Kurdistan Regional Government-Iraq
Ministry of Higher Education & Scientific Research
Salahaddin University
College of Education
Department of physics

A NOTICE OF THE HIGH ORDER DIFFERENTIAL EQUATION NUMERICAL SOLUTION

A Review Article in

Mathematics Physics

In Partial Fulfillment of the Requirements for The Degree of (MSc)

Prepared by: Harem S. Muhammed, harem.mohammed@su.edu.krd

Supervised by: **Sattar O. Hasan,** sattar.hasan@su.edu.krd

(2020 – 2021)

A NOTICE OF THE HIGH ORDER DIFFERENTIAL EQUATION NUMERICAL SOLUTION

[1]Harem S. Muhammed , [2] Sattar O. Hasan

[1,2]University Of Salahuddin-Erbil, College of Education Department of Physics, Erbil, Kurdistan Region Iraq

ABSTRACT

In this article, we analyze the numerical arrangement of differential high-order conditions with cross-border conditions. Spurred on by a discreet approximation of the specific convolution, utilizing fictitious focuses is recommended when additional questions are asked as Lagrange polynomials are carried out locally. As a neighborhood multifaceted differential approach, the suggested strategy should be respected. The proposed vital arrangement is based on two illustrations, a self-worth issue and a constraining esteem issue indicated by the 6th respectful condition and a respectful condition of the eighth arrange.

KEYWORD: high – order differential equation *, DQM , GDQR.*

1. INTRODUCTION

Models the occurrences in the aforementioned instability as opposed to stability. There are many four fields of high-order differential condition. The free vibration study of bar structures is, for example, represented by a differential fourth order, which is the sixth order of ring structures. Moreover, when taking the flimsy conditions into consideration, we see flimsiness as a normal convection and over stability with the sixth order, traditional deferential condition (Tribute) and eighth-order Tribute separately in a finite layer of oil, which are heated up from below and subject to turnover operation. Each of them. In reality, more ordinary tributes can be used if the liquid is joined in the same way as gravity with a uniform, attractive field. A 10th Order and a 12th Order Tribute are separately derived from standard convection and overweight.

Such differential problems are also linked to multi-boundary circumstances, so that the problem is well-positioned. In this case, four boundary conditions exist in the pillar vibration problem, which is a fourth order differential. The number of border conditions for well-posed questions is well known to be the same as the arrangement of the differential condition, and they are given equally at every border.

In addition, three types with executions of multi boundary conditions were suggested when adding polynomials as guess pieces (Boutayeb and Twizell 1991, Wu and Liu 2000, Liu and Wu 2002). The 4th class creates the limiting conditions into the estimate bit, i.e. the addition of polynomials. In N questions a discrete use of a one-dimensional computer space with N hubs takes place. The performance of the management situation at each hub results in N direct equations. These direct requirements should be directly free of limitations but not necessary in order to achieve an agreement in order to provide an individual value issue.

$$g'''' + ag''' = zg'' \qquad -1 < x < 1 \tag{1}$$

$$g(-1) = g'(-1) = g(1) = g'(1) = 0 \tag{2}$$

Where g is the dark work, a can be a real parameter, z a parameter Eigen is and a prime denotes di and erentiation as regards t. M lattice focuses are yielded after discretization. All the boundary conditions are sought by a polynomial expression. Therefore, a few adding polynomials are talked of as the number of .

$$g(r) \approx g^N(r) = \sum_{k=2}^{N-1} u_k h_k(r) \tag{3}$$

Where $g_k = g(r_k)$ *and ,*

$$h_k = \frac{(1-r^2)}{(1-r_k^2)} l_k(r) \tag{4}$$

Here $l_k(r)$ *is the polynomial presented by Lagrange .*

$$l_k(r) = \prod_{i=1,\, i\neq k}^{N} \frac{r-r_i}{r_k-r_i} \tag{5}$$

It is shown that the limits were considered in the construction of the polynomial, i.e.

$$g^N(\pm 1) \approx g^{B'}(\pm 1) = 0 \tag{6}$$

At all events, this arrangement is problematic where restrictive limits are difficult, such as within the granted state of the higher scheme.

The minute and 3rd kind of execution isolated the theory bit from the limits. In this manner, $N + p$ *is gotten in coordinate circumstances where N is the number of center (or questions) and p is the number of boundary conditions, i.e., N is the number of center points (or questions). It is as of now caught on that circumstances at N are still subordinated by the administration's insight. The coefficient organize classifies the standard* $N - p$ *for a* p^{th} *Arrange differential. Take note that there are ordinarily the same number of multi-boundary conditions as the differential state course of action; certain N-p straight conditions can be* *expelled and related with this contrast by coordinate p conditions inferred from boundary conditions. All of typically conceivable by means of the differential procedure on sacredness (DQM) and the streamlined methodology on squaring (GDQ). Refs may allude to this kind of common case (Shu and Wang 1999, Wu and Liu 2000).*

It is known that there's a refinement between the number of questions and the number of straight conditions for the third kind of procedures. On the other side, extra questions can be inquired. The GDQR methodology gives higher-order backup values of the boundary center points for outlining the broad respectful four. The GDQR must utilize the Hermite interpolating polynomials instead of the Lagrange ones to have these higher-order backup values. The GDQR must utilize the Hermite inserting polynomials instead of the Lagrange ones to have these higher-order backup values. As an bizarre case such as the one with a low-order concept may moreover be considered, the four-nite separation procedure created by Boutayeb and Twizell (Boutayeb and Twizell 1993, Djidjeli, Twizell et al. 1993, Twizell, Boutayeb et al. 1994). Liu and his colleagues have appeared a few things almost the GDQR approach to distinguish high-quality differential conditions, such as Refs (Wu and Liu 2000, Wu and Liu 2001).The definitions of the GDQR for 2D problems are somewhat different to one D (Wu and Liu 2001) while for both two-dimensional and one-dimensional concerns the routine definitions of DQM have the same form. In this article, we suggest an elective third-specific approach called the multifaceted quadrature strategy for the neighborhood (La-DQM). Increased with 4ctitious focuses, we grow 4ctitious emphasis outwards of the border, as extra questions, in our distinct specific convolution (DSC) (Wei 1999). The 4th differentiation method is focused on the La-DQM. The 4-point technique, (Chen 2000) said, when he showed his 4-nite comparison approach with difference quadrature. However, it has not been investigated how La-DQM can be established for the determination of high-order differential conditions. The suggested approach, which uses the Lagrange polynomials, is somewhat less complex than the GDQR strategy and is similar to the GDQR Strategy in terms of its accuracy. In addition, it is noted that the use of 4ctious focuses externally ensure that the estimate bit is sound. This can be seen by comparing it with splintering (Akram and Rehman 2013, El-Gamel, Adel et al. 2019) for a few issues of eighth order, which are fast to focus on a few hubs following the boundary.

The document is structured as follows. Area 2 details the suggested solution process, the in equivocal weighting coefficient equation and the multi-boundary conditions use points of interest. This creates a local multi-purpose GDQ technique. Two examples are used for Area 3, with a question of their own importance illustrated by six-order differential conditions and *an eighth-order differential. Compared to some known models, such as the DQM (Gutierrez and Laura 1995), spline (Akram and Rehman 2013) and GDQR (Liu and Wu 2002), are the results of the proposed technique.*

Higher value issues (BVP), in the areas astrophysics, magnet stability, fluid dynamics, astronomy, beam, theory of long waves, application and engineering, occur in the area of hydrodynamics and hydro magnetic stability. Many scholars have examined high-quality BVPs due to their mathematic graphic relevance and their applicability in several disciplines of applied science. The hydrodynamic stability difficulties are modeled by these equations. If we are heated from below, an endless horizontal fluid layer is perpetrated side by side by side, susceptible to rotation. An eighth order BVP(Howard 1962). Bishop et al. model eight-order BVP the torsional vibration phenomena of beams(Bishop, Cannon et al. 1989). As a result, many researchers became more interested in the numerical solution of these equations for the broad scope applications of eight BVPs.

2. FORMULATION

The article concludes by concluding. In this section, in combination with the use of multi-boundary circumstances a scalable differential quadrature technique (La-DQM) is generated using 4 kernels. The problem below is agreed as a one-dimensional p^{th} condition ($p \geq 4$) with p-boundary conditions, counting two types of Dirichlet border conditions (one at each boundary).

2.1 Coefficient Weighting

The La-DQM notes that a weighted direct combination of labor values is used in some neighboring discrete focuses within the course of the space variable within the computer space to presume the subordinate work in respect to a space variable at a given discrete stage. The m^{th} subsidiary $g(r)$ at the point i^{th}, r_i , is approx. as illustrative $g_r^{(m)}(r_i)$,

$$g^{(m)}(r_i) = \sum_{j=-L_i}^{k_i} C_{i,j}^{(m)}\, g(r_{i+j)} \quad (7)$$

Where $C_{i.j}^{(m)}, (j = -z_i, \ldots\ldots, k_i)$, the weighting factors for the m^{th} subsidiary assuming ith point, z_i and k_i represent separately the number and the right of the cleared-out job values of the neighbor. The coefficients of weighting must be calculated in advance. As z_i and k_i are specified, the weighting coefficients of the i^{th} hub can be determined with the information compare to the unequivocal GDQ (Shu 1991). A Lagrange frame is used for each hub, which includes polynomials. In case, one has (Shu 1991) for the point of i^{th}

$$g_{i,j}(r) = \prod_{l=i-z_i,\ l\neq i+j}^{i+k_i} \frac{r-r_l}{r_{i+j}-r_l}\ , \qquad j = -z_i, \dots\dots, k_i \tag{8}$$

The coefficients of weighting can be derived from the exact distinguishing of polynomial structures from the 4th subaltern. Then you've got one .

$$C_{i,j}^{(1)} = g_{i,j}^{(1)}(r_i) \qquad for \quad j = -z_i, \dots\dots, k_i\ ; \quad j \neq 0 \tag{9}$$

So $j = 0$*, Then*

$$C_{i,0}^{(1)} = \sum_{j=-z_i, j\neq 0}^{k_i} C_{i,j}^{(1)} \tag{10}$$

The weighting coefficients can be achieved by means of a repeat equation for higher order subordinates:

$$C_{i,j}^{(m)} = m\left(c_{i,j}^{(1)} c_{i,j}^{(m-1)} - \frac{c_{i,j}^{(m-1)}}{r_i - r_{i+j}}\right)\ ,\ j = -z_i, \dots\dots, k_i\ ; \quad j \neq 0 \tag{11}$$

So

$$C_{i,0}^{(m)} = -\sum_{j=-L_i, j\neq 0}^{k_i} C_{i,j}^{(m)} \tag{12}$$

2.2. Multi-Boundary Terms Implemented Across ßctious Points

The implementation of boundary conditions will change adequately, as a result of the distinction between a boundary value problem and an own value issue. First of all, we deem a question of boundary appreciation. The pth-order differential condition of the single dimension is de4ned in an interim [x,y] , with $p(= p_l + p_r)$ *limits, since* p_l *and* p_r *speak to the number of limits on the removed and right limits individually. Furthermore, we agree that* $p_l \geq 2$ *and* $p_r \geq 2$ *and on each border, there is one Dirichlet-like limitation. The approach presented presents the external left and the right frontiers of* $p_l - 1$ *and* $p_r - 1$ *4ctious focuses, respectively. That is why we have .*

$$r_{2-p_l} < \cdots < r_1 = a < r_2 < \cdots < r_N = b < \cdots < r_{N+p_r-1} \tag{13}$$

In consideration of the status of the two Dirichlet border conditions, we are able to make the following provisions.The fulfillment of a p^{th} *order differential condition under the discrete* $N + p - 2$ *hubs results in* $N + p - 2$ *direct conditions at any point. We pick* $(N + p - 2)p$ *, i.e.,* $N - 2$ *from them, as said early. Here, the parameters we have obtained by fulfilling the administrative requirement are used for the intermediate focal points [x,y], i.e.,*$r_2, \dots r_{N-2}$*.* $N + p - 2$ *have direct conditions to light up the* $N + p - 2$ *questions on the side of the direct* *p conditions from the boundary conditions, which count values for the $p-2$ fictitious emphasis.*

The solution to the framework varies marginally from the limitation of utility for The p-conditions from the limit conditions are illuminated and used to mark the imaginary points and the boundary points in relation to the inner focus. In this respect, we have the question of own worth in the framework if the structure of these variables replaces the $N-2$ straight conditions derived from the administrative conditions.

$$Ar = \lambda r \tag{14}$$

Where A is the square lattice $(N-2)\times(N-2)$, r may be the column vector of $N-2$ internal dots values, i.e. $r_2, \dots, r_{N-2}$ value.

3. APPLICATION

The European Parliament. There are two scenarios utilized to demonstrate what is more around the unused procedure. Both a constrain esteem issue and self-esteem issue which constitute the 8th and the 6th arrange separately are postured.

3.1 Problem 1: A 6th-Order Eigenvalue Problem

In this image, a ring structure with a constant ring diameter and showing variable thickness is considered (see ref. (Gutierrez and Laura 1995, Wu and Liu 2000)). This structure has rectangular cross sections. In view of half the ring structure, this issue is a peculiar value identified by the sixth-order difference:

$$\beta_1 \upsilon^{(6)} + \beta_2 \upsilon^{(5)} + \beta_3 \upsilon^{(4)} + \beta_4 \upsilon^{(3)} + \beta_5 \upsilon^{(2)} + \beta_6 \upsilon^{(1)} = \Gamma^2 (f\upsilon^{(2)} + f^{(1)}\upsilon^{(1)} - \pi^2 f\upsilon) \tag{15}$$

where $\upsilon^{(s)} = \frac{d^s w}{dr^s}$; γ is the dimensionless frequency, υ is the tangential displacement, and

$\beta_1 = \frac{\varphi}{\pi^4}$,

$\beta_2 = \frac{3\varphi^{(1)}}{\pi^4}$,

$\beta_3 = \frac{2\varphi}{\pi^2} + \frac{3\varphi^{(2)}}{\pi^4}$,

$\beta_4 = \frac{4\varphi^{(1)}}{\pi^2} + \frac{\varphi^{(3)}}{\pi^4}$,

$\beta_5 = \varphi + 3\frac{\varphi^{(2)}}{\pi^2}$,

$\beta_6 = \varphi^{(1)} + \frac{\varphi^{(3)}}{\pi^2}$ *(16)*

In which

$\varphi = [f(r)]^3$,

$f = f(r) = -4(s-1)r^2 + 4(s-1)r + 1$ *(17)*

In $x\ \varepsilon\ [0,1]$ *The vector* $\varphi^{(i)} = \frac{d^i\varphi}{dr^i}$; $f^{(i)} = \frac{d^i f}{dr^i}$, *and r is a variable of the ring's thickness. We've rewritten Eq. (15)*

$$(\beta_1 \frac{d^6}{dr^6} + \beta_2 \frac{d^5}{dr^5} + \beta_3 \frac{d^4}{dr^4} + \beta_4 \frac{d^3}{dr^3} + \beta_5 \frac{d^2}{dr^2} + \beta_6 \frac{d}{dr})\upsilon = \Gamma^2(f \frac{d^2}{dr^2} + f^{(1)} \frac{d}{dr} - \pi^2 f)\upsilon \quad (18)$$

Administrators' discretization leads to a widespread problem of self worth. At any conclusion, the ring structure has three limits. It has boundary conditions both within and out. Two distinctive boundary settings are taken into account.

case 1.1

$$\upsilon(0) = \upsilon^{(1)}(0) = \upsilon^{(3)}(0) = 0 \quad , \quad \upsilon(1) = \upsilon^{(1)}(1) = \upsilon^{(3)}(1) = 0 \quad (19)$$

Case 1.2.

This picture is taken from Refs in addition (Gutierrez and Laura 1995, Wu and Liu 2000). A quarter of the ring structure without restrictions is called rather than a half-ring structure. The monitoring status is quite similar to (15) but from n2fw to n2fw=4 the last word is modified. Additional modifications include .

$\beta_1 = 16\frac{\varphi}{\pi^4}$,

$\beta_2 = 48\frac{\varphi^{(1)}}{\pi^4}$,

$\beta_3 = 8\frac{\varphi}{\pi^2} + 48\frac{\varphi^{(2)}}{\pi^4}$,

$\beta_4 = 16\frac{\varphi^{(1)}}{\pi^2} + 16\frac{\varphi^{(3)}}{\pi^4}$,

$\beta_5 = \varphi + 12\frac{\varphi^{(2)}}{\pi^2}$,

$\beta_6 = \varphi^{(1)} + 4\frac{\varphi^{(3)}}{\pi^2}$

$\varphi = [f(x)]^3$,

$$f = f(r) = -(s-1)r^2 + 2(s-1)r + 1 \tag{20}$$

$$\upsilon(0) = \upsilon^{(2)}(0) = 0 \ \ , \ \varphi^{(1)}(0)\left[\upsilon^{(1)}(0) + 4\frac{\upsilon^{(3)}(0)}{\pi^2}\right] + 4\varphi(0)4\frac{\upsilon^{(4)}(0)}{\pi^2} = 0$$

$$\upsilon(1) = \upsilon^{(2)}(1) = 0 \ \ , \quad \varphi^{(1)}(1)\left[\upsilon^{(1)}(1) + 4\frac{\upsilon^{(3)}(1)}{\pi^2}\right] + 4\varphi(1)4\frac{\upsilon^{(4)}(1)}{\pi^2} = 0 \tag{21}$$

To unravel the issue of claim esteem and to induce recurrence Γ a standard self-value solver can be utilized.

2.3 Problem 2

a dilemma with 8th order limits In this contett, an 8th order limit value problem with four distinctive settings in (Liu and Wu 2002, Akram and Rehman 2013) is again managed to make precise within the entire room. You are in the form of

$$f^{(8)} + \emptyset(r)y = \psi(r) \ \ , \quad -\infty < a \le r \le b < \infty \tag{23}$$

$$f(t) = T_0$$

$$f^{(2)}(t) = T_2$$

$$f^{(4)}(t) = T_4$$

$$f^{(6)}(t) = T_6$$

$$f(u) = U_0$$

$$f^{(2)}(u) = U_2$$

$$f^{(4)}(u) = U_4$$

$$f^{(6)}(u) = U_6 \tag{24}$$

When the continuous de4ned functions in $r \in [x, y]$ are $= f(r)$, $\emptyset(r)$ and $\psi(r)$. The finite true constants are T_s and U_s , s= 0,2,4,6). For the four cases, Table 1 lists analytical solutions of different constants and functions.

Table 1: Variable differential conditions in four boundary conditions etamples

Problem	***2.1***	***2.2***	***2.3***	***2.4***
[x,y]	***[0,1]***	***[-1,1]***	***[-1,1]***	***[-1,1]***
$\emptyset(r)$	***t***	***-t***	***-(1)***	***-(1)***
$\psi(r)$	$(-48-15r-r^3)e^r$	$(r^3-r^2-17r-55)e^r$	$2[21\sin(r)-8r\cos(r)]$	$2[21\cos(r)-8r\sin(r)]$
T_0	***0***	***0***	***0***	***0***
T_2	***0***	***2/e***	$-2[2\cos(1)+\sin(1)]$	$-2[2\sin(1)-\cos(1)$
T_4	***-8***	***-4/e***	$-4[2\cos(1)+3\sin(1)]$	$4[2\sin(1)-3\cos(1)$
T_6	***-24***	***-18/e***	$-3[4\cos(1)+10\sin(1)]$	$3[10\cos(1)-4\sin(1)$
U_0	***0***	***0***	***0***	***0***

U_2	***-4e***	***-6e***	$2[2cos(1)+\sin(1)]$	$-2[2\sin(1)-cos(1)]$
U_4	***--16e***	***-20e***	$-2[4cos(1)+6\sin(1)]$	$-2[6cos(1)-4\sin(1)$
U_6	***-36e***	***-42e***	$-6[2cos(1)-5\sin(1)]$	$6[5cos(1)-2\sin(1)$
Analytical solution	$(r-r^2)e^r$	$(r-r^3)e^r$	$(-1+r^2)\sin(r)$	$(-1r^2)\cos(r)$

3. RESULT AND DEBATE

For comfort, all conditions take a uniform lattice into consideration. Moreover, the taking after model applies to z_i and k_i :

$$z_i = min\{M, i-2+p_l\} \ , k_i = min\{M, N+p_r-1-i\} \qquad (25)$$

This means that almost $2M+1$ nett point or both focuses, whichever is smaller was approtimated at one point in the subsidiary. In order to manage the GDQR agreements, a strong M esteem will be chosen in all circumstances (see Tables 2–7).

Tables 2 and 3 assess and list the ring structure recurrence (problem 1.1 and 1.2). Table 2 notes that the show solution follows the same GDQR parameter with N=11 for littler r values r = 1.0 , 1.1 and 1.2. The new solution begins to go wrong with GDQR with totally higher r values. Thereafter, the distinction is satisfactory. The same subject as seen in Table 3. As this problem is not well understood in DQM and Rayleigh Ritz variants, it is generally used.

Table 2: Comparison of the essential frequencies for case 1.1 of the problem of the 6th arrange . (Gutierrez and Laura 1995)

r	*DQM*	*Rayleigh Ritz*	*GDQR*					*Present*				
			7[a]	*8[a]*	*9[a]*	*10[a]*	*11[a]*	*7[a]*	*8[a]*	*9[a]*	*10[a]*	*11[a]*
1.0	*2.267(12[a])*	*2.273*	*2.2630*	*2.2668*	*2.2666*	*2.2666*	*2.2666*	*2.2623*	*2.2646*	*2.2668*	*2.2667*	*2.2666*
1.1	*2.416(12[a])*	*2.415*	*2.4132*	*2.4138*	*2.4136*	*2.4136*	*2.4136*	*2.4134*	*2.4135*	*2.4136*	*2.4136*	*2.4136*
1.2	*2.560(12[a])*	*2.556*	*2.5596*	*2.5564*	*2.5566*	*2.5567*	*2.5567*	*2.5582*	*2.5575*	*2.5569*	*2.5568*	*2.5567*
1.3	*2.700(12[a])*	*2.696*	*2.7138*	*2.6943*	*2.6961*	*2.6965*	*2.6965*	*2.7018*	*2.6994*	*2.6975*	*2.6971*	*2.6967*
1.4	*2.838(14[a])*	*2.836*	*2.8946*	*2.8241*	*2.8317*	*2.8335*	*2.8334*	*2.8451*	*2.8399*	*2.8363*	*2.8352*	*2.8340*
1.5	*2.975(14[a])*	*2.969*	*3.1296*	*2.9406*	*2.9622*	*2.9680*	*2.9677*	*2.9877*	*2.9790*	*2.9737*	*2.9714*	*2.9693*

Number of the grid point , N; $M = N+1$

Table 3: Comparison of the essential frequencies of the 6th arrange of possess esteem problem Case 1.2 . (Gutierrez and Laura 1995)

r	*DQM*	*Rayleigh Ritz*	*GDQR*					*Present*				
			6[a]	*7[a]*	*8[a]*	*9[a]*	*10[a]*	*6[a]*	*7[a]*	*8[a]*	*9[a]*	*10[a]*
1.0	*2.687(12[a])*	*2.688*	*2.6829*	*2.6834*	*2.6834*	*2.6834*	*2.6834*	*2.6957*	*2.6829*	*2.6831*	*2.6834*	*2.6834*
1.1	*2.850(12[a])*	*2.847*	*2.8453*	*2.8453*	*2.8453*	*2.8453*	*2.8453*	*2.8524*	*2.8489*	*2.8489*	*2.8490*	*2.8490*

1.2	***3.011(12[a])***	***3.007***	***3.0063***	***3.0063***	***3.0063***	***3.0063***	***3.0063***	***3.0200***	***3.0182***	***3.0183***	***3.0182***	***3.0182***
1.3	***3.172(12[a])***	***3.168***	***3.1667***	***3.1666***	***3.1666***	***3.1666***	***3.1666***	***3.1918***	***3.1885***	***3.1888***	***3.1885***	***3.1885***
1.4	***3.333(14[a])***	***3.327***	***3.3268***	***3.3264***	***3.3263***	***3.3264***	***3.3264***	***3.3596***	***3.3578***	***3.3580***	***3.3579***	***3.3579***
1.5	***3.494(14[a])***	***3.487***	***3.4862***	***3.4859***	***3.4858***	***3.4859***	***3.4859***	***3.5151***	***3.5252***	***3.5231***	***3.5253***	***3.5249***

Number of the grid point , N; $M = N + 1$

Table 4: The Supreme Misconduct Case 2.1 of the 8th -order (Amin, Shah et al. 2021)

$y_i^{(k)}$	x_4, x_{N-4}	$N = 32^a$ Otherwise	GDQR		Present	
			$N = 7^a$	$N = 11^a$	$N = 7^a$	$N = 11^a$
k =0	811.3×10^{-10}	182.0×10^{1}	29.2×10^{-11}	49.5×10^{-15}	74.7×10^{-10}	52.7×10^{-11}
k =1	246.1×10^{-9}	430.2×10^{2}	99.0×10^{-11}	26.5×10^{-14}	27.7×10^{-9}	17.9×10^{-10}
k =2	845.1×10^{-9}	366.7×10^{4}	28.0×10^{-10}	79.3×10^{-14}	85.6×10^{-9}	54.1×10^{-10}
k =3	273.8×10^{-8}	386.5×10^{7}	12.0×10^{-9}	79.7×10^{-14}	41.1×10^{-8}	21.8×10^{-9}
k =4	115.3×10^{-7}	901.3×10^{11}	50.4×10^{-9}	30.7×10^{-15}	14.1×10^{-7}	69.9×10^{-9}
k =5	487.0×10^{-7}	884.4×10^{13}	11.8×10^{-7}	62.5×10^{-14}	12.0×10^{-6}	44.1×10^{-8}
k =6	530.7×10^{-6}	443.0×10^{15}	47.4×10^{-7}	24.1×10^{-13}	50.9×10^{-6}	17.5×10^{-7}
k =7	114.2×10^{-4}	945.0×10^{16}	20.0×10^{-5}	25.3×10^{-11}	17.9×10^{-4}	54.2×10^{-6}

Number of the grid point , N; $M = N + 2$

Table 5: Case 2.2 of the eighth order limit-value problem most serious mistakes

$y_i^{(k)}$	x_4, x_{N-4}	$N = 32^a$ Otherwise	GDQR		Present	
			$N = 7^a$	$N = 11^a$	$N = 7^a$	$N = 11^a$
k =0	944.3×10^{-7}	494.8×10^{1}	31.3×10^{-7}	15.4×10^{-12}	75.8×10^{-6}	27.1×10^{-10}
k =1	145.4×10^{-6}	116.9×10^{3}	50.9×10^{-7}	27.5×10^{-12}	13.0×10^{-5}	48.3×10^{-10}
k =2	233.5×10^{-6}	498.4×10^{5}	75.2×10^{-7}	40.7×10^{-12}	19.0×10^{-5}	73.1×10^{-10}
k =3	357.8×10^{-6}	105.0×10^{8}	15.0×10^{-6}	92.8×10^{-12}	41.0×10^{-5}	16.6×10^{-9}
k =4	586.7×10^{-6}	245.0×10^{10}	22.0×10^{-6}	19.4×10^{-11}	64.8×10^{-5}	28.2×10^{-9}
k =5	861.2×10^{-6}	240.4×10^{12}	18.3×10^{-5}	1.58×10^{-10}	23.4×10^{-4}	11.8×10^{-8}
k =6	172.9×10^{-5}	120.4×10^{14}	40.9×10^{-5}	32.1×10^{-10}	45.3×10^{-4}	26.9×10^{-8}
k =7	120.6×10^{-4}	256.9×10^{15}	88.1×10^{-4}	17.0×10^{-8}	77.7×10^{-3}	69.3×10^{-7}

Number of the grid point , N; $M = N + 2$

Table 6: Most ettreme supreme blunders in eighth-order constrain esteem issue case 2.3

$y_i^{(k)}$	x_4, x_{N-4}	$N = 32^a$ Otherwise	GDQR		Present	
			$N = 7^a$	$N = 11^a$	$N = 7^a$	$N = 11^a$
k =0	120.3×10^{-8}	161.6×10^{1}	95.8×10^{-9}	90.9×10^{-14}	27.2×10^{-7}	16.5×10^{-10}
k =1	$293.6.4 \times 10^{-8}$	381.9×10^{2}	39.7×10^{-8}	29.9×10^{-13}	99.4×10^{-7}	32.3×10^{-10}

k =2	718.2×10^{-8}	162.8×10^{5}	93.2×10^{-8}	97.2×10^{-13}	27.4×10^{-6}	52.6×10^{-10}
k =3	201.6×10^{-7}	343.0×10^{7}	67.8×10^{-7}	30.8×10^{-12}	10.4×10^{-5}	13.8×10^{-9}
k =4	592.0×10^{-7}	800.0×10^{9}	10.8×10^{-6}	12.8×10^{-11}	29.2×10^{-5}	27.8×10^{-9}
k =5	185.4×10^{-6}	785.1×10^{11}	18.3×10^{-5}	15.2×10^{-10}	13.7×10^{-4}	12.2×10^{-8}
k =6	574.9×10^{-6}	393.2×10^{13}	32.1×10^{-5}	27.3×10^{-10}	33.4×10^{-4}	30.5×10^{-8}
k =7	471.5×10^{-5}	838.8×10^{14}	67.3×10^{-4}	14.5×10^{-8}	60.3×10^{-3}	77.9×10^{-7}

Number of the grid point , N; $M = N + 2$

We prove that the 8th arrangement is limited by the matimum possible blunders, i.e. case 2.1–2.4, as there was no other botch in the tett (Liu and Wu 2002, Amin, Shah et al. 2021). *There are no other botches. The nitty scratchy semblances appear in table 4 – 7. The new protocol is much less accurate than the GDQR. This may be due to the uniformity of show job employments, while the GDQR used a Chebyshev girdle. The Chebyshev brace improves the precision of slowness configurations, but can result in more errors in high-order modes. La-DQM and GDQR both work well on their boundaries, but they did not focus on their borders in a fractured way.*

Table 7: Case 2.4 of the eighth order limit value problem most serious blunders

$y_i^{(k)}$	x_4, x_{N-4}	$N = 32^a$ Otherwise	GDQR		Present	
			$N = 7^a$	$N = 11^a$	$N = 7^a$	$N = 11^a$
k =0	131.3×10^{-6}	223.9×10^{1}	26.1×10^{-7}	14.9×10^{-12}	73.8×10^{-6}	40.5×10^{-10}
k =1	202.4×10^{-6}	550.4×10^{2}	41.6×10^{-7}	23.4×10^{-12}	11.6×10^{-5}	63.8×10^{-10}
k =2	323.8×10^{-6}	469.2×10^{4}	62.3×10^{-7}	36.4×10^{-12}	18.1×10^{-5}	99.7×10^{-10}
k =3	500.8×10^{-6}	494.5×10^{7}	11.3×10^{-6}	58.5×10^{-12}	29.0×10^{-5}	16.0×10^{-9}
k =4	793.7×10^{-6}	115.3×10^{10}	14.7×10^{-6}	84.9×10^{-12}	43.6×10^{-5}	24.1×10^{-9}
k =5	125.8×10^{-5}	113.2×10^{12}	47.5×10^{-6}	18.2×10^{-11}	82.9×10^{-5}	46.2×10^{-9}
k =6	185.3×10^{-5}	566.8×10^{13}	91.1×10^{-6}	46.6×10^{-11}	89.4×10^{-5}	52.1×10^{-9}
k =7	215.0×10^{-5}	120.9×10^{15}	13.0×10^{-4}	16.2×10^{-9}	11.9×10^{-3}	12.2×10^{-7}

Number of the grid point , N; $M = N + 2$

5. CONCLUSION

This article suggests a multi-boundary La-DQM for the structure of high-order differential situations. In the DSC theory 4 relations are used as advanced vague in combination with the La-GDQ handle. In addition, the equation and details on the implementation of multi-boundary conditions for weighting coefficients are given. Two outlines are used in ettension, one of the 6th and one of the 8st arrangement, to demonstrate the implementation of the proposed system.

The GDQR (Wu and Liu 2000, Liu and Wu 2002, Lal and Saini 2020) and the splinter method (Amin, Shah et al. 2021)are linked to numerical effects. The GRQR is widespread. The La-DQM is even higher than the Spline Technique. The latest findings seemed to be in harmony with GDQR etceptionally well. More often than not the GDQR is taking the recommended approach. In any case, in view of the completity of the GDQR approach, like loner polynomials, the suggested arrangement is actually more direct. As a result, the La-DQM can essentially be etpanded to include two-dimensional problems that are relatively complicated for the GDQR structure. We also accept the solution to unraveling the differential terms of the Tall Arrangement is of tremendous benefit.

In etpansion to multi-boundary conditions, it ought to moreover be recollected that the framework dissemination utilized is another imperative calculate deciding the number method's effectiveness, e.g. etactness and steadiness. For occurrence, the DQM and the GDQR strategies alluded to in this report utilize the & technique and Chebyshev lattice separately, which improve the accuracy of arrangements gradually shifting but unfavorable for HF modes. In arrange to be basic, a uniform framework appears the pro-posited approach. Of course, within the proposed arrangement we ought to utilize the Chebyshev focuses on the off chance that distant better; a much better; a higher; a stronger; an improved an improved precision for the lower course possess modes is required.

ACKNOWLEDGMENTS

I am thankful for the ability of the President of the University of Salahaddin to write the analysis and article And I thank Dr. Saeed Omar Abdul Rahim, the Dean of the College of Education, Dr. Hassan Muhammad, head of the Department of Physics for his instructions. I thank Dr.Star Othman for providing details and the chance to write this study on Mathematical Physics.

REFERENCE

Akram, G. and H. U. Rehman (2013). "Numerical solution of eighth order boundary value problems in reproducing Kernel space." Numerical Algorithms **62**(3): 527-540.

Amin, R., et al. (2021). "Efficient Numerical Algorithm for the Solution of Eight Order Boundary Value Problems by Haar Wavelet Method." International Journal of Applied and Computational Mathematics **7**(2): 1-18.

Bishop, R., et al. (1989). "On coupled bending and torsional vibration of uniform beams." Journal of Sound and Vibration **131**(3): 457-464.

Boutayeb, A. and E. Twizell (1991). "Finite-difference methods for twelfth-order." Journal of Computational and Applied Mathematics **35**(1-3): 133-138.

Boutayeb, A. and E. Twizell (1993). "Finite-difference methods for the solution of special eighth-order boundary-value problems." International Journal of Computer Mathematics **48**(1-2): 63-75.

Chen, C.-N. (2000). "Solution of anisotropic nonuniform plate problems by the differential quadrature finite difference method." Computational mechanics **26**(3): 273-280.

Djidjeli, K., et al. (1993). "Numerical methods for special nonlinear boundary-value problems of order 2m." Journal of Computational and Applied Mathematics **47**(1): 35-45.

El-Gamel, M., et al. (2019). "Bernoulli polynomial and the numerical solution of high-order boundary value problems." Mathematics in Natural Science **4**(01): 45-59.

Gutierrez, R. and P. Laura (1995). "Vibrations of non-uniform rings studied by means of the differential quadrature method." Journal of Sound and Vibration **185**(3): 507-513.

Howard, L. N. (1962). "Hydrodynamic and Hydromagnetic Stability. By S. CHANDRASEKHAR. Clarendon Press: Oxford University Press, 1961. 652 pp.£ 5. 5s." Journal of Fluid Mechanics **13**(1): 158-160.

Lal, R. and R. Saini (2020). "Vibration analysis of FGM circular plates under non-linear temperature variation using generalized differential quadrature rule." Applied Acoustics **158**: 107027.

Liu, G. and T. Wu (2002). "Differential quadrature solutions of eighth-order boundary-value differential equations." Journal of Computational and Applied Mathematics **145**(1): 223-235.

Shu, C. (1991). Generalized differential-integral quadrature and application to the simulation of incompressible viscous flows including parallel computation, ProQuest Dissertations & Theses.

Shu, C. and C. Wang (1999). "Treatment of mixed and nonuniform boundary conditions in GDQ vibration analysis of rectangular plates." Engineering structures **21**(2): 125-134.

Twizell, E. H., et al. (1994). "Numerical methods for eighth-, tenth-and twelfth-order eigenvalue problems arising in thermal instability." Advances in Computational Mathematics **2**(4): 407-436.

Wei, G. (1999). "Discrete singular convolution for the solution of the Fokker–Planck equation." The Journal of chemical physics **110**(18): 8930-8942.

Wu, T. and G. Liu (2000). "Application of generalized differential quadrature rule to sixth-order differential equations." Communications in Numerical Methods in Engineering **16**(11): 777-784.

Wu, T. and G. Liu (2001). "The generalized differential quadrature rule for fourth-order differential equations." International Journal for Numerical Methods in Engineering **50**(8): 1907-1929.