Salahaddin University-Erbil
College of Engineering
Department of Architectural Engineering
First Year Students
2nd Semester

Mathematics I
Application of Derivative
Extreme Value of Function (Maxima and Minima)(Ch.3)

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3.2. Maxima, Minima theorem, Local and Absolute

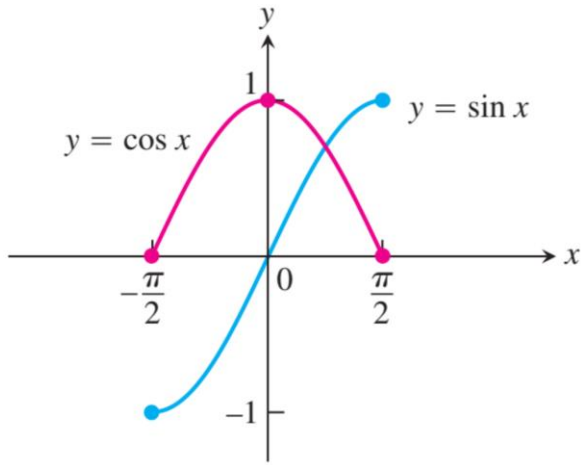
- DEFINITIONS **Absolute Maximum, Absolute Minimum**
- Let ƒ be a function with domain D. Then ƒ has an absolute maximum value on D at a point c if

$$f(x) \leq f(c)$$

and an absolute minimum value on D at c if

$$f(x) \geq f(c)$$

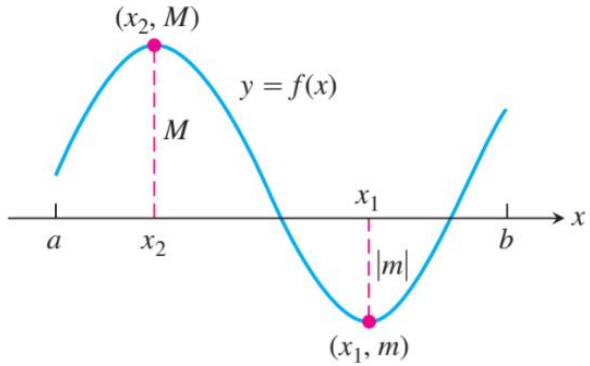
- For example, on the closed interval [-π/2,π/2] the function $f(x) = \cos x$ takes on an absolute maximum value of 1 and an absolute minimum value of 0 (twice). On the same interval, the function $g(x) = \sin x$ takes on a maximum value of 1 and a minimum value of -1.



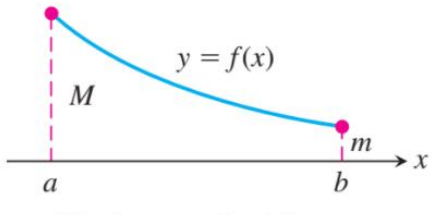
3.2. Maxima, Minima theorem, local and absolute

THEOREM 1 The Extreme Value Theorem

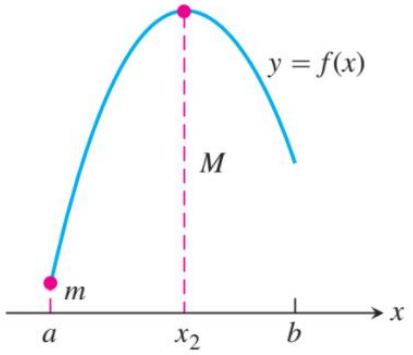
If ƒ is continuous on a closed interval [a, b], then ƒ attains both an absolute maximum value M and an absolute minimum value m in [a, b]. That is, there are numbers x_1 and x_2 in [a, b] with $f(x_1)=m$, $f(x_2)=M$, and $m \le f(x) \le M$ for every other x in [a, b]



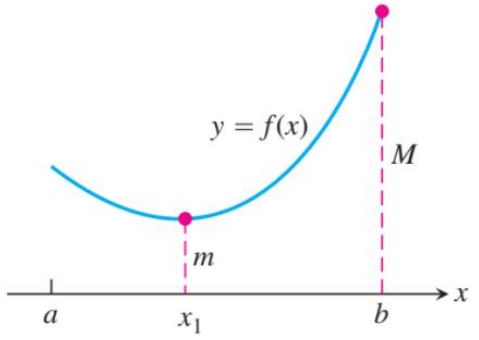
Maximum and minimum at interior points



Maximum and minimum at endpoints



Maximum at interior point, minimum at endpoint



Minimum at interior point, maximum at endpoint

Local Maximum, Local Minimum

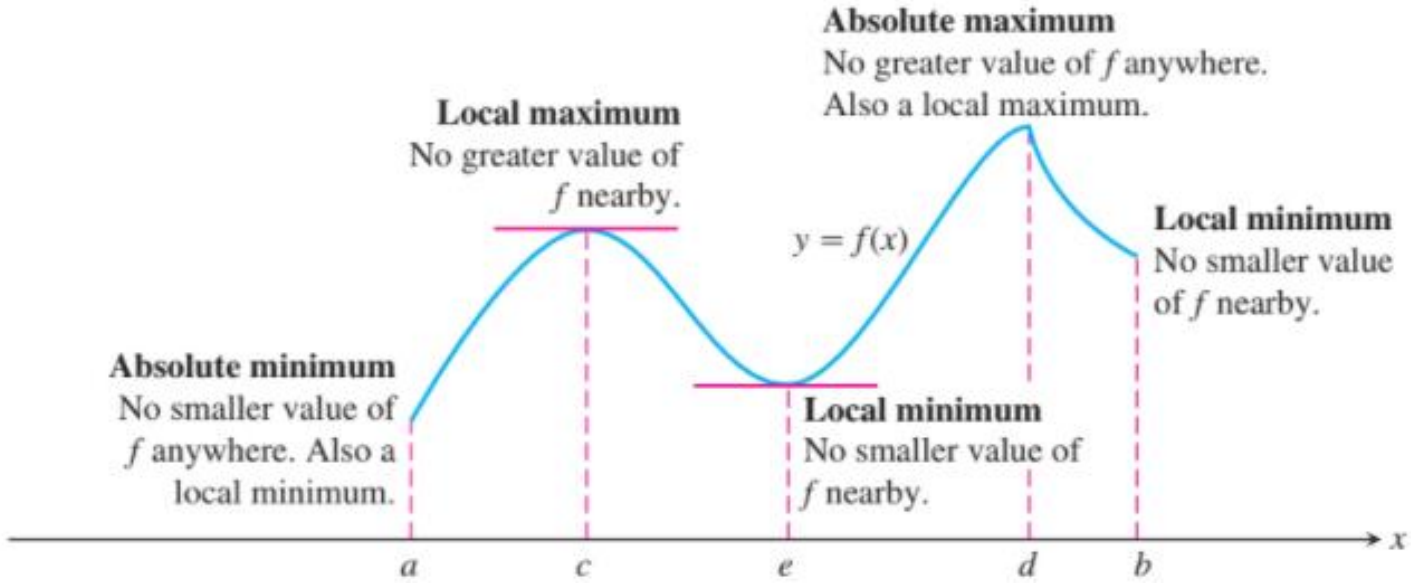
- DEFINITIONS **Local Maximum, Local Minimum**

A function ƒ has a local maximum value at an interior point c of its domain if

$$f(x) \leq f(\mathrm{c}) \text{ for all x in some open interval containing c}$$

A function ƒ has a local minimum value at an interior point c of its domain if

$$f(x) \geq f(\mathrm{c}) \text{ for all x in some open interval containing c}$$



Finding Extrema

- THEOREM 2 **The First Derivative Theorem for Local Extreme Values**

If ƒ has a local maximum or minimum value at an interior point c of its domain, and if is defined at c, then

$$f'c = 0$$

DEFINITION **Critical Point**

An interior point of the domain of a function ƒ where is zero or undefined is a critical point of ƒ.

Examples

1. Find the absolute maximum and minimum values of $f(x) = x^2$ on [-2,1]

2. Find the absolute extrema values of $g(t) = 8t - t^4$ on [-2,1]

3. Find the absolute maximum and minimum vales of $\mathrm{f}(x) = x^{2/3}$ on the interval [-2,3]

Examples

- What is the largest possible area for a right triangle whose hypotenuse is 5 cm long?
- A highway must be constructed to connect Village A with Village B that 150 mi a part. There is a rudimentary roadway that can be upgraded 50 mi south of the line connecting the two villages. The cost of upgrading the existing roadway is $300,000 per mile, whereas the cost of constructing a new highway is $500,000 per mile. Find the combination of upgrading and new construction that minimizes the cost of connecting the two villages. Cleary define the location of the proposed highway.
- A drilling rig 12 mi offshore is to be connected by pipe to a refinery onshore, 20 mi straight down the coast from the rig. If underwater pipe costs $500,000 per mile and land based pipe costs $300,000 per mile, what combination of the two will give the least expensive connection?