Salahaddin University-Erbil
College of Engineering
Department of Chemical Engineering
First Year Students
2nd Semester

Mathematics II
LEC-*04*
Transcendental Function

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Natural Logarithm **(ln)**

The natural logarithm of a positive number x, written as $ln\ x$, is the value of an integral.

Definition:

$$\ln x = \int_1^x \frac{1}{t}\,\mathrm{dt}, \qquad \mathrm{x} > 0$$

If $x > 1$, then $\mathbf{ln}x$ is the area under the curve $y=1/t$ from $t=1$ to $t=x$.

For $0 < x < 1$, $\mathbf{ln}x$ gives the negative of the area under the curve from x to1. The function is not defined for $x \leq 0$.

The Zero Width Interval Rule for definite integrals

$$\ln 1 = \int_1^1 \frac{1}{t}\,dt = 0$$

Natural Logarithm **(ln)**

Notice that we show the graph of $y = 1/x$ but use $y = 1/t$ in the integral.

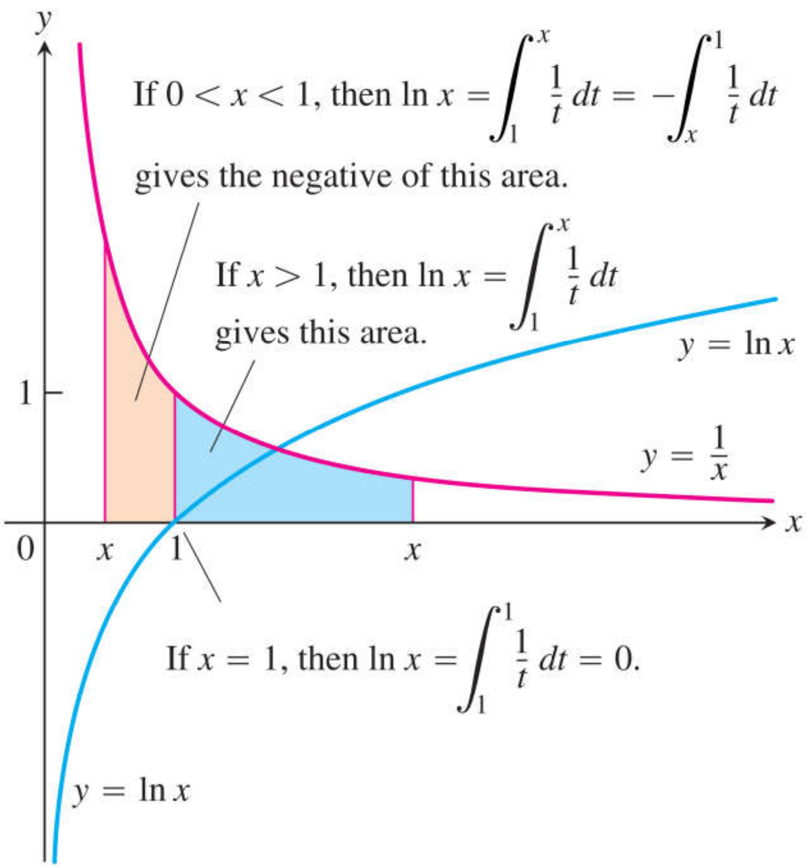
Using x for everything would have us writing.

$$\ln x = \int_1^x \frac{1}{x}\,dx$$

The Number (e) is that number in the domain of the natural logarithm satisfying:

$$\ln(e) = 1$$

the area under the graph of $y = 1/t$ and above the interval $[1, e]$ is the exact area of the unit square {in the figure, the shaded area from 1 to x}, $e = 2.71828.......$



Derivative of $y = \ln x$

$$\frac{d}{dx}\ln x = \frac{d}{dx}\int_1^x \frac{1}{t}\,dt = \frac{1}{x}$$

$$\frac{d}{dx}\ln x = \frac{1}{x}$$

From Chain Rule $\frac{dy}{dx} = \frac{dy}{du} . \frac{du}{dx}$

$$\frac{d}{dx}\ln u = \frac{1}{u}\frac{du}{dx} \qquad u > 0$$

Natural Logarithm **(ln)**

Example (1):

Derivatives of Natural Logarithms

1) $\ln 2x$

2) $\ln(x^2 + 3)$

Properties Natural Logarithm **(ln)**

For any number a > 0 and $x > 0$, the natural logarith satisfies the following rules:

1. Product Rule: $\ln ax = \ln a + \ln x$

2. Quotient Rule: $\ln \frac{a}{x} = \ln a - \ln x$

3. Reciprocal Rule: $\ln \frac{1}{x} = -\ln x$ Rule 2 with a=1

4. Power Rule: $\ln x^r = r \ln x$ r rational

Properties Natural Logarithm **(ln)**

Example (2):

Interpreting the properties of logarithms

1) $\ln 6$
2) $\ln 4 - \ln 5$
3) $\ln \frac{1}{8}$
4) $\ln 4 + \ln \sin x$
5) $\ln \frac{x+1}{2x-3}$
6) $\ln \sec x$
7) $\ln \sqrt[3]{x+1}$

Example (3):

Proof that $\ln ax = \ln a + \ln x$

Integral of Natural Logarithm **(ln)**

When u is a positive differentiable function

$$\int \frac{1}{u} du = \ln u + C$$

But if u is negative

$$\int \frac{1}{u} du = \int \frac{1}{-u} d(-u) = \ln(-u) + C$$

We can combine both equations for +ve and –ve

If u is a differentiable function that is never zero $(u \neq 0)$

$$\int \frac{1}{u} du = \ln|u| + C$$

$$\int u^n \, du = \frac{u^{n+1}}{n+1} + c \qquad n \neq -1$$

Integral of Natural Logarithm **(ln)**

Example (4):

Evaluate the following integrals:

$$\int_0^2 \frac{2x}{x^2 - 5}\,dx$$

$$\int_{-\pi/2}^{\pi/2} \frac{4\cos\theta}{3 + 2\sin\theta}\,d\theta$$

Natural Logarithm **(ln)**

Example (5):

Simplify: $\ln \sqrt[3]{25}$

Example (6):

Integrate: $\displaystyle\int \frac{6x}{3x^2+4}\,dx$

Example (7):

Differentiate:

(1) $y = \ln \dfrac{x\sqrt{x+25}}{(x-1)^3}$

(2) $y^{2/3} = \dfrac{(x^2+1)(3x+4)^{\frac{1}{2}}}{\sqrt[3]{(2x-3)(x^2-4)}}$

The integrals of $\tan x$ and $\cot x$

$$\int \tan u \; du = -\ln|\cos u| + C = \ln|\sec u| + C$$

$$\int \cot u \; du = \ln|\sin u| + C = -\ln|\csc u| + C$$

Example (8):

Evaluate:

$$\int_{0}^{\pi/6} \tan 2x \, dx$$

More Examples

Class Activity:

Evaluate: $$\int \frac{\sec y \tan y}{2 + \sec y}\, dy$$