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**1<sup>st</sup> Semester**  
**2020-2021**

**Mathematics III**  
**Vectors (Chap. 12)**

Lec 02

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# Unit Vectors

A vector  $\mathbf{v}$  of length 1 is called a **unit vector**. The **standard unit vectors** are:

$$\mathbf{i} = (1, 0, 0), \mathbf{j} = (0, 1, 0), \text{ and } \mathbf{k} = (0, 0, 1)$$

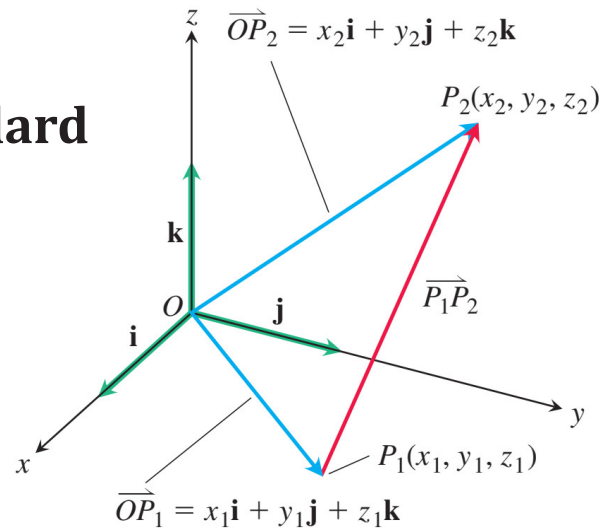
Any vector  $\mathbf{v} = (v_1, v_2, v_3)$  can be written as a *linear combination* of the standard unit vectors as follows:

$$\begin{aligned}\mathbf{v} &= (v_1, v_2, v_3) = (v_1, 0, 0) + (0, v_2, 0) + (0, 0, v_3) \\ &= v_1(1, 0, 0) + v_2(0, 1, 0) + v_3(0, 0, 1) \\ &= v_1\mathbf{i} + v_2\mathbf{j} + v_3\mathbf{k}\end{aligned}$$

We call the scalar (or number)  $v_1$  the **i-component** of the vector  $\mathbf{v}$ ,  $v_2$  the **j-component**, and  $v_3$  the **k-component**.

(Vector between two points) the component form for the vector from  $P_1(x_1, y_1, z_1)$  to  $P_2(x_2, y_2, z_2)$  is:

$$\overrightarrow{P_1P_2} = (x_2 - x_1)\mathbf{i} + (y_2 - y_1)\mathbf{j} + (z_2 - z_1)\mathbf{k}$$



# Direction of vector

If  $\mathbf{v} \neq \mathbf{0}$ , then its length  $|\mathbf{v}|$  is not zero:  $\left| \frac{1}{|\mathbf{v}|} \mathbf{v} \right| = \frac{1}{|\mathbf{v}|} |\mathbf{v}| = 1$

That is,  $\mathbf{v}/|\mathbf{v}|$  is a unit vector in the direction of  $\mathbf{v}$ , called **the direction** of the nonzero vector  $\mathbf{v}$ .

## Summary

If  $\mathbf{v} \neq \mathbf{0}$ , then

1.  $\frac{\mathbf{v}}{|\mathbf{v}|}$  is a unit vector called the **direction** of  $\mathbf{v}$ ;
2. the equation  $\mathbf{v} = |\mathbf{v}| \frac{\mathbf{v}}{|\mathbf{v}|}$  expresses  $\mathbf{v}$  as its length times its direction.

## EXAMPLE 3

Find a unit vector  $\mathbf{u}$  in the direction of the vector from  $P_1(1, 0, 1)$  to  $P_2(3, 2, 0)$ .

# Direction of vector (Cont.)

## EXAMPLE 4

A force of 6 newtons is applied in the direction of the vector  $\mathbf{v} = 2\mathbf{i} + 2\mathbf{j} - \mathbf{k}$ . Express the force  $\mathbf{F}$  as a product of its magnitude and direction.

# Midpoint of a Line Segment

The **midpoint**  $M$  of the line segment joining points  $P_1(x_1, y_1, z_1)$  and  $P_2(x_2, y_2, z_2)$  is the point:

$$M = \left( \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}, \frac{z_1 + z_2}{2} \right)$$

## EXAMPLE 5

Find the midpoint of the segment joining

$P_1(3, -2, 0)$  and  $P_2(7, 4, 4)$

