Salahaddin University-Erbil
College of Engineering
Department of Water Resources Engineering
1st Semester
2020-2021

Mathematics III

Differential Equation (Chap. 15*)

11th lecture

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2rd23z

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Ordinary Differential Equation ODE

Many real-world problems, when formulated mathematically, lead to differential equations.

Differential equations are the language of much of physics and Engineering Phenomena.

In general, modeling of the variation of a physical quantity and engineering subjects, such as **temperature**, **pressure**, **displacement**, **velocity**, **stress**, **strain**, **current**, **voltage**, **concentration of a pollutant**, **dynamics of rigid bodies**, **flow of fluids**, **heat transfer**, **mechanical vibration** or **structural dynamic** with *the change of time* or *location*, or *both* would result in differential equations

Ordinary Differential Equation ODE (Cont.)

A differential equation is an equation involving differentials or differential coefficients

$$\frac{dy}{dx} = x^2 - 1 \qquad \ldots(1)$$

$$\frac{d^2 y}{dx^2} + 2\left(\frac{dy}{dx}\right)^2 + y = 0 \qquad \ldots(2)$$

$$(x + y^2 - 3y)\, dx = (x^2 + 3x + y)\, dy \qquad \ldots(3)$$

$$y = x\frac{dy}{dx} + \frac{c}{\frac{dy}{dx}} \qquad \ldots(4)$$

$$\frac{d^3 y}{dx^3} + 2\,\frac{d^2 y}{dx^2} \cdot \frac{dy}{dx} + x^2\left(\frac{dy}{dx}\right)^3 = 0 \qquad \ldots(5)$$

$$\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{\frac{3}{2}} = k \cdot \frac{d^2 y}{dx^2} \qquad \ldots(6)$$

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = nu \qquad \ldots(7)$$

$$\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} = x + y \qquad \ldots(8)$$

Ordinary Differential Equation ODE (Cont.)

- Differential equations which involve only one independent variable and the differential coefficients with respect to it are called **ordinary differential equations**. (equations (1) to (6) are all ordinary differential equations)

- Differential equations which involve two or more independent variables and partial derivatives with respect to them are called **partial differential equations**. (equations (7) and (8) are partial differential equations)

- The **order** of a differential equation is the order of the highest order derivative occurring in the differential equation. (equations (1), (3) and (4) are of first order ; equations (2) and (6) are of the second order while equation (5) is of the third order)

Ordinary Differential Equation ODE (Cont.)

- The **degree** of a differential equation is the degree of the highest order derivative which occurs in the differential equation provided the equation has been made free of the radicals and fractions as far as the derivatives are concerned. (equations (1), (2), (3) and (5) are of the first degree)

Equation 4 is the second degree $y\frac{dy}{dx} = x\left(\frac{dy}{dx}\right)^2 + c$

Equation (6) is of the second degree $\left[1+\left(\frac{dy}{dx}\right)^2\right]^3 = k^2\left(\frac{d^2y}{dx^2}\right)^2$

Order 2

Degree 3

$$\left(\frac{d^2y}{dx^2}\right)^3 + \frac{dy}{dx} + y = 4x^5$$

Ordinary Differential Equation ODE (Cont.)

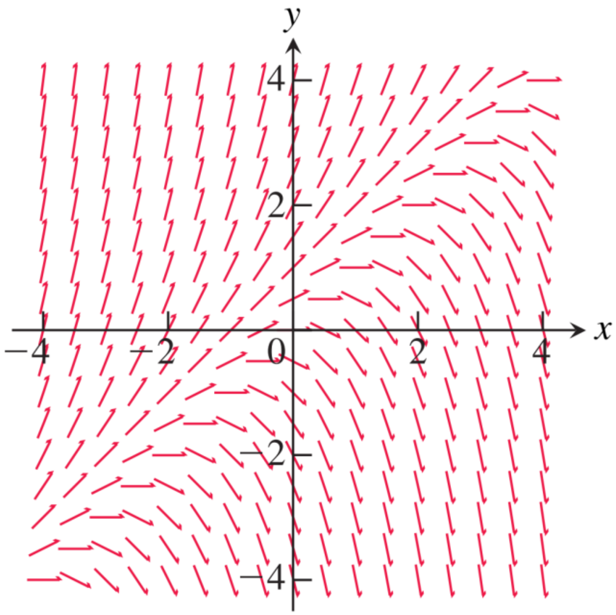
- **Solution of a Differential Equation.** A solution (or integral) of a differential equation is a relation, free from derivatives, between the variables which satisfies the given equation.
  - **The general (or complete) solution** of a differential equation is that in which the number of independent arbitrary constants is equal to the order of the differential equation.

    For example, $y = c_1 \cos x + c_2 \sin x$ (involving two arbitrary constants c_1, c_2) is the general solution of the differential equation $\frac{d^2y}{dx^2} + y = 0$ of second order.

  - **A particular solution** of a differential equation is that which is obtained from its general solution by giving particular values to the arbitrary constants.
  - **A singular solution** of a differential equation is that solution which satisfies the equation but cannot be derived from its general solution.

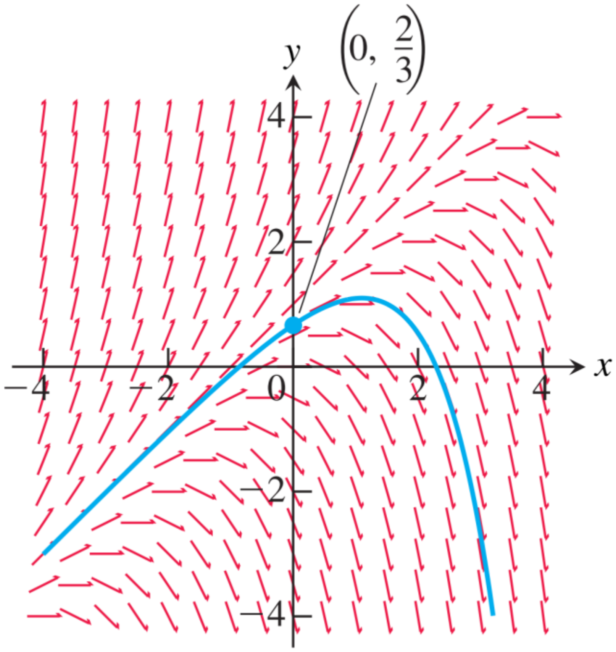
Ordinary Differential Equation ODE (Cont.)

- **Solution of a Differential Equations**



(a)

General Solution of $\frac{dy}{dx} = y - x$



(b)

Particular Solution of $\frac{dy}{dx} = y - x$ at (0, 2/3)

Formation of ODE

Differential equations are formed by elimination of arbitrary constants. Elimination of ***n*** arbitrary constants leads us to ***n***th order derivatives and hence a differential equation of the ***n***th order.

$f(x, y, c_1, c_2, \ldots, c_n)$ equation containing ***n*** arbitrary constants $c_1, c_2, \ldots, c_n$ (sometimes called parameters)

$$\left.\begin{aligned} f_1\left(x, y, c_1, c_2, \cdots, c_n, \frac{dy}{dx}\right) &= 0 \\ f_2\left(x, y, c_1, c_2, \cdots, c_n, \frac{dy}{dx}, \frac{d^2y}{dx^2}\right) &= 0 \\ f_n\left(x, y, c_1, c_2, \cdots, c_n, \frac{dy}{dx}, \frac{d^2y}{dx^2}, \cdots, \frac{d^ny}{dx^n}\right) &= 0 \end{aligned}\right\}$$

<u>Hence</u> *n*th order differential equation has exactly *n* arbitrary constants in its general solution.

$$\phi\left(x, y, \frac{dy}{dx}, \frac{d^2y}{dx^2}, \cdots, \frac{d^ny}{dx^n}\right) = 0$$

which is ***n***th order differential equation

Formation of ODE

EXAMPLE 1

Eliminate the constants from the following equations (Form DE):

1) $y = cx + c^2$

2) $(x - h)^2 + y^2 = h^2$

Solution of 1st Order and Degree ODE

All differential equations of the first order and first degree cannot be solved. Only those among them which belong to one of the following categories can be solved by the standard methods:

1. Equations in which variables are separable.
2. Differential equation of the form $\frac{dy}{dx} = f(ax + by + c)$
3. Homogeneous equations
4. Linear equations
5. Exact equations

Solution by Separation of Variables

If a differential equation of the **first order** and **first degree** *can be put in the form where* ***dx*** *and all terms containing* ***x*** *are at one place*, also ***dy*** *and all terms containing* ***y*** *are at one place*, then the variables are said to be **separable**

Thus the general form of such an equation is $f(x)\,dx + f(y)\,dy = 0$

Integrating, we get $\int f(x)\,dx + \int f(y)\,dy = c$ which is the ***general solution,*** c being an arbitrary constant.

Note. Any equation of the form $f_1(x)\phi_2(y)dx + f_2(x)\phi_1(y)dx = 0$ can be expressed in the above form by dividing throughout by $f_2(x)\phi_2(y)$ thus:

$$\frac{f_1(x)}{f_2(x)}\,dx + \frac{\phi_1(y)}{\phi_2(y)}\,dy = 0 \quad \text{or} \quad f(x)\,dx + \phi(y)\,dy = 0$$

Separation of Variables of ODE

EXAMPLE 2

Solve:

1) $x\,dx - y^2\,dy = 0$

2) $x\cos x\cos y + \sin y\dfrac{dy}{dx} = 0$