Salahaddin University-Erbil
College of Engineering
Department of Architectural Engineering
First Year Students
2nd Semester

Mathematics I

The Rate of Change of a Function(Ch.1)

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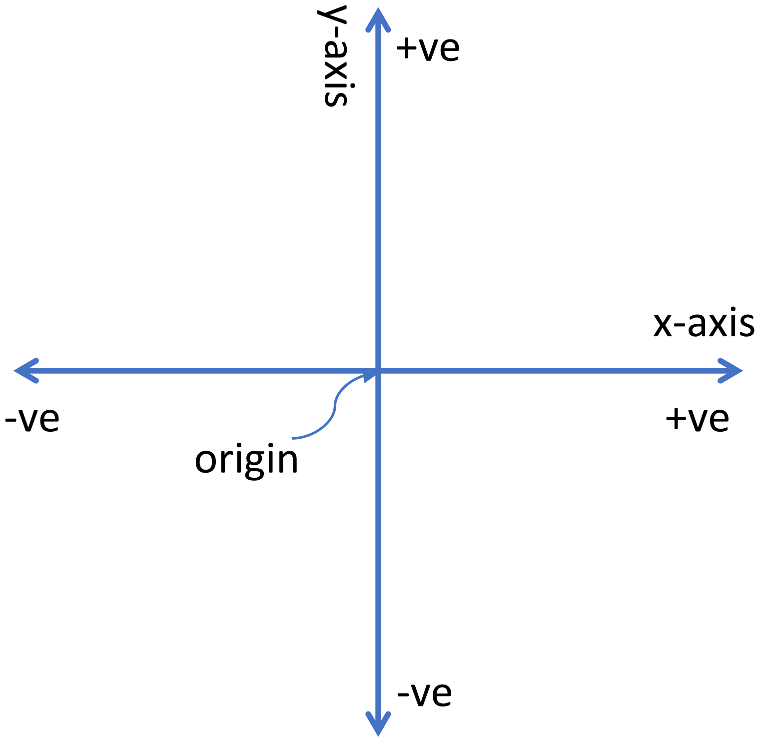
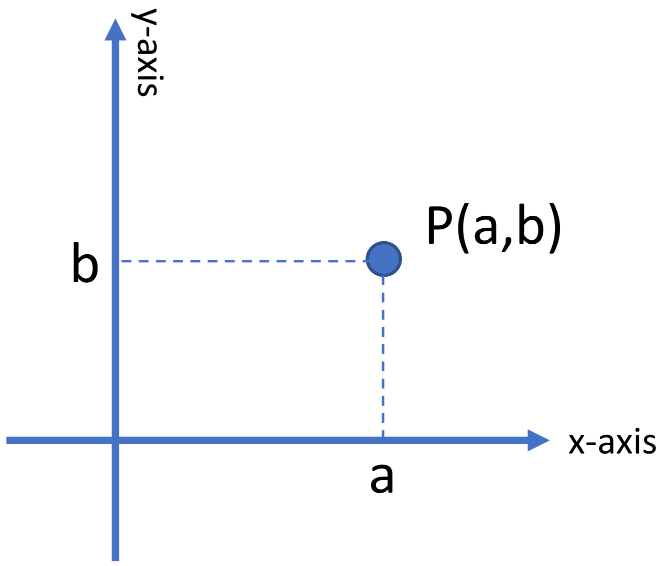
1.1. Cartesian Coordinate and Equation for Lines

- Coordinates: A pair of numbers that describe the ***position of a point*** on a coordinate plane by using the horizontal and vertical distances from the two reference axes. Usually represented by **(x,y)** the x-value and y-value.
- Coordinates make possible to describe lines, curves with coordinate equation
- Origin: The Starting point. The point where the reference axes in a coordinate system meet. The values of coordinates are normally defined as ***zero***.



1.1. Cartesian Coordinate and Equation for Lines(Cont.)

- The horizontal axis called **x-axis**, the vertical axis called **y-axis**.
- On x-axis, the **+ve number** lies to the right of origin.
- On y-axis, the **+ve number** lies above origin.
- The ordered pair (a,b) corresponds to the point P, where perpendicular to x-axis at (a) crosses the perpendicular to the y-axis at (b)



1.1. Cartesian Coordinate and Equation for Lines(Cont.)

- Increment

Definition: if a particle moves from the point (x_1,y_1) to the point (x_2,y_2) the **increments** in its coordinates are

$$\Delta x = x_2 - x_1 \quad and \quad \Delta y = y_2 - y_1$$

- Slope of a Line

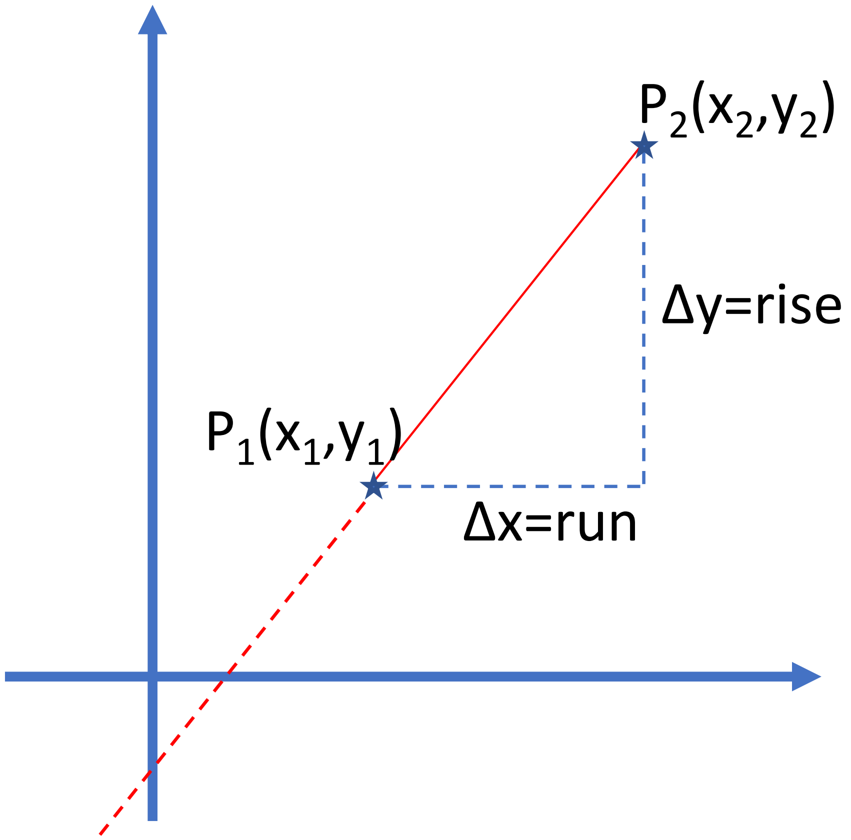
Each non-vertical line has a slope, which we can calculate from increments in coordinates.

- Let L be a non-vertical line in the plane and $P_1(x_1,y_1)$ and $P_2(x_2,y_2)$ two points on L.
- We call $\Delta y = y_2 - y_1$ the rise from P_1 to P_2 and $\Delta x = x_2 - x_1$ the run from P_1 to P_2. Since L is not vertical, $\Delta x \neq 0$

- Slope

Definition: let $P_1(x_1,y_1)$ and $p_2(x_2,y_2)$ be points on a non-vertical line L. The slope of L which is denote by m is

$$m = \frac{rise}{run} = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$$



1.1. Cartesian Coordinate and Equation for Lines(Cont.)

- Note:
  - A line that goes **uphill** as x increases has a **+ve** slope
  - A line that goes **downhill** as x increases it has **–ve** slope
  - A horizontal line has zero slope

- Example

Slope of L_1

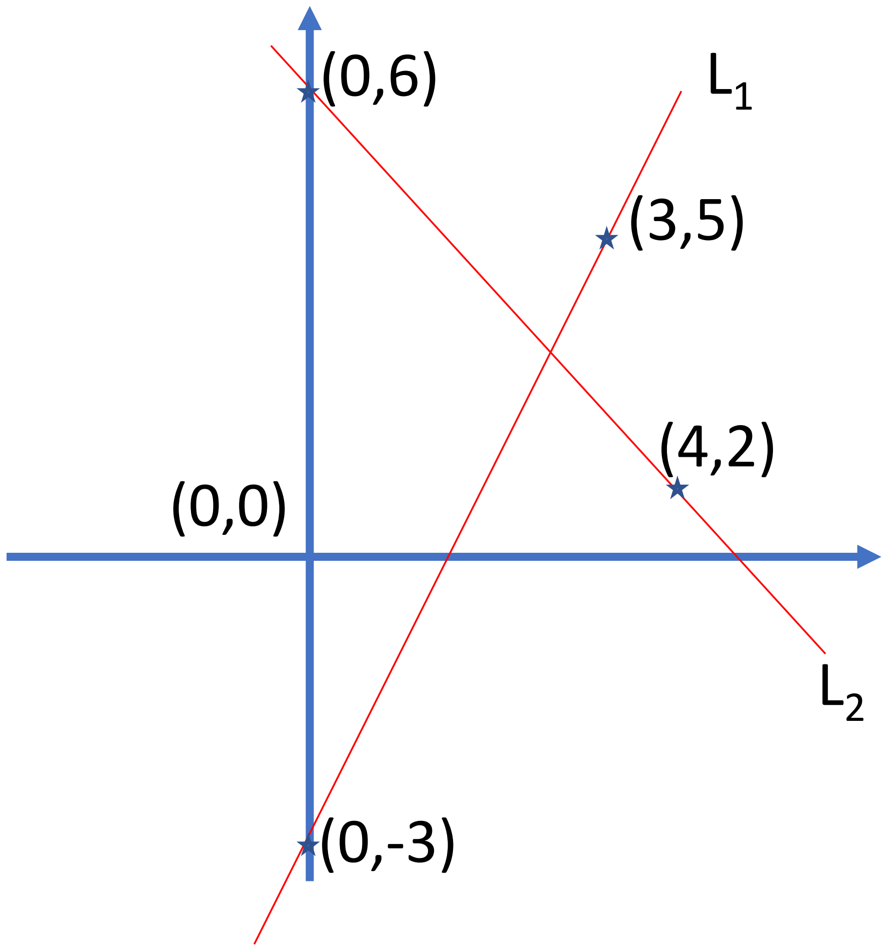
$$m_1 = \frac{\Delta y_1}{\Delta x_1}$$

$$= \frac{5-(-3)}{3-0} = \frac{8}{3}$$

Slope of L_2

$$m_2 = \frac{\Delta y_2}{\Delta x_2}$$

$$= \frac{2-6}{4-0} = \frac{-4}{4} = -1$$



1.1. Cartesian Coordinate and Equation for Lines(Cont.)

- Angle of inclination

The slope of non-vertical line is the tangent of its angle of inclination

Or $m = \tan\theta = \frac{\Delta y}{\Delta x}$

- Parallel lines

Parallel lines have the same slope and the equal angles.

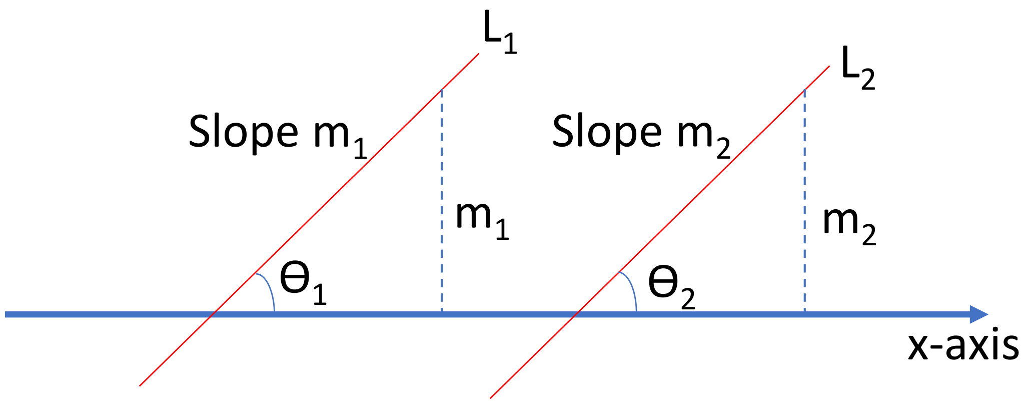
- Perpendicular lines

If two non-vertical lines L_1 and L_2 are perpendicular, their slopes m_1 and m_2 satisfy $m_1 m_2$=-1, so each slope is the negative reciprocal of the other

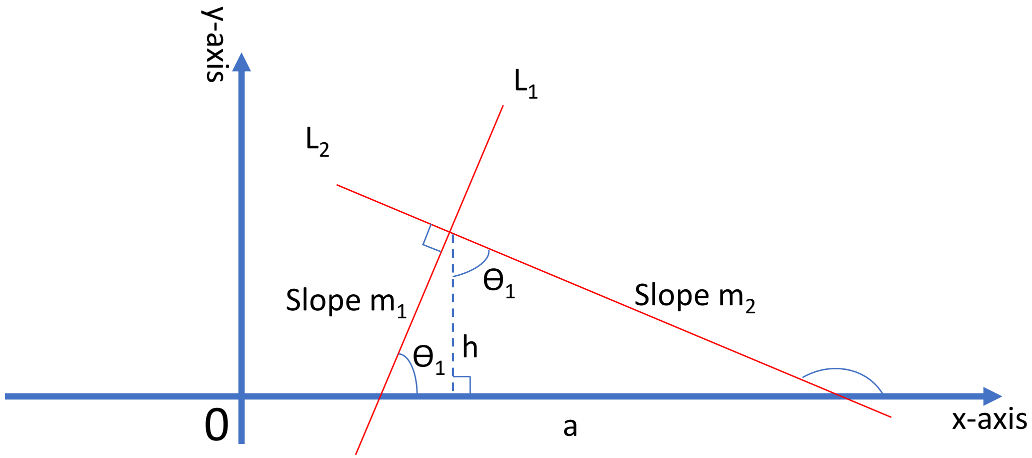
$$m_2 = -\frac{1}{m_1} \quad , \quad m_1 = -\frac{1}{m_2}$$

$$m_1 = \tan\theta_1 = \frac{a}{h} \quad , while \quad m_2 = \tan\theta_2 = -\frac{h}{a}$$

Hence, $m_1 m_2 = \frac{a}{h} \times (-\frac{h}{a})$=-1



Parallel lines



Perpendicular lines

1.1. Cartesian Coordinate and Equation for Lines(Cont.)

- Equations for lines
- These come in many useful forms:

- Point-Slope Form
- The equation of the line passing though the point $P_{1}=(x_1,y_1)$ with slope m is

$$y - y_1 = m(x - x_1)$$

- Thus given the point $P_1 = (1, 2)$ and the slope $m = -1/3$ the equation of the line is

$$y - 2 = -\frac{1}{3}(x - 1)$$

- Example

Write the equation of the line with slope m=−3 and passing through the point (4,8). Write the final equation in slope-intercept form.

1.1. Cartesian Coordinate and Equation for Lines(Cont.)

- Point-Point Form
- The equation of the line passing through the points $P_1 = (x_1, y_1)$ and $P_2 = (x_2, y_2)$ is

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

- Example

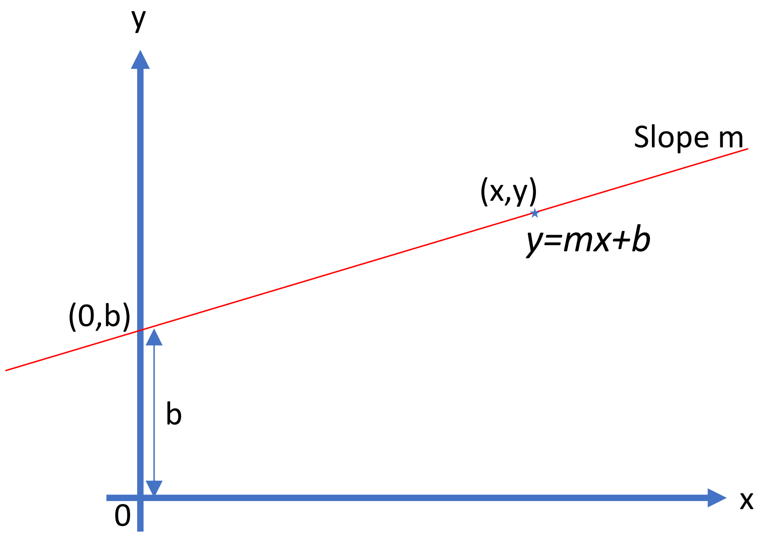
Find the slope of a line that passes through the points (2,−1) and (−5,3).

1.1. Cartesian Coordinate and Equation for Lines(Cont.)

- Slope- Intercept Form
- The equation of the line passing through the y-axis at the point (0, b) with slope m is

$$y = mx + b$$

For example, if m = −1/2 and b = 2, the equation is $y = -\frac{1}{2}x + 2$



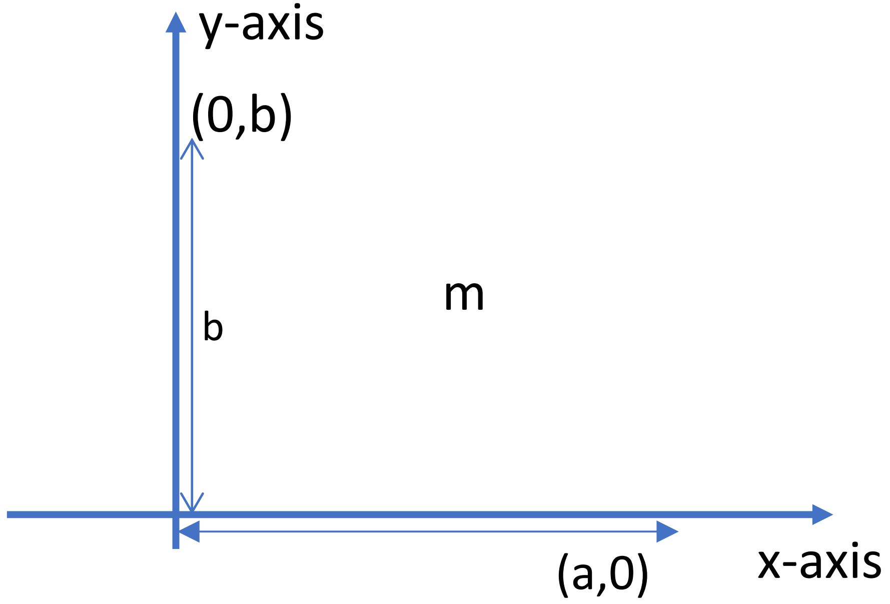
- Example

Identify the slope and y-intercept given the equation $y = -\frac{3}{4}x - 4$

1.1. Cartesian Coordinate and Equation for Lines(Cont.)

- Intercept-Intercept Form
- The equation of the line passing through the intercepts (a, 0) and (0, b) is

$$\frac{x}{a} + \frac{y}{b} = 1$$



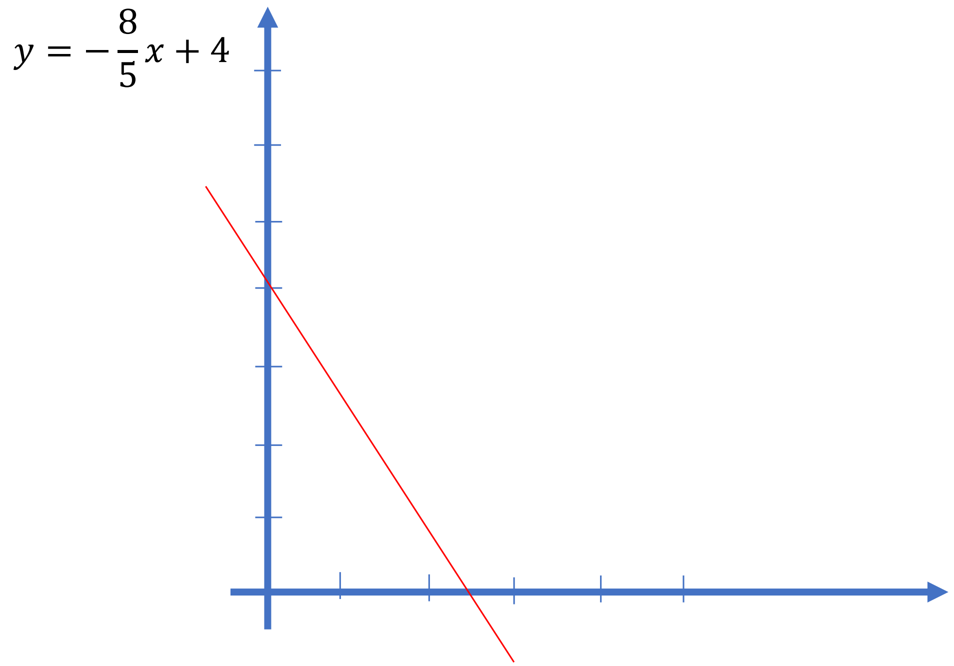
- Example
- Find the equation of the line through (4, 0) and (0, 3)

1.1. Cartesian Coordinate and Equation for Lines(Cont.)

- General Linear Equation

Every line has infinitely many equation of the form

$Ax + By = C$ (A and B not both 0)



- Example

Find the slope and y-intercept of the line 8x + 5y = 20. Graph the line

For the next lecture we will learn:

- Function and their Graphs