CALCULUS

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Question Bank

1. Prove that for all real numbers x and y
$$|x + y|^2 + |x - y|^2 = 2|x|^2 + 2|y|^2.$$

2. Prove that for real numbers x and y
$|x + y| = |x| + |y|$ if and only if $xy \geq 0$.

3. Prove that for real numbers x and y

$|x + y| < |x| + |y|$ if and only if $xy < 0$.

4. Which of the following statements are true?
   a) $\{x : |3 - x| < 4\} = \{x : -1 < x < 7\}$
   b) $\{x : |4 - x| < 1\} = \{x : 3 < x < 5\}$
   c) $\{x : |1 - x| < 2\} = \{x : 1 < x < 3\}$

5. Find the domains and ranges of the function $y = f(x) = \sec(x)$.

6. Find the domains and ranges of the function $y = f(x) = \ln(x)$.

7. Find the domains and ranges of the function
$$y = f(x) = \begin{cases} x & if \quad x \leq 3 \\ 3 & if \quad x > 3 \end{cases}.$$

8. Let $f : R \to R$ and $f(x) = x^3 + 3$. Find $f^{-1}(x)$ if possible?

9. For each of the following functions find its domain.
$y = \sin(\frac{1}{x})$, $y = \ln(x + 3)$ and $y = \ln(\cos(x))$.

10. Let $f : R \to R$ and $f(x) = x^3 + 3$. Find $f^{-1}(x)$ if possible?

11. Which of the following functions is even, odd or neither?

1) $f(x) = x^4 + 3x^2 - 1$ 2) $f(x) = x + sinx$ 3) $f(x) = x^2 + 2x.$

12. Determine the range of the given function $f(x) = \frac{|x|}{x}$.

13. Determine the range of the given function $f(x) = sin^2(x)$.

14. Determine the range of the given function $f(x) = e^x.$

15. Find the solution of the following inequality $x^2 - 3x + 2 > 0$.

16. Find the solution of the following inequality $0 \leq |x - 4| \leq 4$.

17. Find the solution of the following inequality $\frac{-x}{3} \geq 2x - 1.$

18. Find the domain of the following function $f(x) = e^x$.

19. Find the domain of the following function $f(x) = \csc(x)$.

20. Find the domain of the following function $f(x) = \begin{cases} 1 - x^2 & if & x \le 0 \\ x & if & x > 0 \end{cases}$.

21. Determine the range of the given function $f(x) = \frac{|x|}{x}$.

22. Determine the range of the given function $f(x) = \ln(x)$.

23. Determine the range of the given function $f(x) = \cos^2(x)$.

24. Which of the following functions is even, odd or neither?

1) $f(x) = x + \cos x$ 2) $f(x) = x^4 + 3x^2 - 1$ 3) $(x) = -x|x|$.

25. Find the solution of the following inequality $x^2 - 3x - 4 < 0$.

26. Find the solution of the following inequality $0 \le |x - 2| \le 4$.

27. Let $f: R \to R$ and $f(x) = x^3 + 6$. Find $f^{-1}(x)$ if possible?

28. Evaluate the following limit: $\lim\limits_{x \to \infty} \frac{\ln(2+e^{3x})}{\ln(1+e^x)}$.

29. Evaluate the following limit: $\lim\limits_{x \to \infty} \frac{x^2-3x+7}{x^3+10x-4}$.

30. Evaluate the following limit: $\lim\limits_{x \to 0} \frac{4x}{\tan 3x+\sin 2x}$.

31. Evaluate the following limit: $\lim\limits_{x \to 0} x^2\cos(\frac{1}{x})$.

32. Evaluate the following limit if exist: $\lim\limits_{x \to \infty} \frac{\sqrt{16x^2+5}}{2x-3}$

33. Evaluate the following limit if exist: $\lim\limits_{x \to -\infty} \frac{3x^2+4x+3}{x^3+x+14}$

34. Evaluate the following limit if exist: $\lim\limits_{x \to \infty} \frac{2x^2-3x+5}{x-4}$.

35. Evaluate the following limit if exist: $lim_{x\to\infty} \dfrac{10x^3-3x^2+10}{\sqrt{25x^6+x^4+2}}$.

36. Find the following limits by using squeezing theorem:

1) $lim_{x\to\infty} \dfrac{\cos(x)}{x}$.

2) $lim_{x\to-\infty} \dfrac{\sin(x)}{x}$.

3) $lim_{x\to-\infty} \dfrac{\cos(x)}{x}, lim_{x\to-\infty} \dfrac{(\cos(x))^2}{x}$.

4) $lim_{x\to0^-} x^3\cos(\frac{2}{x})$.

5) $lim_{x\to-\infty} \dfrac{2-\cos(x)}{x+3}$.

37. Find the following limits:

1) $lim_{x\to\infty}(\dfrac{x+\cos(x)}{x})^{x\sec(x)}$.

2) $lim_{x\to1} \dfrac{\sin(x-1)}{x^2+x-2}$.

3) $lim_{\theta\to0} \dfrac{\sin(\theta)}{\theta+\tan(\theta)}$.

4) $lim_{\theta\to0} \dfrac{\cos(\theta)-1}{\sin(\theta)}$.

5) $lim_{x\to0} \dfrac{\sin(x^2)}{x}$.

38. Find the points of discontinuity of the function $f(x) = \begin{cases} 0 & \text{if } x < -2 \\ -x & \text{if } -2 \le x < 0 \\ x^2 - 1 & \text{if } x \ge 0 \end{cases}$.

39. Find the points of discontinuity of the function $f(x) = \begin{cases} \dfrac{x^2-9}{x-3} & , & x \ne 3 \\ 4 & , & x = 3 \end{cases}$.

40. Find the value of the constant k so that the function f defined below is continuous at $x = 0$

$$f(x) = \begin{cases} \dfrac{1-\cos(4x)}{8x^2} & x \ne 0 \\ k & x = 0 \end{cases}$$

41. Prove that $\cosh^2 x - \sinh^2 x = 1$.

42. Find the following limits:

1- $\lim_{x\to\infty} \dfrac{\ln(\ln x)}{x}$.

2- $\lim_{x\to\frac{\pi}{4}} [(1-\tan x)\sec 2x]$.

3- $\lim_{x \to 0} (1 + sin2x)^{\frac{1}{x}}$

43. Find the following limits:

1- $\lim_{x \to 1} \frac{lnx}{x-1}$.

2- $\lim_{x \to \infty} x\, e^{-x}$.