Gamma and Beta Function and Its Application in Physics: A Review

Abstract

Formulas in gamma reschedule as well as hypergeometric properties are very good rates for mathematicians as well as researchers, as well as newly proven id entities for these tasks that help find services for different formulas as well as separators. We will map a brief background for the progress of From the beginning to the present extra years, gamma and logarithm works, exhibit tight links among them, and provide a variety of beneficial housing traits and identities, in this review. While continuous research on hypergeometric identity has produced numerous novel results, some of these can be found in a recognized id type, according to our literary assessments. For this aim, we're primarily concerned with computer algorithms that have been created to create and check such identities, as well as to seek for invention and be right. Existence in mathematics. The gamma matrix task is alive because of its interactions with special Differential equations in matrices and other matrix characteristics Engineers are one of Matrix's primary efforts. Matrix works with projected programs that have significant work in science. The beta function was the first known scattering amplitude in string theory. It also arises in the theory of the preferred attachment process, which is a kind for stochastic urn process. This incomplete beta equation substitutes that definite integral with an indeterminate integral in an extension of the beta function. Special functions are common in mathematical analysis and lend themselves well to Applications in science and mathematics Of the special functions, the gamma report will comprise to be the most often used. The purpose of this thesis is to examine the gamma function's numerous characteristics and use them, together with its definition, to deduce and solve various integration issues that arise often in applications. It is important to note that we will be dissatisfied if simple tactics such as replacement and part integration are used to tackle the bulk of integration challenges. As a result, the relevance of the gamma function cannot be overstated. The mathematical computation of this effort will be the topic of this essay. In matrix mode, the well-known gamma scale approach is carefully expanded. There are numerous tough issues in this expansion, as well as numerous ways for counting matrix activities. Much more should be included in styles that are far more effective than they are now. We also present a third approach based on the common gamma properties that are disclosed for the other two carefully matched techniques, with the benefit of recreating matrix stamina and the weaknesses of these offered approaches, they set an example of mathematics. The limitations that cross the boundary for errors, as well as other limitations related with that Gamma Matrix feature, has undoubtedly beem examined.

Keyword: gamma function, beta function , special function , hypergeometric, integral.

1 Introduction

A equation is a mathematical connection between a number of objects and a set of allowable outputs in which each input corresponds to exactly one result. Each real number x is connected to its square by this function x^2 is a simple example. f (x) denotes the a functions f output matching to an input x. (It's pronounced " f of x ".) In most branches for modern mathematics, "the primary objectives of investigation" was that diverse functions. A function can be described or represented in a variety of ways. A function and procedure the specifies what is the best way to calculate the output for a given input may be used to define some functions(Carlson 1977). Others are represented by a graph, sometimes known as the function graph. In mathematics, a list that specifies the outcomes generates a collection of sources is sometimes used to define a function. An implicit description of a function is also possible. In circumstances when the output is a number, addition, subtraction, multiplication, and division of functions can be defined similarly to arithmetic. Functional composing, which converts the outcome with one functions into the inputs of the other, is another important function activity. Special functions have more or less standardized names and notes due to their significance in quantitative equations, functional analysis, physics, and other applications. It's frequently named after an early researcher who discovered its qualities(Freeden and Gutting 2013). Special functions include the gamma function, beta structure, super geometry feature, elliptic function, and geometrical function. In the year 1720, Euler launched his principal industrial endeavor in the realm of calculus. He then went on to discuss a slew of additional common special functions. He discovered that the gamma equation is a continuation of the factorial function. In addition, he devised Bessel functions for studying circular drums. Then he looked at elliptic integrals in a methodical way. He didn't name the functions very often. However, as time went on, more and more of the functions he mentioned began to be employed by several people. After a few repetitions, he began to have defined notations and definable notations. Properties that are important enough to provide their names with 'Special Properties'. These include well-known logarithmic work, speeds and trigonometric works, as well as access to gamma, beta and characteristics of zeta function, crotch, as well as the characteristics of the alligoric pipe pipe, as well as its polynomial courses(Carlson 1977). Ortahogonal, among many others, has a significant area of these tasks, including a number of solutions and identities used by mathematicians and designers as well as physicists, and has great special work in clean mathematics, except for the areas used as ten Negatology, electrical presence, liquid properties, hot media, wave equation solving, minutes of powerlessness, and quantum automatic mechanics (Bell 2004). Applied problems constantly require a property solution related to properties, otherwise only in connection with a change, and this type of option will fully participate in the nature of the hypergeometric property parameter. After that, hypergeometric features can be used to solve physical problems in various applied mathematical locations(Temme 1996). In the pure mathematics sector, they explain that hypergeometric selection provides a principle of non-integration to deal with a communal binary range, so playing a valuable task especially in mixing(Wimp 1965). Hypergeometric actions are actually shown in a more effective way that it has solutions in group concepts, mathematical size, K-Jaberi theory, and formal industry theory. The wide q-hypergeometric set is actually related to epilepsy and titanic action, and is actually useful in the idea of distribution, discriminatory equations and algebra(Seaborn 2013). Mathematicians and scientists are interested in formulae with hypergeometric properties, and recently discovered identities for these actions help uncover choices for a variety of varied and fundamental formulas. There are many such identities, images, and repairs for hypergeometric qualities, general content (Prudnikov, Brychkov et al. 1990) providing over 400 complements and also picturing collections for these activities As a result, hypergeometric challenges provide a rich industry for ongoing education, which continues to yield new ultimate outcomes(Prudnikov, Brychkov et al. 1990). The gamma function is important to the notion of hyperg. If the hypergeometric function is at the heart of the concept of a special function, the gamma function is at the core of the notion of hyperg. According to Davis, "in the jaberi tasks that are "in such a state," the gamma attribute is "probably the most vivid (Davis 1959) Factory rise establishes a direct connection between gamma and hypergeometric functions, as well as a number of hypergeometric identities that may be provided more often in gamma action terms. "Gamma's actions and completeness are truly important to understanding the hypergeometric jobs," Andrew and Setre claim(Andrews, Askey et al. 1999) .As a consequence, our specialists' gamma action groups, as well as the analysis and reward for various picture inspections, swiftly transition certain fundamental notions and underlying ideas to other mathematical areas, paving the way for ultimate outcomes in gamma mixing and hypergeometric work. Gamma's activity has a lengthy and interesting history, and it is now almost employed as a factor characteristic in scientific study. He describes how this work began as a simple childbirth from The Euler's perspective to the natural resources factory, and is now progressing (Chaudhry, Qadir et al. 1997).Our experts provide a number of widely used photographs and identities, particularly those with hypergeometric characteristics, as well as how interesting use is being used (Parmar 2013).Indeed, having these classic hypergeometric findings gives our organization a review of literary customers for more recent queries. The focus of this customer assessment is on how well the new hypergeometric ID is planned. Because of the 1970s trend to look for this sort of spread, N skin of this sort of spread is being studied. The computer system's formulae were utilized to generate, match, and assess various sorts of identities. A couple of these mathematical approaches are provided by our professionals ((Kuczma 1968, Simsek 2015)).

2 Generalization gamma and beta equation

Gamma characteristic has actually of be started for the Leonardo Euler in 1729, as it was added in a different term and eventually assembled together as an untrue importance of certainty, such as (Arfken and Weber 1999)

$$\Gamma(n) = \int_0^\infty x^{n-1} e^{-x} dx \qquad Re(n) > 0 \qquad (1)$$

Gamma's work has many amazing features and is utilized in almost every aspect of scientific inquiry as well as design. A year later, Euler offered to speak about beta work for a large sum of money, as well as b with the true helpful structure(Arfken and Weber 1999).

$$\beta(p,q) = \int_0^1 x^{p-1}(1-x)^{q-1}\, dx \qquad Re(p) > 0, Re(q) > 0 \qquad (2)$$

Beta function is a symmetric $\beta(p,q) = \beta(p,q)$, as well as its own relationship to the gamma feature

$$\beta = \frac{\Gamma(p)\Gamma(q)}{\Gamma(p+q)} \qquad (3)$$

In the concept of analysis cycle, gamma and beta properties are actually widely used. Using gamma suspended and beta functions, gamma and beta quality features are generally described.

The domains of the gamma and beta equations have recently been spread across the entire complex plane by introducing integrands of (1) and (2), the factors exp(-σ/x) and exp[-σ/x(1-x)], respectively, where Re(σ) > 0. The equation so defined have been named extended gamma equation and extended beta equation. The extended gamma equation defined Γ(n,σ) as

$$\Gamma(n,\sigma) = \int_0^\infty x^{n-1} e^{(-x-\frac{\sigma}{x})} dx \qquad (4)$$

Where Re(σ) > 0 and n is any complex number. for Re(n) > 0 and σ=0,its clean the above extention for this gamma equation decreases near gamma (classical) equation, $\Gamma(n,0) = \Gamma(n)$. The extended gamma equation, which is a specific instance of the generalized gamma equation (extended), has shown to be particularly beneficial in a variety of engineering and scientific problems. (Chaudhry, Qadir et al. 2004)

$$\Gamma(p,q;\sigma) = \int_0^1 x^{p-1}(1-x)^{q-1} e^{[-\frac{\sigma}{x(1-x)}]} \qquad (5)$$

Where Re(σ) > 0 and parameters p and q are complex numbers. When σ = 0, it is clear that for Re(p) > 0 and Re(q) > 0 that extended beta equation reduces to classical beta equation $\beta(p,q)$. Recently have further generalized the extended gamma and beta equation

$$\Gamma^{(n,\beta)}(n,\sigma) = \int_0^\infty x^{n-1} \Phi(n,\beta,-x-\frac{\sigma}{x})\, dx \qquad (6)$$

$$\Gamma^{(p,\beta)}(p,q,\sigma) = \int_0^\infty x^{p-1}[1-x]^{q-1}\Phi(p,\beta,-\frac{\sigma}{x(1-x)})\,dx \tag{7}$$

Where $\Phi(n,\beta)$ is the type 1 conAuent hypergeometric equation. The gamma equation, the extended gamma equation , the beta equation, the extended beta equation The gamma and beta distributions have been adapted to the matrix case in a variety of methods, and some of their features have been discovered. The gamma outs, on the other hand, are long, with extended beta functions specified in (6) and (7) to the matrix scenario that have yet to be identified and evaluated. As a result, there is a lot of interest in assigning extended gamma and beta functions to matrix instances and exploring their topologies. (Chaudhry et al., 1997), acquiring a variety of important images and linking these circulars with other circulars A cause of conflict was identified as a functional exclusivity. Bargains having certain well-known meanings and outcomes in matrix algebra, multi-border regions, and also unique source difference characteristics. The beta-changing feature expanded matrix's meaning and numerous vital pictures Some integrations, such as multi-border areas and generalized large-scale variable matrix beta feature distributions of total generalized generalized Wishart sources, have been calculated using a long generalized extended source variable beta function.

3 Gamma function

If $n > 0$, that gamma equation Γ is given by the erroneous integral

$$\Gamma(n) = \int_0^\infty x^{n-1}\,e^{-x}\,dx \tag{8}$$

This integral converges for all $n > 0$.

It is sometimes called the second Eulerian integral

3.1 History of gamma function

The gamma equation Γ(x) is almost as common in exact sciences as the well-known factorial sign x! It was proposed by the great mathematician L. Euler (1729) as a logical expansion of the factorial operation x! from positive integers x to real and even complex numbers(Artin 1964). The gamma function Γ(x) has been studied and employed since 1729, when Euler derived his famous integral version of the factorial function. The history of the gamma function is discussed in the section "Gamma function," paragraph "General." Mathematicians have employed the logarithm of the gamma function log(Γ(x)) for their studies of the gamma function since J. Stirling (1730), who was the first to utilize series for log Γ(x!) to develop the asymptotic formula for x! Γ(x). Legendre, K. F. Gauss, C. Siegel, A. M. Siegel, A. M. Siegel, A. M. Siegel, A. M. Siegel, A. M J. Malmstén, O. Schlömilch, J. P. M. Binet

(1843), E. E. Kummer (1847), and G. Plana are among the renowned investigators (1847). Stirling's series for the log(Γ(x)) derivative (Erdglyi, Magnus et al.). The function log Γ(x) was used in a number of research that used or studied the gamma function during the twentieth century. The rise of computer systems towards the end of the twentieth century necessitated a greater focus on the form of branch cuts for fundamental mathematical functions in order to ensure the validity of mathematical connections throughout the complex plane. As a result, the log-gamma function log Γ(x) arose as a multivalued analytic function that is similar to the logarithm of the gamma function log(Γ(x)) but has a distinct branch cut structure and primary sheet. For Mathematica, J. Keiper (1990) designed the log-gamma function Γ(x). There are several identities a (Moll 2007).

3.2 Property of gamma function

1-If n > 0 then $\Gamma(n+1) = n\Gamma(n)$

Proof:

The gamma function definition

$$\Gamma(n) = \int_0^\infty x^{n-1}\, e^{-x}\, dx$$

If 0 < a < b, we can the integral part

$$\int_a^b x^n\, e^{-x}\, dx = [x(-e^{-x})]_a^b - \int_a^b n x^{n-1}(-1)e^{-x}\, dx$$

$$= -b^n e^{-b} + a^n e^{-a} + n\int_a^b x^{n-1} e^{-x}\, dx$$

Take the limit of this equation a→0 and b→∞ to get

$$= \Gamma(n+1) = n\int_0^\infty x^{n-1}\, e^{-x}\, dx = n\Gamma(n)$$

2- if n > 0, then

$$\frac{\Gamma(n)}{z^n} = \int_0^\infty e^{-zx}\, x^{n-1}\, dx$$

Prove:

We know that

$$\Gamma(n) = \int_0^\infty x^{n-1}\, e^{-x}\, dx$$

On putting x = az we get

$$\begin{aligned}\Gamma(n) &= \int_0^{\infty} e^{-az}(az)^{n-1} a\, dz \\ &= a^n \int_0^{\infty} e^{-az} z^{n-1} a\, dz \\ &= a^n \int_0^{\infty} e^{-ax} x^{n-1}\, dx\end{aligned}$$

Replacing a by z above equation we get

$$\frac{\Gamma(n)}{z^n} = \int_0^{\infty} e^{-zx}\, x^{n-1}\, dx$$

3- $$\Gamma\left(\tfrac{1}{2}\right) = \sqrt{\pi}$$

Prove: by definition gamma function

Put $x^n = y$ and $nx^{n-1}\, dx = dy$, we get

$$\begin{aligned}\Gamma(n) &= \int_0^{\infty} y^{n-1/n} e^{-y^{1/n}} \frac{1}{nx^{n-1}}\, dy \\ &= \int_0^{\infty} y^{n-1/n} e^{-y^{1/n}} \frac{1}{ny^{n-1/n}}\, dy \\ \Gamma(n) &= \frac{1}{n} \int_0^{\infty} e^{-y^{1/n}}\, dy\end{aligned}$$

Now pu n = 1/2 in above equation we get

$$\Gamma\left(\tfrac{1}{2}\right) = \tfrac{1}{2}\int_0^\infty e^{-y^{1/2}}\,dy = 2(\tfrac{1}{2}\sqrt{\pi}) = \sqrt{\pi}$$

We can solve several integral problems by utilizing the features of the gamma function. As an example,

Example 1: Evaluate the following integral by using gamma equation

$$I = \int_0^\infty x^{n-1} e^{-h^2x^2} dx$$

Solution:

Putting $h^2x^2 = t$ or $x = \frac{\sqrt{t}}{h}$ and $dx = \frac{dt}{2h\sqrt{t}}$

$$I = \int_0^\infty \left(\frac{\sqrt{t}}{h}\right)^{n-1} e^{-t}\frac{dt}{2h\sqrt{t}} = \frac{1}{2h^n}\int_0^\infty t^{\frac{n}{2}-1} e^{-t} dt$$

$$= \frac{1}{2h^n}\int_0^\infty t^{\frac{n-2}{2}} e^{-t} dt = \frac{1}{2h^2}\int_0^\infty t^{\frac{n}{2}-1} e^{-t} dt$$

$$= \frac{1}{2h^n}\Gamma\left(\frac{n}{2}\right)$$

Example 2: evaluate the following integral by using gamma equation

$$\Gamma(n) = \int_0^\infty x^{n-1}\, e^{-x}\, dx$$

$$I = \int_0^\infty \frac{x^a}{a^x} dx$$

Solution:

Putting $a^x = e^t$ *or* $xln(a) = t$ *and* $dx = \frac{dt}{\ln(a)}$

Thus,

$$I = \int_0^\infty \frac{t^a}{[\ln(a)]^a} e^{-t}\frac{dt}{\ln(a)} = \frac{1}{[\ln(a)]^{a+1}}\int_0^\infty t^a e^{-t} dt = \frac{1}{[\ln(a)]^{a+1}}\Gamma(a+1)$$

4 Beta function

The timeless beta feature B (p,q) is without a doubt one of the most essential special functions. , thanks to its important role in a variety of scientific fields, including physics, mathematics, physical science, and analytical design. Various sorts of unique features have become a sought-after tool for professionals and developers alike in many areas of Managed Math. Many beneficial and helpful extensions of distinct unique features (including gamma and beta features, Gauss hypergeometric feature, etc.) have been introduced during the previous two decades or more(Artin 1931, Yalcin and Simsek 2020) .

$$\beta(p,q) = \frac{\Gamma(p)\Gamma(q)}{\Gamma(p+q)} \tag{9}$$

There is an additional type to represent the beta feature, which is actually insufficiently experimental and also the most common beta function type

$$\beta(p,q) = \int_0^1 x^{p-1}(1-x)^{q-1}\,dx \tag{10}$$

The full beta function and the incomplete beta function are consistent for x = 1, and Legendre, Whittaker, and Watson term the following equation the Eulerian integral of the first sort. The beta function B (p + 1; q + 1) is the solution to this integral.

$$\beta(p,q) = \frac{\Gamma(p)\Gamma(q)}{\Gamma(p+q)} = \frac{(p-1)!(q-1)}{(p+q-1)!}$$

4.1 Transformation and property of beta function

There are a number of simple transformation of the general beta function by:

$$\beta(p,q) = \int_0^1 x^{p-1}(1-x)^{q-1}\,dx = \frac{\Gamma(p)\Gamma(q)}{\Gamma(p+q)} \qquad p > 0 \ \ and \ \ q > 0$$

1 interchanging the location of (q , p)

$$\beta(p,q) = \int_0^1 x^{q-1}(1-x)^{p-1}\,dx = \frac{\Gamma(p)\Gamma(q)}{\Gamma(p+q)}$$

2 transformation of integration range

$$\beta(p,q) = \int_0^1 x^{p-1}(1-x)^{q-1}\,dx = \frac{1}{a^{p+q-1}}\frac{\Gamma(p)\Gamma(q)}{\Gamma(p+q)}$$

3 transformation trigonometric from beta equation:

$$\beta(p,q) = \int_0^{\frac{\pi}{2}} (sin\theta)^{2p-1}(cos\theta)^{2q-1}\,dx = \frac{1}{2}\frac{\Gamma(p)\Gamma(q)}{\Gamma(p+q)}$$

4- transformation to rational from of beta function

$$\beta(p,q) = \int_0^{\infty} \frac{x^{p-1}}{(1+x)^{p+q}}\,dx = \int_0^1 x^{p-1}(1-x)^{q-1}\,dx = \frac{1}{a^{p+q-1}}\frac{\Gamma(p)\Gamma(q)}{\Gamma(p+q)}$$

Example(3):Evaluate the following integral by using Beta-function.

$$\int_0^1 [1-x^3]^{-\frac{1}{2}}dx$$

Solution:

Putting $y = x^3$ or $dy = 3x^2dx$ or $dx = \frac{1}{3x^2}dy = \frac{1}{3y^{2/3}}\,dy$

$$\beta\,(p,q) = \int_0^1 [1-x^3]^{-\frac{1}{2}}dx = \int_0^1 [1-y]^{-\frac{1}{2}}\,\frac{1}{3y^{2/3}}\,dy$$

$$\beta\,(p,q) = \frac{1}{3}\int_0^1 y^{\frac{-2}{3}}\,[1-y]^{\frac{-1}{2}}\,dy$$

comparing to:

$$\beta\,(p,q) = \int_0^1 y^{p-1}\,[1-y]^{q-1}\,dy = \frac{\Gamma(p)\Gamma(q)}{\Gamma(p+q)}$$

we get: $p-1= -\frac{2}{3}$ then $p=\frac{1}{3}$ and $q-1=-\frac{1}{2}$ then $q=\frac{1}{2}$

$$\beta\,(p,q) = \frac{1}{3}\int_0^1 y^{\frac{-2}{3}}\,[1-y]^{\frac{-1}{2}}\,dy = \frac{1}{3}\,\beta\,(\tfrac{1}{3},\tfrac{1}{2})$$

$$\beta\,(p,q) = \frac{1}{3}\frac{\Gamma(\frac{1}{3})\Gamma(\frac{1}{2})}{\Gamma(\frac{1}{3}+\frac{1}{2})}$$

$$\beta\,(p,q) = \frac{1}{3}\frac{\Gamma(\frac{1}{3})\Gamma(\frac{1}{2})}{\Gamma(\frac{5}{6})}$$

Example (4): Evaluate the following integral by using Beta-function.

$$\int_0^1 \frac{x^3}{(1+x)^5}\,dx$$

Solution:

Putting p-1=3 then p=4 and p + q=5 then q=1

$$\beta\ (p,q) = \int_0^1 \frac{x^3}{(1+x)^5}\,dx = \frac{\Gamma(p)\Gamma(q)}{\Gamma(p+q)} = \frac{\Gamma(4)\Gamma(1)}{\Gamma(5)} = \frac{3!\ 0!}{4!} = \frac{1}{4}$$

Example(5): show that:

$$\int_0^{\frac{\pi}{2}} \sqrt{\cot\theta}\ d\theta = \frac{1}{2}\ \Gamma(\frac{1}{4})\Gamma(\frac{3}{4})$$

Solution: we know that:

$$\beta\ (p,q) = \int_0^{\frac{\pi}{2}} (\sin\theta)^p\ (\cos\theta)^q\ d\theta = \frac{1}{2}\frac{\Gamma(\frac{p+1}{2})\Gamma(\frac{q+1}{2})}{\Gamma(\frac{p+q+2}{2})}$$

Therefore the integral of the above equation is reduces to(Dass 2008):

$$\int_0^{\frac{\pi}{2}} (\sin\theta)^{-\frac{1}{2}}\ (\cos\theta)^{\frac{1}{2}}\ d\theta = \frac{1}{2}\frac{\Gamma(\frac{p+1}{2})\Gamma(\frac{q+1}{2})}{\Gamma(\frac{p+q+2}{2})}$$

Putting $p=-\frac{1}{2}$ and $q=\frac{1}{2}$

Then,

$$\int_0^{\frac{\pi}{2}} (\sin\theta)^{-\frac{1}{2}}\ (\cos\theta)^{\frac{1}{2}}\ d\theta = \frac{1}{2}\frac{\Gamma(\frac{-\frac{1}{2}+1}{2})\Gamma(\frac{\frac{1}{2}+1}{2})}{\Gamma(\frac{-\frac{1}{2}+\frac{1}{2}+2}{2})} = \frac{1}{2}\frac{\Gamma(\frac{1}{4})\Gamma(\frac{1}{4})}{\Gamma(1)}$$

Thus,

$$\int_0^{\frac{\pi}{2}} (\sin\theta)^{-\frac{1}{2}}\ (\cos\theta)^{\frac{1}{2}}\ d\theta = \frac{1}{2}\ \Gamma(\frac{1}{4})\Gamma(\frac{3}{4})$$

5 Relationship between gamma and beta function

It is possible to find a straightforward derivation for the relationship. Create the final result. of two factorials as to get the integral form of the beta function (Artin 1931).

$$\Gamma(x)\Gamma(y) = \int_{u=0}^{\infty} e^{-u}u^{x-1}\,du \cdot \int_{v=0}^{\infty} e^{-v}v^{y-1}\,dv$$
$$= \int_{v=0}^{\infty}\int_{u=0}^{\infty} e^{-u-v}u^{x-1}v^{y-1}\,du\,dv.$$

Changing the variables u = f(z,t) = zt and v = g(z,t) = z(1, t) reveals that this is the case.

$$\begin{aligned}\Gamma(x)\Gamma(y) &= \int_{z=0}^{\infty}\int_{t=0}^{1} e^{-z}(zt)^{x-1}(z(1-t))^{y-1}\big|J(z,t)\big|\,dt\,dz \\ &= \int_{z=0}^{\infty}\int_{t=0}^{1} e^{-z}(zt)^{x-1}(z(1-t))^{y-1}z\,dt\,dz \\ &= \int_{z=0}^{\infty} e^{-z}z^{x+y-1}\,dz\cdot\int_{t=0}^{1} t^{x-1}(1-t)^{y-1}\,dt \\ &= \Gamma(x+y)\,\mathrm{B}(x,y),\end{aligned}$$

wherein |J(z,t)| is indeed the actual value of a Jacobian determinants for u = f(z,t) where v = g(z,t) and |J(z,t)| is the absolute value of the Jacobian determinants for u = f(z,t) as v = g(z,t) and (z,t). The aforementioned identity can be thought of as a special instance of the identity for a convolution's integral. Taking the initiative

$$\Gamma(x)\Gamma(y) = \int_{\mathbb{R}} f(u)\,du\cdot\int_{\mathbb{R}} g(u)\,du = \int_{\mathbb{R}} (f*g)(u)\,du = \mathrm{B}(x,y)\,\Gamma(x+y).$$

6 Application of gamma and beta function

The gamma function is "arguably the most common special function, or the least'special' of all," according to one source. Other transcendental functions are called'special' because some of them may be avoided by bypassing a few specialized mathematics topics. The gamma function, on the other hand, y = Γ(x) A circumstance like this is one of the most difficult things to avoid. (Chaudhry and Zubair 2002). The beta function is important for determining and explaining the scattering amplitude for Regge trajectories. The beta function is critical in statistics for the Beta distribution and Beta prime distribution. The beta function, as previously noted, is closely related to the gamma function and plays a significant part in calculus. (Riddhi 2008).

7 Conclusion

The authors gave a fresh justification for the traditional beta feature in their assessment of the current paper. It has been used to investigate a number of features of the new extended experimental feature. The Gauss hypergeometric function expansion, as well as the combined hypergeometric functions, were also released at the request of this all-new extended experimental feature. The common integrals of long hypergeometric features, as well as their image approaches, are generated. The gamma function was introduced as one of the world's most important and well-known private functions. This specific function is also often used in mathematics. We began by discussing the particular function. For further analysis, we used the gamma function. Following that, a brief history of the gamma function is offered, as well as a quick explanation of the gamma function. After some time, we were able to grasp the characteristics of the gamma function as well as certain ideas. Furthermore, knowing the integral and differentiation that underpin the equation can aid in the solution of the gamma function problem. It will be simple to understand and solve a business problem using properties and theories. Finally, we were able to solve the problem analytically, with some of the solutions effectively implemented.

8 Reference

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