Cont.)

- product $k\mathbf{u}$ of the scalar k and vector $\mathbf{u}$:

If $k > 0$, then $k\mathbf{u}$ has the same direction as $\mathbf{u}$; if $k < 0$, then the direction of $k\mathbf{u}$ is opposite to that of $\mathbf{u}$. The length of $k\mathbf{u}$ is the absolute value of the scalar k times the length of $\mathbf{u}$.

$$\begin{aligned} |k\mathbf{u}| &= \sqrt{(ku_1)^2 + (ku_2)^2 + (ku_3)^2} = \sqrt{k^2({u_1}^2 + {u_2}^2 + {u_3}^2)} \\ &= \sqrt{k^2}\sqrt{{u_1}^2 + {u_2}^2 + {u_3}^2} = |k||\mathbf{u}|. \end{aligned}$$

- The vector (-1)**u** = -**u** has the same length as **u** but points in the opposite direction.
- The **difference u - v** of two vectors is defined by
  $\mathbf{u} - \mathbf{v} = \mathbf{u} + (-\mathbf{v})$.
  If $\mathbf{u} = (u_1, u_2, u_3)$ and $\mathbf{v} = (v_1, v_2, v_3)$, then $\mathbf{u} - \mathbf{v} = (u_1 - v_1, u_2 - v_2, u_3 - v_3)$

Properties of Vector Operations

Let $\mathbf{u}$, $\mathbf{v}$, $\mathbf{w}$ be vectors and a, b be scalars.

$\mathbf{u} + \mathbf{v} = \mathbf{v} + \mathbf{u}$

$(\mathbf{u} + \mathbf{v}) + \mathbf{w} = \mathbf{u} + (\mathbf{v} + \mathbf{w})$

$\mathbf{u} + \mathbf{0} = \mathbf{u}$

$\mathbf{u} + (-\mathbf{u}) = \mathbf{0}$

$0\ \mathbf{u} = \mathbf{0}$

$1\mathbf{u} = \mathbf{u}$

$a(b\ \mathbf{u}) = (ab)\mathbf{u}$

$a(\mathbf{u} + \mathbf{v}) = a\ \mathbf{u} + a\mathbf{v}$

$(a + b)\ \mathbf{u} = a\mathbf{u} + b\mathbf{u}$

Properties of Vector Operations (Cont.)

We can take advantage of vector properties to make operations, for example:

$$\mathbf{u} + \mathbf{v} = (u_1, u_2, u_3) + (v_1, v_2, v_3)$$
$$= (u_1 + v_1, u_2 + v_2, u_3 + v_3)$$
$$= (v_1 + u_1, v_2 + u_2, v_3 + u_3)$$
$$= (v_1, v_2, v_3) + (u_1, u_2, u_3)$$
$$= \mathbf{v} + \mathbf{u}$$

EXAMPLE 2

Let $\mathbf{u} = (-1, 3, 1)$ and $\mathbf{v} = (4, 7, 0)$. Find the components of

(a) $2\mathbf{u} + 3\mathbf{v}$, (b) $\mathbf{u} - \mathbf{v}$, (c) $\left|\frac{1}{2}\mathbf{u}\right|$.

Unit Vectors

A vector **v** of length 1 is called a **unit vector**. The **standard unit vectors** are:
i = (1, 0, 0), **j** = (0, 1, 0), and **k** = (0, 0, 1)

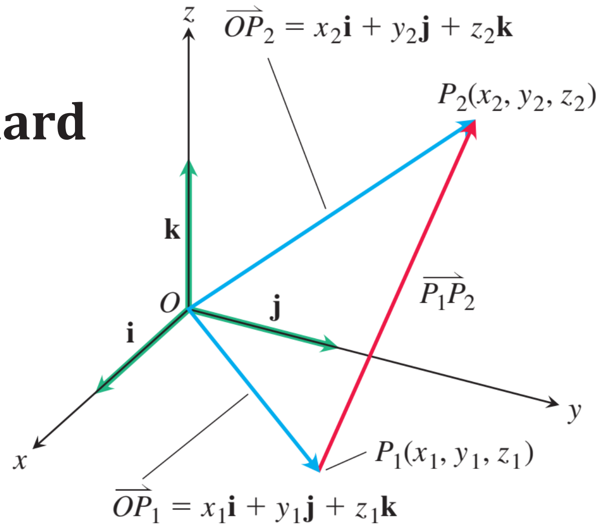
Any vector **v** = (v_1, v_2, v_3) can be written as a *linear combination* of the standard unit vectors as follows:

$$\mathbf{v} = (v_1, v_2, v_3) = (v_1, 0, 0) + (0, v_2, 0) + (0, 0, v_3)$$
$$= v_1(1,0,0) + v_2(0,1,0) + v_3(0,0,1)$$
$$= v_1\mathbf{i} + v_2\,\mathbf{j} + v_3\,\mathbf{k}$$

We call the scalar (or number) v_1 the **i-component** of the vector **v**, v_2 the **j-component**, and v_3 the **k-component**.

(<u>Vector between two points</u>) the component form for the vector from $P_1(x_1, y_1, z_1)$ to $P_2(x_2, y_2, z_2)$ is:

$$\overrightarrow{P_1P_2} = (x_2 - x_1)\mathbf{i} + (y_2 - y_1)\mathbf{j} + (z_2 - z_1)\mathbf{k}$$



Direction of vector

If $\mathbf{v} \neq \mathbf{0}$, then its length $|\mathbf{v}|$ is not zero: $\left| \frac{1}{|\mathbf{v}|} \mathbf{v} \right| = \frac{1}{|\mathbf{v}|} |\mathbf{v}| = 1$

That is, $\mathbf{v}/|\mathbf{v}|$ is a *unit vector* in the direction of **v**, called **the direction** of the nonzero vector **v**.

Summary

If $\mathbf{v} \neq \mathbf{0}$, then

1. $\frac{\mathbf{v}}{|\mathbf{v}|}$ is a unit vector called the **direction** of **v**;

2. the equation $\mathbf{v} = |\mathbf{v}| \frac{\mathbf{v}}{|\mathbf{v}|}$ expresses **v** as its length times its direction.

EXAMPLE 3

Find a unit vector **u** in the direction of the vector from $P_1(1, 0, 1)$ to $P_2(3, 2, 0)$.

Direction of vector (Cont.)

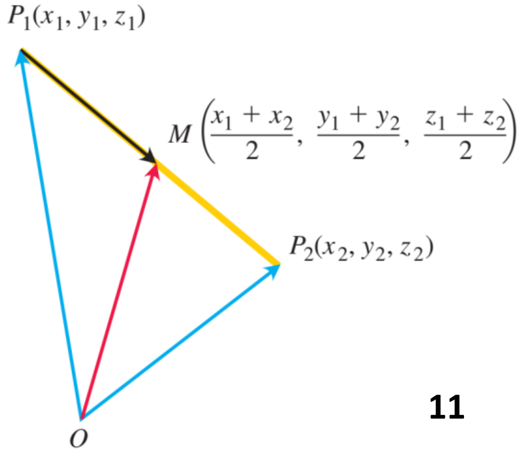
EXAMPLE 4

A force of 6 newtons is applied in the direction of the vector v = 2**i** + 2**j** - **k**. Express the force **F** as a product of its magnitude and direction.

Midpoint of a Line Segment

The **midpoint** M of the line segment joining points $P_1(x_1, y_1, z_1)$ and $P_2(x_2, y_2, z_2)$ is the point:

$$M = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}, \frac{z_1+z_2}{2}\right)$$



EXAMPLE 5

Find the midpoint of the segment joining

$P_1(3, -2, 0)$ and $P_2(7, 4, 4)$