Salahaddin University-Erbil
College of Engineering
Department of Architectural Engineering
First Year Students
2nd Semester

Mathematics I
Differentiation (Ch.2)

Shawnm Mudhafar Saleh

shawnm.saleh@su.edu.krd

2. Differentiation

- Derivative

Definition: The derivative of the function f(x) with respect to the variable x is the function f'whose value at x is

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

Provided the limit exists.

- Different Rules

1. Derivative of a constant

Suppose $y = c$ then $y' = \frac{dy}{dx} = 0$

- Example: find y' for $y = 5$

2. Integer power of x

If $y = (x)^n$

Then $y' = \frac{dy}{dx} = n(x)^{n-1}$

- Example: find y' for $y = x^2$

2. Differentiation

3. The constant multiple

If u is differentiable function of x and k is constant, then

$$\frac{d}{dx}(ku) = k.\frac{d}{dx}(u)$$

- Example: find y' for $y = 7x^4$

4. The sum and difference

If $y = u \pm v$

Then $y' = \frac{du}{dx} \pm \frac{dv}{dx}$

- Example: find y' for $y = 3x^2 + 4x - 4$

2. Differentiation

5. Products

The product of two differentiable function u and v is differentiable, for $y = u.v$, then

$$\frac{dy}{dx} = \frac{d}{dx}(u.v) = u.\frac{dv}{dx} + v\frac{du}{dx}$$

- Example: find y' for $y = \frac{1}{x}(x^2 + \frac{1}{x})$

6. The quotient

If u and v are differentiable at x and if $v(x) \neq 0$, then the quotient u/v is differentiable at x, and

$$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$$

- Example: Find y' for $y = \frac{t^2-1}{t^2+1}$

2. Differentiation

- Second and higher order derivatives

The derivative $y' = \frac{dy}{dx}$ is the first derivative of y with respect to x, the first derivative may also be a differentiable function of x, if so its derivative is second derivative

$$y'' = \frac{d'y}{dx} = \frac{d}{dx}(y') = \frac{d}{dx}\left(\frac{dy}{dx}\right)$$

$$y'' = \frac{d^2y}{dx^2}$$

Or it may be Third derivative

$$y''' = \frac{d}{dx}\left(\frac{d^2y}{dx^2}\right)$$

2. Differentiation

- Example: find the first and second derivatives

1. $s = 5t^3 - 3t^5$
2. $w = 3z^7 - 7z^3 + 21z^2$
3. $r = \frac{1}{3s^2} - \frac{5}{2s}$

Example: Find y' for

1. $y = \left(x + \frac{1}{x}\right)\left(x - \frac{1}{x} + 1\right)$
2. $y = \frac{1}{(x^2-1)(x^2+x+1)}$
3. $r = 2(\frac{1}{\sqrt{\theta}} + \sqrt{\theta})$

- Example: does the function $y = x^4 - 2x^2 + 2$ have any horizontal tangent? If so, where?
- Example: find the equation of the tangent to the curve $y = x + \frac{2}{x}$ at the point (1,3)

Examples:

$$y = \frac{x^4}{2} - \frac{3}{2}x^2 - x$$

$$s = \frac{t^2 + 5t - 1}{t^2}$$

For next lecture we will learn

- The derivatives as a rate of change