Salahaddin University-Erbil
College of Engineering
Department of Water Resources Engineering
Second Year Students
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Mathematics III

Polar Coordinates (Chap. 10)

5th lecture

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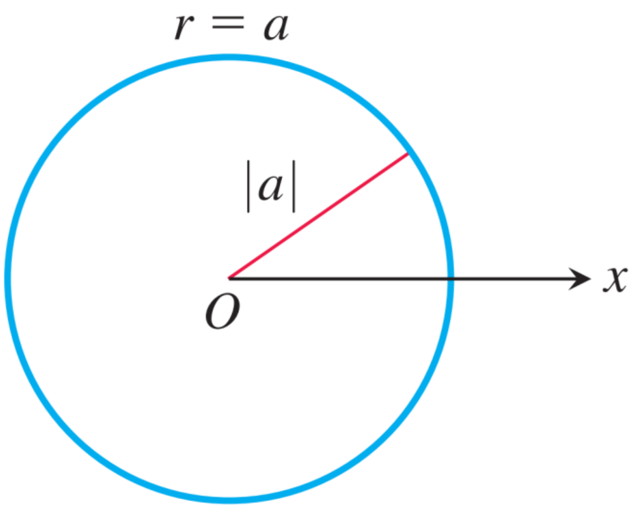
What we learned from previous class

- Polar Coordinates

Polar Equations

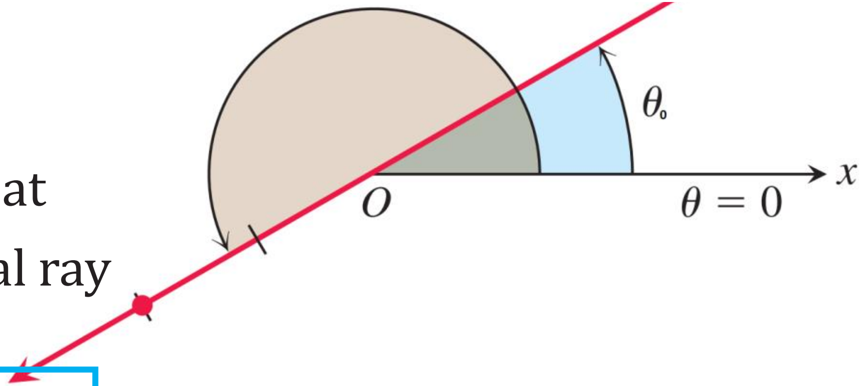


If hold r fixed at a constant value $r = a \neq 0$,

the point $P(r, \theta)$ will lie $|a|$ units from the origin O.

As θ varies over any interval of length 2π

P then traces a circle of radius $|a|$ centered at O



If hold θ fixed at a constant value $\theta = \theta_0$

let r vary between -∞ and ∞,

The point $P(r, \theta)$ traces the line through O that makes an angle of measure θ_0 with the initial ray

$r = a$ circle radius $|a|$ centered at O

$\theta = \theta_0$ Line from O making θ_0 with the initial ray

Polar Equations (Cont.)

EXAMPLE 1

Graph the sets of points whose polar coordinates satisfy the following conditions:

(1) $1 \leq r \leq 2$ and $0 \leq \theta \leq \frac{\pi}{2}$

(2) $-\mathbf{3} \leq r \leq 2$ and $\theta = \frac{\pi}{4}$

(3) $r \leq 0$ and $\theta = \frac{\pi}{4}$

(4) $\frac{2\pi}{3} \leq \theta \leq \frac{5\pi}{6}$ no restriction on r

Relating Polar and Cartesian Coordinates

- Both polar and Cartesian coordinates in a plane, place the two origins together.
- Let the *initial polar ray* be the *positive x-axis*.
- The ray $\theta = \pi/2$, r > 0, becomes the positive y-axis

The two coordinate systems are then related by the following equations.

If x and y are given, the third equation gives two possible choices for r (a positive and a negative value)

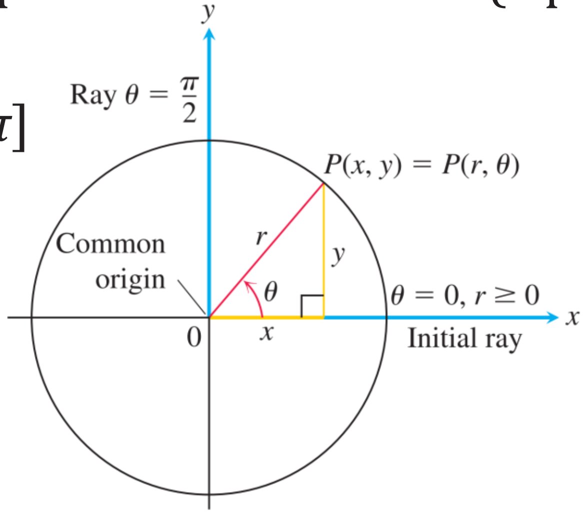
For each $(x, y) \neq (0, 0)$ there is a unique $\theta \in [0, 2\pi]$

$$x = r\cos\theta \quad \text{-------------}(1)$$

$$y = r\sin\theta \quad \text{-------------}(2)$$

$$r^2 = x^2 + y^2 \quad \text{-----------}(3)$$

$$\tan\theta = \frac{y}{x} \quad \text{--------------}(4)$$



Relating Polar and Cartesian Coordinates (Cont.)

EXAMPLE 2

Convert the following Polar equation to Cartesian (x, y) equivalent equations:

Polar equation
(1) $r \cos \theta = 2$
(2) $r^2 \cos \theta \sin \theta = 4$
(3) $r^2 \cos^2 \theta - r^2 \sin^2 \theta = 1$
(4) $r = 1 + 2r \cos \theta$
(5) $r = 1 - \cos \theta$

Relating Polar and Cartesian Coordinates (Cont.)

EXAMPLE 3

Find a polar equation for the circle $x^2 + (y - 3)^2 = 9$

EXAMPLE 4

Replace the following polar equations by equivalent Cartesian equations:

(a) $r \cos \theta = -4$

(b) $r^2 = 4r \cos \theta$

(c) $r = \dfrac{4}{2 \cos \theta - \sin \theta}$