CALCULUSI

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Question Bank First Semester 2023-2024

1. Prove that for all real numbers x and y
$$|x+y|^2+|x-y|^2=2|x|^2+2|y|^2.$$
2. Prove that for real numbers x and y
$|x+y|=|x|+|y|$ if and only if $xy\geq 0$.
3. Prove that for real numbers x and y

$|x+y|<|x|+|y|$ if and only if $xy<0$.

4. Which of the following statements are true?
   a) $\{x:|3-x|<4\}=\{x:-1<x<7\}$
   b) $\{x:|4-x|<1\}=\{x:3<x<5\}$
   c) $\{x:|1-x|<2\}=\{x:1<x<3\}$
5. Find the domains and ranges of the function $y=f(x)=\sec(x)$.
6. Find the domains and ranges of the function $y=f(x)=\ln(x)$.
7. Find the domains and ranges of the function $y=f(x)=$
$$\begin{cases} x & if \quad x\leq 3 \\ 3 & if \quad x>3 \end{cases}.$$
8. Let $f:R\to R$ and $f(x)=x^3+3$. Find $f^{-1}(x)$ if possible?
9. For each of the following functions find its domain.
$y=\sin(\frac{1}{x})$, $y=\ln(x+3)$ and $y=\ln(\cos(x))$.
10. Let $f:R\to R$ and $f(x)=x^3+3$. Find $f^{-1}(x)$ if possible?
11. Which of the following functions is even, odd or neither?

1) $f(x)=x^4+3x^2-1$ 2) $f(x)=x+sinx$ 3) $f(x)=x^2+2x.$

12. Determine the range of the given function $f(x)=\frac{|x|}{x}$.
13. Determine the range of the given function $f(x)=sin^2(x)$.
14. Determine the range of the given function $f(x)=e^x$.
15. Find the solution of the following inequality $x^2-3x+2>0$.
16. Find the solution of the following inequality $0\leq|x-4|\leq 4$.
17. Find the solution of the following inequality $\frac{-x}{3}\geq 2x-1.$

18. Find the domain of the following function $f(x) = e^x$.

19. Find the domain of the following function $f(x) = \csc(x)$.

20. Find the domain of the following function $f(x) = \begin{cases} 1 - x^2 & if \quad x \leq 0 \\ x & if \quad x > 0 \end{cases}$.

21. Determine the range of the given function $f(x) = \frac{|x|}{x}$.

22. Determine the range of the given function $f(x) = \ln(x)$.

23. Determine the range of the given function $f(x) = cos^2(x)$.

24.Which of the following functions is even, odd or neither?

1) $f(x) = x + cosx$ 2) $f(x) = x^4 + 3x^2 - 1$ 3) $f(x) = -x|x|$.

25. Find the solution of the following inequality $x^2 - 3x - 4 < 0$.

26. Find the solution of the following inequality $0 \leq |x - 2| \leq 4$.

27. Let $f: R \to R$ and $f(x) = x^3 + 6$. Find $f^{-1}(x)$ if possible?

28. Evaluate the following limit: $\lim_{x\to\infty} \frac{ln\,(2+e^{3x})}{ln\,(1+e^{x})}$.

29. Evaluate the following limit: $\lim_{x\to\infty} \frac{x^2-3x+7}{x^3+10x-4}$.

30. Evaluate the following limit: $\lim_{x\to 0} \frac{4x}{tan3x+sin2x}$.

31. Evaluate the following limit: $\lim_{x\to 0} x^2 \cos\left(\frac{1}{x}\right)$.

32. Evaluate the following limit if exist: $\lim_{x\to\infty} \frac{\sqrt{16x^2+5}}{2x-3}$

33. Evaluate the following limit if exist: $\lim_{x\to-\infty} \frac{3x^2+4x+3}{x^3+x+14}$

34. Evaluate the following limit if exist: $\lim_{x\to\infty} \frac{2x^2-3x+5}{x-4}$.

35. Evaluate the following limit if exist: $\lim_{x\to\infty} \frac{10x^3-3x^2+10}{\sqrt{25x^6+x^4+2}}$.

36. Find the following limits by using squeezing theorem:

1) $\lim_{x\to\infty} \frac{\cos(x)}{x}$.
2) $\lim_{x\to-\infty} \frac{\sin(x)}{x}$.
3) $\lim_{x\to-\infty} \frac{\cos(x)}{x}$, $\lim_{x\to-\infty} \frac{(\cos(x))^2}{x}$.
4) $\lim_{x\to0^-} x^3\cos(\frac{2}{x})$.
5) $\lim_{x\to-\infty} \frac{2-\cos(x)}{x+3}$.

37. Find the following limits:

1) $\lim_{x\to\infty} (\frac{x+\cos(x)}{x})^{xsec(x)}$.
2) $\lim_{x\to1} \frac{\sin(x-1)}{x^2+x-2}$.
3) $\lim_{\theta\to0} \frac{\sin(\theta)}{\theta+\tan(\theta)}$.
4) $\lim_{\theta\to0} \frac{\cos(\theta)-1}{\sin(\theta)}$.
5) $\lim_{x\to0} \frac{\sin(x^2)}{x}$.