

A REVIEW OF APPLICATION OF LAPLACE TRANSFORMATION IN SCIENCES

Abstract: The Laplace transformation is an important mathematical technique that is used in a variety of engineering and science fields. One of the most common methods used by researchers and scientists to find solutions to their problems is the Laplace transform. We investigate the features and a broad variety of Laplace implementations changes in different fields in this article. Laplace transforms have many advantages and are useful for solving electrical and mechanical problems including discontinuous force functions or non-periodic functions, as well as how to locate the transition function of a mechanical device and how to use it in nuclear physics, and we'll talk about how the Laplace Transform is used in various engineering fields. Also discuss how to solve differential equations by Laplace transformation.

Keyword: Laplace transformation; integral transformation; harmonic oscillator.

1. INTRODUCTION

The importance of the transformation method is that it simplifies the structure such that a solution can be extracted from it. Experts benefit from transform theory and methods with a variety of theoretical foundations. There are several types of problems that are difficult to solve or that are algebraic expressions very uncomfortable in their representations. In applied math's, the Laplace transformation is critical for completing the solution led by the complex integral function. This integral transform is easier to use than the variance process with constant and undetermined coefficients. The Laplace transformation is a way for solving linear ordinary differential equations that uses integral transforms. It can be used in a variety of fields, including mechanics, electrical engineering, control engineering., optics, geometry, and signal processing, among other fields (Sawant, 2018).

P. Simon Laplace was a French physicist and mathematician who's influenced many ideas of arithmetic, statistics, physics, and astronomy, and he called the Laplace Transform after him. He made a significant contribution to physical mechanics by transforming previous geometrical research to calculus-based analysis, allowing his calculations to be applied to a broader variety of problems. It is capable of solving both ordinary and partial linear

differential equations. It solves a differential equation by converting it to an algebraic expression. The Laplace transformation method can efficiently solve ordinary linear differential equations with constant and variable coefficients without the need to locate the general solution or an arbitrary constant. It's used to solve physical issues. The integral equation and differential ordinary equations with constant and variable coefficients are involved. It has applications in almost every engineering discipline, such as System Modeling, Digital Signal Processing, Electrical Circuit Analysis , Process Controls, Nuclear Physics,, Probability, Physics, Power Systems Load Frequency Control, Mat lab, and so on (Verma, 2019). In a governing engineering scheme, Laplace is a major quantity. In addition, there is a tool called the inverse Laplace transformation that is related to the Laplace. In systems that use evaluating dynamic power, both the Laplace transform and the inverse Laplace transform have specific characteristics. Using algebraic techniques, the Laplace transform can also be used to analyze linear structures(Htay, Khaing, & Win).

Laplace transformations have quite a number of advantages, and they can be used to; i) we can used to solved problem in electrical and mechanical for a problem it consistence of non-periodic function or discontinues function of force other than sinusoidal and cosine functions such as a step function or impulse. ii) It can be used to solve nonhomogeneous differential equations without having to calculate the complement integral. iii) A specific solution can be obtained without suggesting the general solution while using the Laplace transform. iv) It can be used to solve differential and partial differential equation (PDE). Furthermore, applying the Laplace transform to a boundary value problem, such as an ordinary differential equation (ODE) with initial conditions (IC), could be reduced to a problem of solving algebraic formula with any specified initial case (Ramana, 2006).

Each function of times $f(t)$ must satisfy the following Dirichlet requirements in order to be Laplace transformable. (Poularikas, 2018): 1) For $t > 0$, $f(t)$ Piecewise continuous means it must be single valued but can have a finite number of discrete finite discontinuities. 2) $f(t)$ need to be of exponential order, which means that as t approaches ∞ , $f(t)$ must remain less than Se^{-ta_0} , when a_0 is a real positive number and S is a positive constant.

If the Dirichlet conditions are satisfied by some function $f(t)$, then,

$$F(s) = \int_0^{\infty} f(t)e^{-st} dt$$
The Laplace transform of $f(t)$ is written as $L(f(t))$. s may be a real number or a dynamic quantity in this case (Adhikari, 2015).

2. DEFINE THE LAPLACE TRANSFORM

A Laplace transformation can be defined as:

$$f(s) = \int_0^{\infty} f(t)e^{-at} dt \dots \dots \dots (a)$$

It's called the Laplace transform of $f(t)$ if the integral exists, and it's written as:

$$L[f(t)] = F(s) = \int_0^{\infty} e^{-at} f(t) dt \dots \dots \dots (b)$$

3. SOME IMPORTANT PROPERTIES OF LAPLACE TRANSFORMS

3.1 Multiplied by a Constant:

If a is a constant and $f(t)$ is a function of t then

$$L[af(t)] = aL\{f(t)\}$$

3.2 Linear Property:

If $f(t)$ and $g(t)$ are functions of t and (b, a) are constants:

$$L\{af(t) + bg(t)\} = aL\{f(t)\} + bL\{g(t)\}$$

3.3 Property of Scale Change:

If $L\{f(t)\} = F(s)$ then

$$L\{f(at)\} = \frac{1}{a} F\left(\frac{s}{a}\right)$$

3.4 Shifting Property:

If $L\{f(t)\} = F(s)$ then

$$L\{e^{at} f(t)\} = F(s - a)$$

3.5 Time Multiplication:

If $\{f(t) = F(s)\}$, therefore

$$L[tf(t)] = -f'(s) = -\frac{d}{ds}F(s)$$

3.6 First order Derivative:

If $\{ f(t) = F(s) \}$ therefore

$$L\{f'(t)\} = SF(s) - f(0)$$

3.7 Integration of Functions:

If $L\{f(t) = F(s)\}$ then:

$$L \left[\int_0^t f(t) dt \right] = \frac{F(s)}{s}$$

4. METHODS OF LAPLACE TRANSFORMATION

We can Laplace transformation to solved many problems including; impulse function, function of hyperbolic sine, function of sine, function of exponential decay function, cosine hyperbolic function, shifted unit step function, function of unit step, function of Bessel , ram functions, function of cosine , and function of natural logarithm,. There will be two important theorems that accompany the Laplace transform; the first is the Original Value Hypothesis, which is abbreviated as IVT, and the second is the Final Value Theorem, which is abbreviated as FVT (electrical4u, 2021).

To transformation of any function using the Laplace transformations, two important steps must be followed to ensure a successful transform. The first step is to multiply a function $g(t)$ by $e^{(-st)}$ when s is a complex number, which equals $(s = \sigma + jw)$. Where j is an imaginary number that equals to $\sqrt{-1}$.

With respect to the time the integration of the result of the first step through zero to infinity is the second stage. In addition, the inverse Laplace transform, denoted by L^{-1} , can be used to reverse this transformation L^{-1} .

5. APPLICATION OF LAPLACE TRANSFORMATION

The Laplace transformation can be used in many field areas like: physics, mathematics, physics engineering, economic engineering and control system. I'll try to list a few of the applications below:

5.1 Application in *problem initial value*

The ordinary differential equation at first-order is considered.

$$d \frac{dx}{dt} + px = f(t), \quad t > 0, \dots \dots \dots (1)$$

with the initial condition

$$x(t = 0) = a,$$

$f(t)$ is input external function and p and a both are constant, therefore they have Laplace transformation.

The Laplace transformation $\bar{x}(s)$ of the $x(t)$ gives

$$s \bar{x}(s) - x(0) + p \bar{x}(s) = \bar{f}(s), \dots \dots \dots (2)$$

Or

$$\bar{x}(s) = \frac{a}{s+p} + \frac{\bar{f}(s)}{s+p}$$

The convolution theorem and inverse Laplace Transforms both together have the solution

$$x(t) = ae^{-pt} + \int_0^t f(t-\tau)e^{-p\tau}d\tau \dots \dots \dots (3)$$

As a result, the solution is naturally divided into two terms. The first phrase is the initial condition's reaction, while the second term is solely attributed to the external input function $f(t)$.

Specifically, if $f(t) = q = c$ (when c constant) the solution of Eq.(3) is:

$$x(t) = \frac{q}{p} + \left(a - \frac{q}{p}\right) e^{-pt} \dots \dots \dots (4)$$

The steady-state solution is the first concept of this solution, which is independent of time t . The transient solution is the second term, which is dependent on time t . If $p > 0$, if t increases, the transient answer decays to none, and the steady state solution is achieved. When $p < 0$, on the other hand, the transient solution increases exponentially as $t \rightarrow \infty$, making the unstable solution.

Eq. (1) defines the natural law of growth or decaying in the presence of an external force functions $f(t)$ with $p > 0$ or $p < 0$. In chemical kinetics, the corresponding equation (1) appears often when $f(t) = 0$ and $p > 0$. The rate of chemical reactions is defined by such an equation(Debnath & Bhatta, 2014)..

5.2 Application in differential equation second order ordinary

General form for second order linear ordinary is:

$$\frac{d^2x}{dt^2} + 2p \frac{dx}{dt} + qx = f(t), \quad \text{if } t > 0 \dots \dots \dots (5)$$

The basic conditions are as follows:

$$x(t) = a, \frac{dx}{dt} = x(t) = b \text{ at } t = 0, \dots\dots\dots(6)$$

where p, q, a and b are constants.

The Laplace transform to the problem of this general initial value is:

$$s^2 \bar{x}(s) - s x(0) - \dot{x}(0) + 2p\{s \bar{x}(s) - x(0)\} + q\bar{x}(s) = \bar{f}(s).$$

Can be use of equation (6) for get solution $\bar{x}(s)$ as

$$\bar{x}(s) = \frac{(s+p)a+(b+pa)+\bar{f}(s)}{(s+p)^2+n^2}, n^2 = q - p^2 \dots\dots\dots(7)$$

The inverse Laplace transformation gives the solution forms depending on $q > = < p^2$, in three distinct and they are

$$x(t) = ae^{-pt} \cos nt + \frac{1}{n} (b + pa) e^{-pt} \sin nt + \frac{1}{n} \int_0^t f(t - \tau) e^{-p\tau} \sin n\tau d\tau,$$

When

$$n^2 = q - p^2 > 0, \dots\dots\dots(8)$$

$$x(t) = a e^{-pt} + (b + pa) t e^{-pt} + \int_0^t f(t - \tau) \tau e^{-p\tau} d\tau, \text{ when } n^2 = q - p^2 = 0 \dots\dots\dots(9)$$

$$x(t) = ae^{-pt} \cosh mt + \frac{1}{m} (b + pa) e^{-pt} \sinh mt + \frac{1}{m} \int_0^t f(t - \tau) e^{-p\tau} \sinh m\tau d\tau, \text{ when } m^2 = p^2 - q > 0 \dots\dots\dots(10)$$

(Debnath & Bhatta, 2014)

5.3 Application in medical of carbon nanotubes flow

Hypothesis: In nano medication, endeavors of utilizing the Carbon Nano Tubes (CNT) as medication bearers are attempted particularly in the treatment of malignancy. This Carbon Nano Tubes (CNT) is infused into the blood which arrives at tumor site with the activity of waves spread by the dividers of supply route with outside power, for example, attractive field or laser bar. The outcomes pronounced that the temperature profiles are delicate with respect to the worth appointed to the convective parameter.

Issue took a shot at: A viable logical strategy to reason careful numerical model depicting the impact of a convective warmth condition on the stream and the warmth move of carbon nanotube suspended nano liquids within the sight of pull/infusion. How Laplace Transformation is utilized: By applying Laplace Transform, the warmth move condition has

been explained and the arrangement has been communicated as far as the summed up deficient gamma work.

Method: The stream and warmth move of CNTs are generally depicted by arrangement of nonlinear differential conditions. The present arrangements decrease to those in writing without the pull/infusion and the connective parameters are likewise demonstrated. SWCNT nanofluids are of lower temperature than MWCNT nanofluids yet it is opposite if there should arise an occurrence of nonattendance of the two slip parameters (Saleh, Alali, Ebaid, & sciences, 2017).

5.4 Laplace Transform in the harmonic oscillation of a beam which is support at two ends

If we have the beam with length l and with the uniform cross sections which parallel to plan of zy , so the deflection of the $w(x,t)$ usually is determined downward, if beam facing to the x axis. The following is the fundamental equation that describes this phenomenon:

$$\frac{EId^4w}{dx^4} - m w\omega^2 = 0 \dots\dots\dots(11)$$

when I is the cross section's moment of inertia with respect to the y axis, the Young's modulus of elasticity is E , mass per unit length is m , and ω is angular frequency is ω .

So, equation (11) we can rewriting by set $\alpha^4 = \frac{m\omega^2}{EI}$, get,

$$\frac{d^4 w}{dx^4} - \alpha^4 w = 0 \dots\dots\dots(12)$$

Apply the Laplace transform to equation (12),

$$s^4 f(s) - s^3 F(+0) - s^2 F'(+0) - s F''(+0) - F'''(+0) - \alpha^4 f(s) = 0.$$

These problems need to know the boundary conditions therefore the boundary is:

$$F(+0) = 0 ; ; F'(l) = 0 ; ; F''(+0) = 0 ; ; F'''(l) = 0 ;$$

We get,

$$f(s) = s^2 F'(+0) + F'''(+0) / s^4 - \alpha^4 \dots\dots\dots(13)$$

After that the inverse Laplace transform is given,

$$\omega = \frac{F'(+0)}{2\alpha} \sinh \alpha x + \sin \alpha x + \frac{F'''(+0)}{2\alpha^3} \sinh \alpha x - \sin \alpha x \dots \dots \dots (14)$$

We get,

$$\omega = A_1 \sinh \alpha x + A_2 \sin \alpha x .$$

When

$$x = l, \quad A_2 \sin l\alpha + A_1 \sinh l\alpha = 0; \quad A_1 \sinh l\alpha - A_2 \sin l\alpha = 0;$$

If $A_1 = A_2 = 0$ are satisfied *i. e* $\sinh l\alpha = \sin l\alpha = 0$. This will give, $l\alpha = \pi n$, for integral values of n. Hence, $A_1 = 0$ and A_2 is unknown and the result is:

$$w_n = A_n \sin\left(\frac{\pi n x}{l}\right), \quad \dots \dots \dots (15)$$

Therefore the frequencies are

$$\omega_n = \frac{n^2 \pi^2}{l^2} \sqrt{\frac{EI}{m}} \quad \dots \dots \dots (16)$$

In equation (16), if $n = 1$, it equal to fundamental vibration and if $n = 2$ the first harmonic oscillation and so on (Adhikari, 2015).

5.5 Application in Power Systems Load Frequency control

Generation, transmission, and delivery networks make up power systems. A generation system consists of a turbo generator set in which the electrical generator is driven by a turbine, and the generator serves the loads through transmission and distribution lines. It is necessary to keep the device voltage and frequency under those limits, such as those set by the manufacturer. The frequency should be 50 to 60 Hz, and the voltage magnitude should be 0.950-1.050 per band Load Frequency Control (LFC) and Automatic Voltage Regulator (AVR) equipment is mounted for each generator in an integrated power system.

The controllers are programmed for a certain working state and track minor variations in load demand to keep the frequency and voltage within predetermined limits. Modification in real power is influenced by rotor angle, and hence system frequency and reactive power are influenced by voltage magnitude, or generator excitation. The modeling of the engine, load, prime mover (turbine), and governor is the first step in designing the control system (Saadat, 1999) .

5.6 Application in theory of Electric Circuit

RL, RC, and RLC circuits in series or parallel, can be used the Laplace transformation to solve the switch transient phenomenon. The following is a clear explanation of how to demonstrate this program.

Consider the following series RLC circuit, as seen in Figure 1, to which a DC voltage V_o is added abruptly. Can be apply Kirchhoff's Voltage Law (KVL) to the circuit is the next step, we have,

$$Ri + Ldi/dt + 1/C \int i dt = V_o \dots \dots \dots (15)$$

Differentiating both sides,

$$Ld^2i/dt^2 + 1/Ci + Rdi/dt = 0;$$

Or we can write as a following:

$$d^2i/dt^2 + (R/L)di/dt + (1/LC)i = 0 \dots \dots \dots (16)$$

Applying Laplace transformation to this equation (16), let's pretend the resolution to this formula is $i(t)$ Kest when K is constant also s is a constant too, which may be only a real, or imaginary some time may be a complex. Now, from eq. (16),

$$LKs^2est + RKest + 1/CKest = 0$$

By simplification we get,

$$s^2 + (R/L)s + 1/LC = 0$$

After taken a root for this equation we get:

$$s_1, s_2 = R/2L \pm (R^2/4L^2 - (1/LC))^{1/2} \dots \dots \dots (17)$$

The differential equation's general solution is equal to,

$$i(t) = K_1e^{s_1t} + K_2e^{s_2t} \dots \dots \dots (18)$$

Where the value of (K_1, K_2) are calculated from basic condition:

Now, if we defined the Natural Frequency, ω_n = which is also known as un damped natural frequency or resonance frequency.

Now if we define the $n\omega = 1/\sqrt{LC}$ = Natural Frequency as known to damping natural frequency or resonance frequency and $\alpha = R/2L$ = Damping Coefficient.

Root of the equation is:

$$s_2, s_1 = -\alpha \pm (\alpha^2 - \omega^2)^{1/2}$$

The end form of the answer is determined

$$\left(\frac{R^2}{4L^2}\right) > \frac{1}{LC};$$

$$(R^2/4L^2) = 1/LC \dots\dots\dots(19a)$$

And

$$(R^2/4L^2) < 1/LC \dots\dots\dots(19b)$$

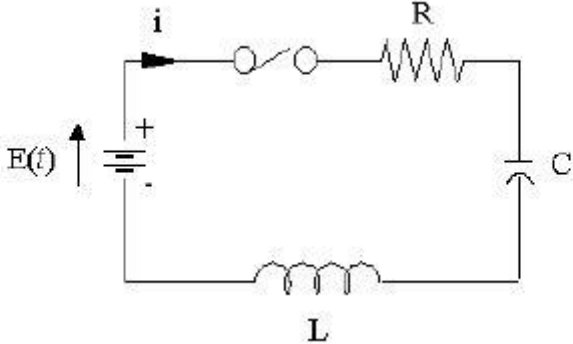


Fig.1 : RLC circuit

We can classification of cases in to three initial conditions of the circuit by the following: case of under damping if $\alpha < \omega n$, critically damping case if $\alpha = \omega n$ and over damping case if $\alpha > \omega n$ (Sh. Sharma, 2014).

5.7 Oscillation of power systems and electrochemical disruption propagation

D. Wang and X. Wang analyze electromechanical disruption propagate and oscillate analysis focused on a uniform multi segment shift power grid (Delin wang, 2012). The reflected and transmit formulae of electrochemical disruption are interpreted in the same way as they are for electrochemical wave propagation in a continuum model. From the standpoint of electro mechanic disruption propagates power oscillation and frequency is also derived. The electromechanical disruption propagates through the chain control structures, according to the analytical expressions of Bessel functions. Electro mechanic disruption propagation causes electromechanical control oscillations in a power field, and the oscillate frequency modes may be determined using grid structures and parameters. The power increment and angles of the machine rotors at both bases are calculated using the Laplace Transform under a unit phase function disturbance.

5.8 Application in a new unilateral nuclear magnetic resonance sensor to determine the age of silicone rubber insulators

Ji Weng et al. proposed a new unilateral nuclear magnetic resonance (NMR) approach for measuring insulator deterioration due to aging. The NMR transverse relaxation time decreases with age and the more severe the ageing, the greater the decline. For sufficiently broad depth, the values of all SRIs with varying service time appear to be approximately the same is. To achieve respective activity measurements, a sensor static field and a phase motor device are used. The ageing of a synthetic rubber insulator puts the power grid's safety in jeopardy. As a result, accurate estimate of the insulator aging status is needed for taking precautions against insulation loss, which can be found using the proposed process(Ji, Wang, Wu, Cui, & He, 2016).

5.9 Application in nuclear physics

Consider the following linear differential equation, which is first-order:

$$\frac{dN}{dt} = -\lambda N \dots\dots\dots (20)$$

The basic relationship defining radioactive decay is this equation,

$$N = N(t)$$

Where the un decayed atom remaining in a sample of a radioactive isotope is denoted by N, the decay constant represent by λ and the time t (in seconds)

Now, to solve this equation we can apply the Laplace transformation. Rearranging the equation to one side,

$$\frac{dN}{dt} + \lambda N = 0 \dots\dots\dots (21)$$

Then , we apply the Laplace transform for both sides:

$$(S.\check{N} (s) - N_0) + \lambda.\check{N} (s) = 0 \dots\dots\dots (22)$$

If, $\check{N} (s) = L\{N (t)\}$ and $N_0 = L\{N (t)\}$

Solving is, $\check{N} (s) = \frac{N_0}{s+\lambda}$

Finally, to find the general solution for this equation we most applied the inverse Laplace transformation:

$$N (t) = L^{-1}\{ N(s)\} = L^{-1} \left\{ \frac{N_0}{s+\lambda} \right\} \dots\dots\dots (23)$$

$$N(t) = N_0 e^{-\lambda t} \dots\dots\dots (24)$$

which is indeed the correct form for radioactive decay (Rahul M. Jetwani, 2017).

5.10 Application in harmonic oscillator without resisting medium:

In the presence of an external applied force $G g(t)$, the differential in a harmonic oscillator could be written as follows (Debnath & Bhatta, 2014):

$$\frac{dh^2}{dt^2} + w^2 h = Gg(t) \dots\dots\dots (25)$$

In which h denotes displacement, w denotes frequency, and F denotes constant. There would be if we give the equation initial conditions.

$$h(t) = a, \dot{h}(t) = U \text{ at } t = 0 \dots\dots\dots (26)$$

The constants a and U are used in this equation. We get the following results by applying the Laplace transform and initial conditions to equation (2):

$$(s^2 + w^2)\bar{h}(s) = sa + U + G\bar{g}(s)$$

The above is transformed using the convolution theorem.

$$h(t) = a \cos(wt) + \frac{U}{w} \sin(wt) + \frac{G}{w} \int_0^t g(t - \tau) \sin(w\tau) d\tau$$

$$h(t) = A \cos(wt - \varphi) + \frac{G}{w} \int_0^t g(t - \tau) \sin(w\tau) d\tau \dots\dots\dots (27)$$

where $A = (a^2 + \frac{U^2}{w^2})^{1/2}$ and $\varphi = \tan^{-1}(\frac{U}{wa})$

There are two terms in equation (27) This term is also known as natural frequency because it relates to the first term, since φ is phase, A is amplitude, and w is angular frequency. The forced oscillation, which is caused by an outward force is represented by the second expression. In order to analyze the resolution of equation (27) we must first understand the following examples:

5.10.1 Function of zero forcing:

The second term of the formula (27) will be omitted in this situation, and the equation will be reduced to:

$$h(t) = A \cos(wt - \varphi) \dots\dots\dots (28)$$

Equation (28), with angular frequency w , phase φ , and amplitude A , is known as a harmonic oscillator.

5.10.2 Function of steady forcing:

In this case is, equation (27) becomes if $g(t) = 1$,

$$h - \frac{G}{\omega^2} = A \cos(\omega t - \varphi) - \frac{G}{\omega^2} \cos(\omega t) \dots\dots\dots (29)$$

When the particle is at rest, exclude it from the equation such that $U=0$:

$$h - \frac{G}{\omega^2} = (a - \frac{G}{\omega^2}) \cos(\omega t) \dots\dots\dots (30)$$

Equation (30) is a free oscillation of normal frequency ω , which is displaced from of the origins and transferred from $\frac{G}{\omega^2}$.

5.10.3 Function of periodic forcing:

This case, if $g(t)\cos(\omega_0 t)$, equation (27) becomes:

$$h(t) = A \cos(\omega t - \varphi) + \frac{G}{\omega_0^2 - \omega^2} \cos(\omega_0 t) \dots\dots\dots (31)$$

When $A = \sqrt{\left(a + \frac{G}{\omega_0^2 - \omega^2}\right)^2 + \frac{U^2}{\omega^2}}$ and $\tan(\varphi) = \frac{U}{w(a + \frac{G}{\omega_0^2 - \omega^2})}$

Equation (30) includes free oscillation with a duration of $(\frac{2\pi}{\omega})$ and induced oscillation with a period of $(\frac{2\pi}{\omega_0})$. The external force and the forced oscillation are assumed to be equal if $\omega_0 < \omega$ is similar. However, if $\omega_0 > \omega$, the forced oscillation concept experiences a 2π phase transition. This implies where force oscillation and external forced are out of movement phase or in motion phase, depending on how we describe it. However, if $\omega_0 = \omega$, the following equation (30) can be written:

$$h(t) = A \cos(\omega t - \varphi) + \frac{Gt}{2\omega} \sin(\omega t) \dots\dots\dots (9)$$

when $A = \sqrt{a^2 + \frac{U^2}{\omega^2}}$ and $\tan(\varphi) = \frac{U}{a\omega}$

The amplitude of the forced oscillation increases with time t , as seen above. This can be demonstrated by this, this implies that the pressuring frequency is equivalent to the normal frequency, which in this situation means that the oscillation is physically unwelcome and undesirable. Resonant frequency is the name given to these phenomena. Furthermore, for a long time, this resonant frequency would be mathematically invalid and physically impossible.

6. CONCLUSION

This review shows how to apply the Laplace transform in a variety of situations to solve a wide variety of issues. The purpose of this review paper is to present a clear overview of the Laplace transform in different fields. An examination of the transformation's implementations demonstrates how methods can be used to solve a variety of problems.

Researchers have used a variety of approaches to change the domains, when the domains are functions of frequency, time, or complicated angular frequency. A procedure known as the Laplace transform is one of many techniques used in mathematics. The Laplace transform could be used to transform of outputs and inputs functions that are functions of frequency or time into the time domain. The Laplace transform is a mathematic formula that is use to convert a vector (x, or, y, or z, or t) into a parameters (s) in the time domain.

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