Salahaddin University-Erbil
College of Engineering
Department of Architectural Engineering
First Year Students
2nd Semester

Mathematics I
Application of Derivative
Concavity and curve sketching(Ch.3)

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3.2. Graphing with y' and y''

- It means how to use derivative when it exists to show the curve behavior.

First Derivative Test for Monotonic Functions

Suppose that f is continuous on $[a, b]$ and differentiable on (a, b).
If $f'x>0$ at each point x (a,b) then f is increasing on $[a, b]$.
If $f'x<0$ at each point x (a,b), then f is decreasing on $[a, b]$

- Concavity
- DEFINITION **Concave Up, Concave Down**

The graph of a differentiable function is

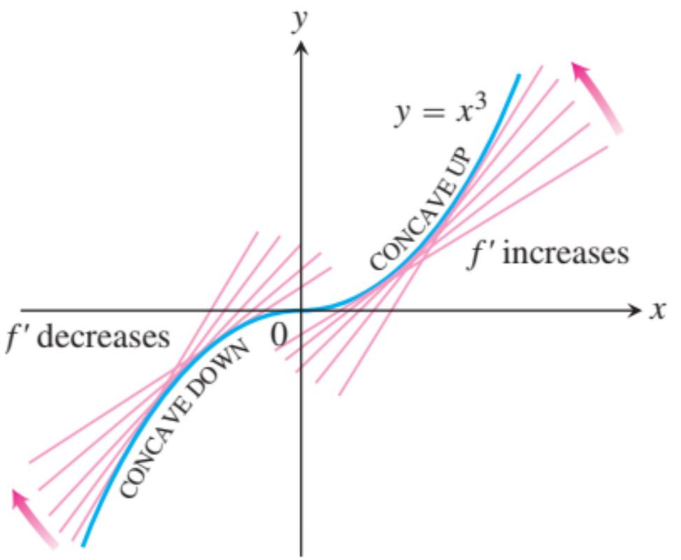
(a) concave up on an open interval I if f' is increasing on I

(b) concave down on an open interval I if f' is decreasing on I.

- The Second Derivative Test for Concavity

Let $y = f(x)$ be twice-differentiable on an interval I.

1. If $f'' > 0$ on I, the graph of f over I is concave up.
2. If $f'' < 0$ on I, the graph of f over I is concave down.

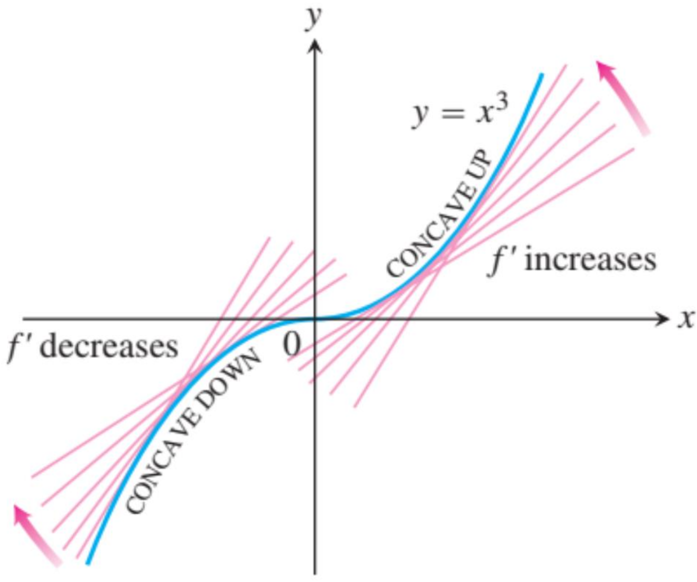


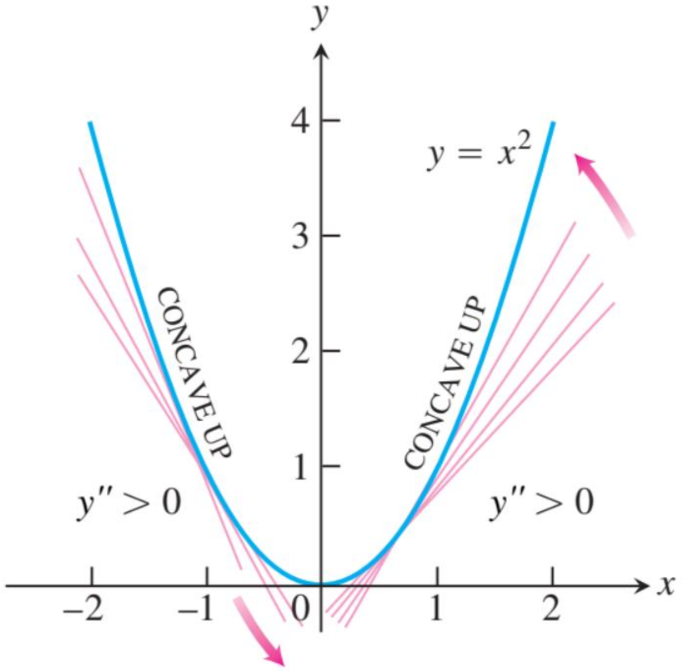
Examples

Ex.1 Applying the concavity test

a) The curve $y=x^3$ is concave down on $(-\infty,0)$ where $y''=6x<0$ and concave up on $(0,\infty)$ where $y''=6x>0$

b) The curve $y=x^2$ is concave up on $(-\infty,\infty)$ because its second derivative $y''=2$ is always positive





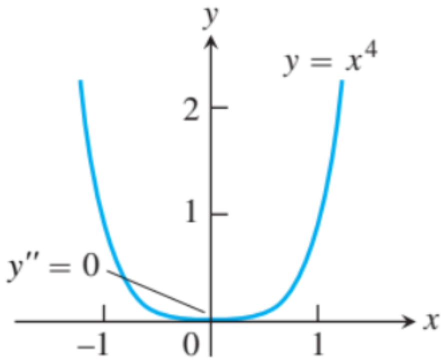
Ex.2. Determine the concavity of $y = 3 + \sin x$ on $[0,2\pi]$

- # Points of Inflection

DEFINITION Point of Inflection

A point where the graph of a function has a tangent line and where the concavity changes is a point of inflection.

Examples



- An Inflection Point May Not Exist Where y'' = 0

The curve $y=x^4$ has no inflection point at x=0. Even though $y''=12x^2$ is zero there, it does not change sign.

Example

- A particle is moving along a horizontal line with position function

$$s(t) = 2t^3 - 14t^2 + 22t - 5 \quad t \geq 0$$

- Find the velocity and acceleration, and describe the motion of the particle.

Strategy for Graphing *y* f(*x*)

1. Identify the domain of f and any symmetries the curve may have.
2. Find y' and y''
3. Find the critical points of f, and identify the function's behavior at each one.
4. Find where the curve is increasing and where it is decreasing.
5. Find the points of inflection, if any occur, and determine the concavity of the curve.
6. Identify any asymptotes.
7. Plot key points, such as the intercepts and the points found in Steps 3–5, and sketch the curve.

Example:

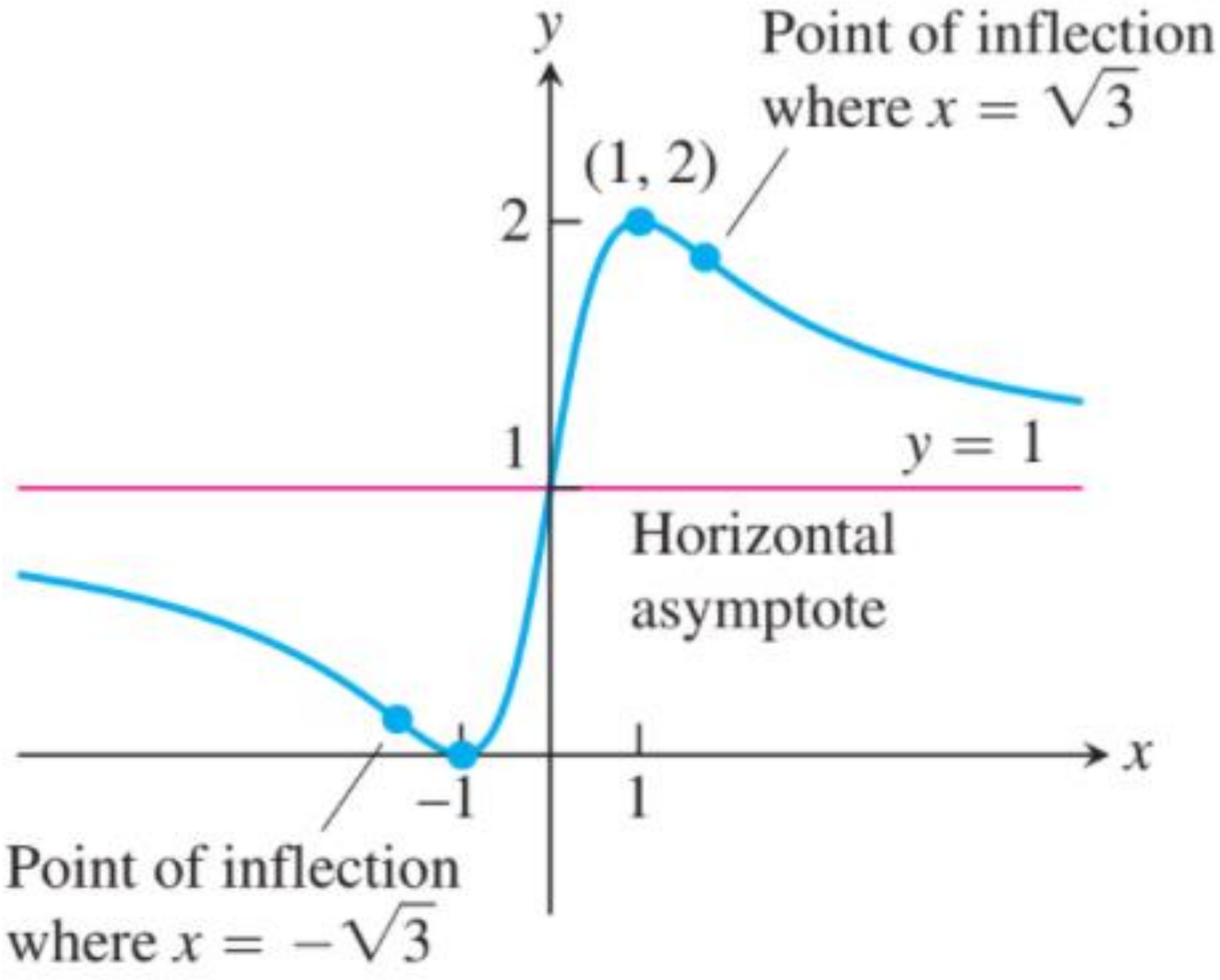
Sketch a graph of the function

$$f(x) = x^4 - 4x^3 + 10$$

Example

Sketch the graph of

$$f(x) = \frac{(x+1)^2}{1+x^2}$$



Next lecture we will learn:

- Optimization