

Salahaddin University-Erbil
College of Engineering
Department of Water Resources Engineering
First Year Students
2nd Semester



Mathematics II

LEC-01

Transcendental Function

Sarkawt H. Muhammad

Sarkawt.muhammad@su.edu.krd

Introduction

- Functions that are *not algebraic* are called *transcendental*.
- The *trigonometric*, *exponential*, *logarithmic*, and *hyperbolic* functions are transcendental, as are their *inverses*.
- Transcendental functions occur frequently in many calculus settings and applications, including growths of populations, vibrations and waves, efficiencies of computer algorithms, and the stability of engineered structures.

The Inverse Function

Definition

- Suppose that f is a function on a domain \mathbf{D} with range \mathbf{R} . The inverse function f^{-1} is defined by

$$f^{-1}(a) = b \quad \text{if} \quad f(b) = a$$

The domain of f^{-1} is \mathbf{R} and the range of f^{-1} is \mathbf{D}

- The Domains and Ranges of f and f^{-1} are interchanged.

The Inverse Function

The process of passing from f to f^{-1} can be summarized as a two-step process:

1. Solve the equation for x . This gives a formula where x is expressed as a function of y .
2. Interchange x and y , obtaining a formula where y is expressed in the conventional format with x as the independent variable and y as the dependent variable.

Example (1):

Find an inverse function for the following functions:

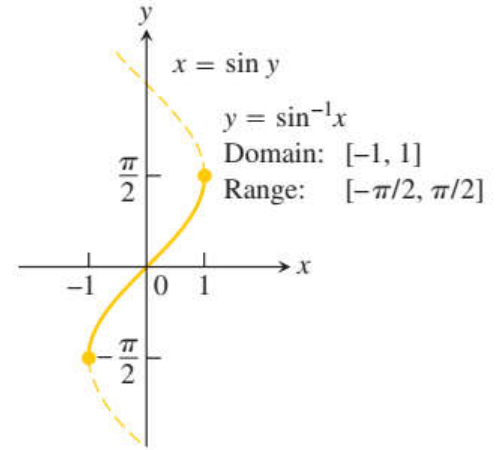
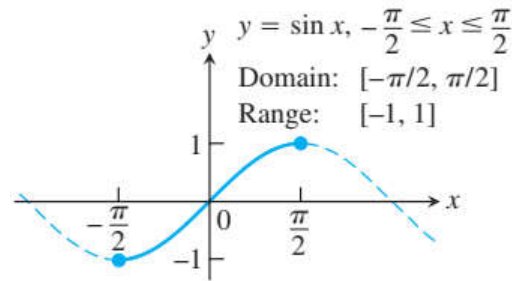
1) $y = \sqrt{x}$

2) $y = \frac{1}{2}x + 1$

3) $y = 8x^3$

The inverse of Trigonometric Function

- The Arcsine (\sin^{-1})



Common values of \sin^{-1}

- $\sin^{-1} 0 = 0$
- $\sin^{-1} \frac{\sqrt{3}}{2} = \frac{\pi}{3}$
- $\sin^{-1} 1 = \frac{\pi}{2}$
- $\sin^{-1} \frac{\sqrt{2}}{2} = \frac{\pi}{4}$

The inverse of Trigonometric Function(Cont.)

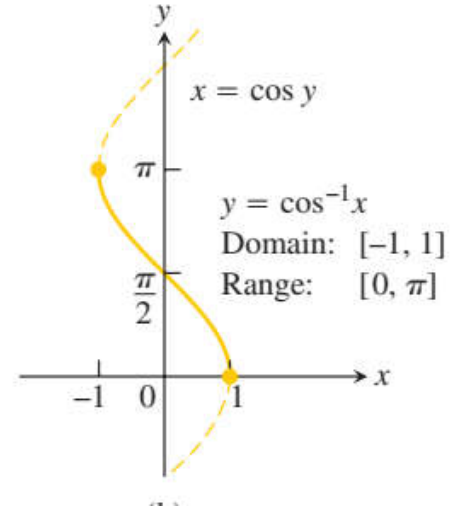
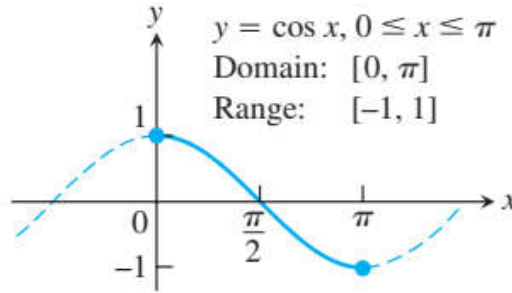
- The **Arccosine** (\cos^{-1})

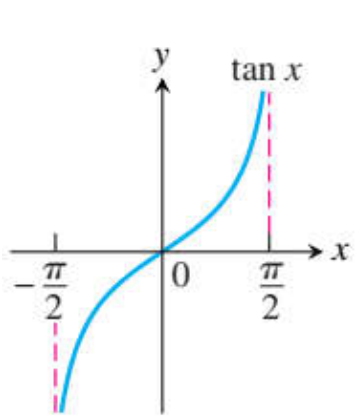
- Common values of \cos^{-1}

- $\cos^{-1} \frac{\pi}{6} = \frac{\sqrt{3}}{2}$

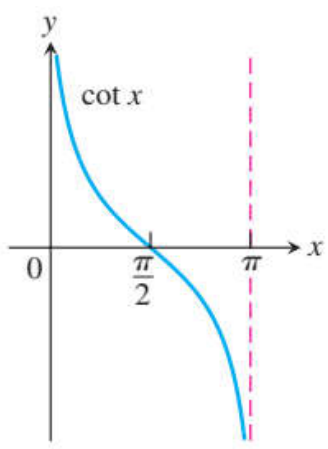
- $\cos^{-1} \frac{1}{\sqrt{2}} = \frac{\pi}{4}$

- $\cos^{-1} \left(-\frac{1}{2}\right) = \frac{2}{3}\pi$

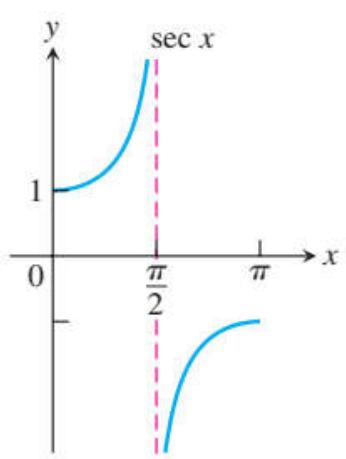




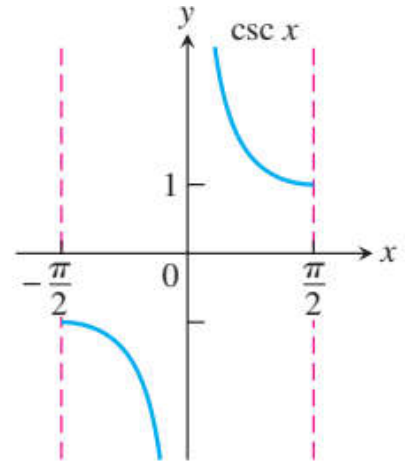
$\tan x$ $D(-\pi/2, \pi/2)$ $R(-\infty, \infty)$



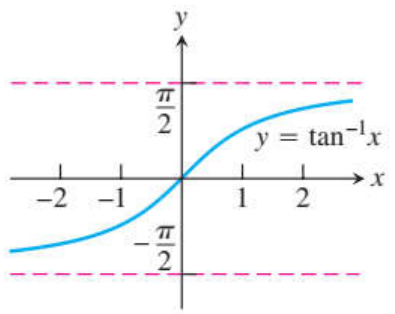
$\cot x$ $D(0, \pi)$ $R(-\infty, \infty)$



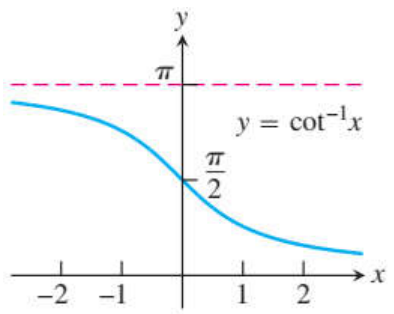
$\sec x$ $D[0, \pi/2) \cup (\pi/2, \pi]$ $R(-\infty, -1] \cup [1, \infty)$



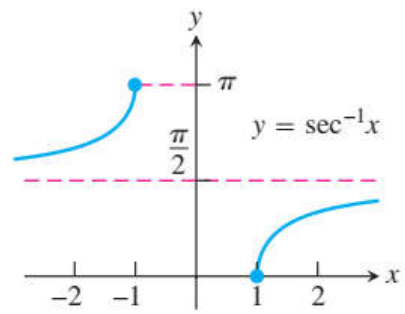
$\csc x$ $D[-\pi/2, 0) \cup (0, \pi/2]$ $R(-\infty, -1] \cup [1, \infty)$



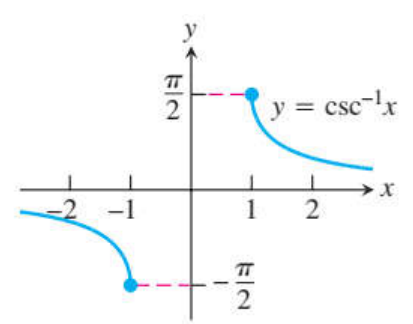
$\tan^{-1} x$ $D(-\infty, \infty)$ $R(-\pi/2, \pi/2)$



$\cot^{-1} x$ $D(-\infty, \infty)$ $R(0, \pi)$

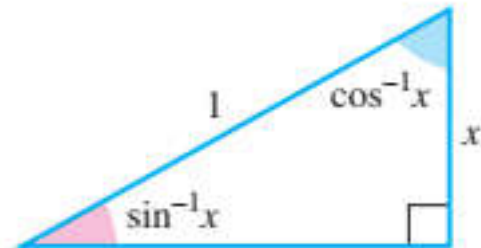


$\sec^{-1} x$ $D(-\infty, -1] \cup [1, \infty)$ $R[0, \pi/2) \cup (\pi/2, \pi]$



$\csc^{-1} x$ $D(-\infty, -1] \cup [1, \infty)$ $R[-\pi/2, 0) \cup (0, \pi/2]$

$$\sin^{-1} x + \cos^{-1} x = \pi/2$$



- There is no general agreement about how to define $\sec^{-1} x$ for negative values of x , so:

$$\sec^{-1} x = \cos^{-1} \frac{1}{x}$$

$$\csc^{-1} x = \sin^{-1} \frac{1}{x}$$

Derivative of Inverse Trigonometric Function

Derivatives:

$$\frac{d}{dx} (\sin^{-1} u) = \frac{1}{\sqrt{1-u^2}} \frac{du}{dx}$$

$$\frac{d}{dx} (\cos^{-1} u) = -\frac{1}{\sqrt{1-u^2}} \frac{du}{dx}$$

$$\frac{d}{dx} (\tan^{-1} u) = \frac{1}{1+u^2} \frac{du}{dx}$$

$$\frac{d}{dx} (\cot^{-1} u) = -\frac{1}{1+u^2} \frac{du}{dx}$$

$$\frac{d}{dx} (\sec^{-1} u) = \frac{1}{|u|\sqrt{u^2-1}} \frac{du}{dx}$$

$$\frac{d}{dx} (\csc^{-1} u) = -\frac{1}{|u|\sqrt{u^2-1}} \frac{du}{dx}$$

Derivative of Inverse Trigonometric Function

Differentials:

$$d(\sin^{-1} u) = \frac{du}{\sqrt{1-u^2}}$$

$$d(\cos^{-1} u) = -\frac{du}{\sqrt{1-u^2}}$$

$$d(\tan^{-1} u) = \frac{du}{1+u^2}$$

$$d(\cot^{-1} u) = -\frac{du}{1+u^2}$$

$$d(\sec^{-1} u) = \frac{du}{|u|\sqrt{u^2-1}}$$

$$d(\csc^{-1} u) = -\frac{du}{|u|\sqrt{u^2-1}}$$

Derivative of Inverse Trigonometric Function

Example (1):

Find derivative of the following inverse function:

$$y = \sin^{-1} x$$

$$y = \cos^{-1} x$$

$$y = \tan^{-1} x$$

$$y = \sec^{-1} x$$

Derivative of Inverse Trigonometric Function

Example (2):

Find dy/dx for the followings:

$$y = \sin^{-1} x^2$$

$$y = \tan^{-1} \sqrt{x + 1}$$

$$y = \sec^{-1} 3x$$