## Engineering Analysis

Civil Engineering Department Sazan Nariman Abdulhamid

1- Solve the DE $\quad x y=\left(1-x^{2}\right) \frac{d y}{d x} ; \quad$ given $y=1$ when $\mathrm{x}=0$

2- Solve $x y^{3} d y=\left(x^{4}+y^{4}\right) d x$

3- Solve $\quad x . d y-y . d x=0$

4- Solve $\sinh (x) \cdot \sinh (y)+\cosh (x) \cdot \cosh (y) \cdot y^{\prime}=0$

5- Solve $\cos (x+y)+\left(3 y^{2}+2 y+\cos (x+y)\right) \cdot y^{\prime}=0$

6- Solve $(\cos \omega x+\omega \cdot \sin \omega x) d x+e^{x} d y=0$

7- Solve $[\cos x \cdot \tan y+\cos (x+y)] d x+\left[\sin x \cdot \sec ^{2} y+\cos (x+y)\right] d y=0$

8 - Solve $2 x y \cdot y^{\prime}=x^{2}+y^{2}$

9- Solve $\left(1+y^{2} \sin 2 x\right) d x-2 y \cos ^{2} x d y=0$

10- Solve $y^{\prime}+2 \cdot \sin 2 x \cdot y=2 e^{\cos 2 x}$

11- Solve $y^{\prime}=6(y-2.5) \tanh (1.5 x)$

12- Solve $y^{\prime} \cdot \cos ^{2} x+3 y=1$

13- Solve $y^{\prime}+2 y=4 \cos 2 x$

14- Solve $y^{\prime}+y=y^{3}$

15- Solve $y^{\prime}=5.7 y-6.5 y^{2}$

16- A tank initially contains 50 gallons of brine, with 30 lb of salt in solution. A brine of $1 / 6$ pounds of salt per gallon of water is added to the tank at the rate of $\mathbf{3}$ gallons per minute. The mixture in the tank is kept uniform by stirring. Brine runs out (drained) from the tank at the same rate .

Find the amount of salt in the tank at any time t?

17- A tank initially contains $\mathbf{6 0}$ gallons of brine, with 20 lb of salt in solution. A brine of $1 / 10$ pounds of salt per gallon of water is added to the tank at the rate of $\mathbf{5}$ gallons per minute. The mixture in the tank is kept uniform by stirring. Brine runs out (drained) from the tank at the rate $\mathbf{2}$ gallons per minute.
Find;
a) The amount of salt in the tank at any time t?
b) Time of overflow?
c) Amount of salt at overflow?

18- A boiling solution $\left(100^{\circ} \mathrm{C}\right)$ is set in room of temp $\left(20^{\circ} \mathrm{C}\right)$, after 5 min the solution cooled to $\left(60^{\circ} \mathrm{C}\right)$.
When will the temp. of the solution be $\left(22^{\circ} \mathrm{C}\right)$.

19- In winter season, the daytime temperature in a particular building is maintained at $70^{\circ} \mathrm{F}$. The heating shut off at 10 P.M. and turned on at 6 A.M.
On a certain day the temperature inside the building at 2 A.M. was found to be $65^{\circ} \mathrm{F}$.

The outside temperature was $50^{\circ} \mathrm{F}$ at $10 \mathrm{P} . \mathrm{M}$. and had dropped to $40^{\circ} \mathrm{F}$ by $6 \mathrm{~A} . \mathrm{M}$.
What was the temp. inside the building when the heat turned on at 6 A.M.

20- A tank contains 8 L (liters) of water in which is dissolved 32 g (grams) of chemical. A solution containing $2 \mathrm{~g} / \mathrm{L}$ of the chemical flows into the tank at a rate of $4 \mathrm{~L} / \mathrm{min}$, and the well-stirred mixture flows out at a rate of $2 \mathrm{~L} / \mathrm{min}$.
a- Determine the amount of chemical in the tank after 20 minutes.
b- What is the concentration of chemical in the tank at that time?

21- A cold juice initially at $35^{\circ} \mathrm{F}$ warms up to $40^{\circ} \mathrm{F}$ in 3 min while sitting in a room of temperature $70^{\circ} \mathrm{F}$. How warm will the juice be if left out for 20 min ?

22- $\quad$ Suppose we have an inverted conical tank with height H and radius $R$, suppose fluid is flowing through a hole in the bottom with cross sectional area a with velocity given by $\mathrm{V}(\mathrm{t})=\mathrm{k}[2 \mathrm{~g} \mathrm{~h}(\mathrm{t})]^{1 / 2}$; where $\mathrm{h}(\mathrm{t})$ is the height of fluid in the tank.


Find the time required to empty the tank.

23- Find the lateral displacement of a hinged-hinged column (or hinged-hinged beam) of length (L) subjected to an axial load (P) and a uniform lateral distributed load (W).

24- Find the Buckling of a hinged-hinged column

25- Find the lateral displacement of a hinged-hinged column (or hinged-hinged beam) of length ( $L$ ) subjected to an axial load ( $P$ ) and a uniform lateral distributed load (W).

26- Solve $y^{\prime \prime}-y^{\prime}-12=0$

27- Solve $y^{\prime \prime}+5 y^{\prime}+4 y=0$

28- Solve $y^{\prime \prime}-5 y^{\prime}=0$

29- Solve $y^{\prime \prime}-9 y=0$

30- Solve $4 y^{\prime \prime}+y=0$

31- Solve $y^{\prime \prime}+y=0$

32- Solve $4 y^{\prime \prime}+y^{\prime}=0$

33- Solve $y^{\prime \prime}+y^{\prime}+y=0$

34- A spring with a mass 2 kg has natural length 0.5 m . A force of 25.6 N is required to maintain it stretched to a length of 0.7 m . If the spring is stretched to a length of 0.7 m and then released with initial velocity zero, find the position of the mass at any time t .

35- Solve the differential equation:

$$
y^{\prime \prime}+3 y^{\prime}+2 y=x^{2} \ldots \ldots \ldots . .(1)
$$

36- Solve the differential equation:

$$
\begin{equation*}
y^{\prime \prime}+9 y=e^{-4 x} \tag{1}
\end{equation*}
$$

37- Solve the differential equation:

$$
y^{\prime \prime}-4 y^{\prime}-5 y=\cos (2 x)
$$

38- Solve the differential equation:

$$
y^{\prime \prime}-y=e^{x}
$$

39- Solve the differential equation:

$$
y^{\prime \prime}-6 y^{\prime}+9 y=6 e^{3 x}
$$

40- Solve the differential equation:

$$
y^{\prime \prime}+y^{\prime}+y=x^{2}+e^{x}
$$

41- Solve the differentail equ:

$$
y^{\prime \prime}-2 y^{\prime}-3 y=e^{2 t}+3 t^{2}+4 t-5+5 \cos (2 t)
$$

42- $\quad$ Solve the differentail equ:

$$
y^{\prime \prime}+9 y=-4 x \sin (3 x)
$$

43- Write down the form of the particular solution to:

$$
y^{\prime \prime}+y^{\prime}+y=G(t)
$$

For the following $G(t)^{\prime}$ s:

$$
\begin{aligned}
& G(t)=16 e^{7 t} \cdot \sin (10 t) \\
& G(t)=\left(9 t^{2}-103 t\right) \cos t
\end{aligned}
$$

44- Write down the form of the particular solution to the following G(t):

$$
y^{\prime \prime}+y^{\prime}+y=G(t)
$$

$G(t)=e^{-2 t}(3-5 t) \cos 9 t$
$G(t)=4 \cos (6 t)-9 \sin (6 t)$
$G(t)=-2 \sin t+\sin (14 t)-5 \cos (14 t)$

45- $\quad$ Find the maximum deflection of simply supported beam subjected to uniform distributed load:


Deflection shape
(elastic curve)
46- Buckling of the hinged-hinged columns

47- Find the lateral displacement of a hinged-hinged column (or hinged-hinged beam) of length ( $L$ ) subjected to an axial load (P) and a uniform lateral distributed load (W) .


48- Buckling of the fixed-hinged columns


49- Solve; $L\left(5 e^{-3 t} \sinh \quad 2 t\right)$
50- So $L\left\lfloor 2 e^{3 t}(4 \cos 2 t-5 \sin 2 t)\right\rfloor$
51- Solve;

$$
L\left\lfloor 3 e^{-2 t}(\sinh \quad 2 t-2 \cosh \quad 2 t)\right]
$$

52- Use the Laplace transform of the Second derivative to derive:

$$
L(\sin a t)=\frac{a}{s^{2}+a^{2}}
$$

53- Use the Laplace transform of the Second derivative to derive:

$$
L(\sinh \quad a t)=\frac{a}{s^{2}-a^{2}}
$$

54- Use the Laplace transform of the Second derivative to derive:

$$
L(\cosh \quad a t)=\frac{s}{s^{2}-a^{2}}
$$

55- Use the Laplace transform of the Second derivative to determine the Laplace of:

$$
L\{t \cdot \cos \omega t\}
$$

56- Solve; $\quad \mathcal{L}^{-1}\left\{\frac{4 s-3}{s^{2}-4 s-5}\right\}$

57- Solve; $L^{-1}\left\{\frac{2(s+1)}{s^{2}+2 s+10}\right\}$

58- Solve; $L^{-1}\left\{\frac{7 s+13}{s\left(s^{2}+4 s+13\right)}\right\}$

59- Determine the following inverse Laplace transforms

$$
\mathcal{L}^{-1} \frac{(s+3)}{s(s-1)(s+2)}
$$

60- Determine the following inverse Laplace transforms

$$
\mathcal{L}^{-1} \frac{(s-1)}{s^{2}+2 s-8}
$$

61- Determine the following inverse Laplace transforms

$$
\mathcal{L}^{-1} \frac{3 s+7}{s^{2}-2 s+5}
$$

62- Determine the following inverse Laplace transforms

$$
\mathcal{L}^{-1} \frac{e^{-7 s}}{(s+3)^{3}}
$$

63- Use Laplace transforms to solve the differential equation

$$
\frac{d^{2} y}{d x^{2}}+6 \frac{d y}{d x}+13 y=0, \text { given that when } x=0, y=3 \text { and } \frac{d y}{d x}=7
$$

64- Use Laplace transforms to solve the differential equation

$$
\frac{d^{2} y}{d x^{2}}-7 \frac{d y}{d x}+10 y=e^{2 x}+20, \text { given that when } x=0, y=0 \quad \text { and } \quad \frac{d y}{d x}=-\frac{1}{3} .
$$

65- Use Laplace transforms to solve the differential equation

$$
\frac{d^{2} y}{d x^{2}}+16 y=10 \cos 4 x \quad \text {, given that when } \quad y(0)=3 \quad \text { and } \quad y^{\prime}(0)=4
$$

66- Use the convolution of the Laplace transforms to solve the differential equation:

$$
\frac{d y}{d x}-a y=e^{c t}, \text { at } \mathrm{y}(0)=0
$$

67- Find the inverse of the Laplace transform using Convolution theorem for function:

$$
H(s)=\frac{2 s}{\left(s^{2}+1\right)^{2}}
$$

68- Use the convolution of the Laplace transforms to solve the differential equation:

$$
\begin{array}{lr}
y^{\prime \prime}+2 y^{\prime}+2 y=\sin \alpha t & y(0)=0 \\
y^{\prime}(0)=0
\end{array}
$$

69- Apply convolution theorem to evaluate:

$$
L^{-1}\left\{\frac{1}{s^{2}(s-1)}\right\}
$$

70- Apply convolution theorem to evaluate:

$$
L^{-1}\left\{\frac{s}{\left(s^{2}+4\right)^{2}}\right\}
$$

71- Apply convolution theorem to evaluate:

$$
L^{-1}\left\{\frac{1}{s\left(s^{2}+4\right)^{1}}\right\}
$$

72- Apply convolution theorem to evaluate:

$$
L^{-1}\left\{\frac{1}{(s+a)(s+b)}\right\}
$$

73-

