Salahaddin University-Erbil
College of Engineering
Department of Architectural Engineering
First Year Students
2nd Semester

Mathematics I

The derivative as a rate of change (Ch.2)

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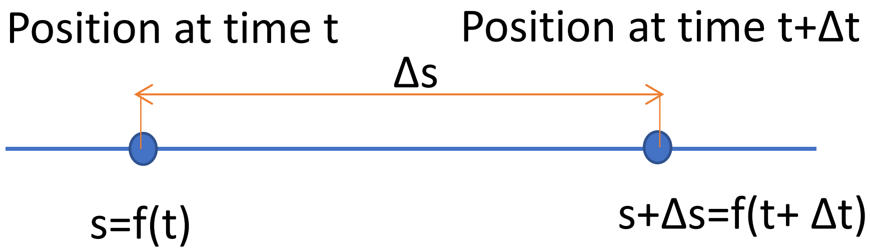
• Instantons rates of change

- Suppose that an object is moving along a coordinate line (say an s-axis) so that we know its position s on that line as a function of time t

$$s = f(t)$$

The displacement of that object over the time interval from t to t+Δt is

$$\Delta s = f(t + \Delta t) - f(t)$$



Definition: the instantaneous rate of change of f with respect to x at x_0 is the derivative

$$f'(x_0) = \lim_{h \to 0} \frac{f(x_0 + h) - f(x_0)}{h}$$

Provided the limit exists.

- Motion along a line: Displacement, Velocity, Speed, and Acceleration
- Velocity

Definition: velocity (instantaneous velocity) is the derivative of position with respect to time. If a body's position at time t is s=f(t), then the body's velocity at time t is

$$v(t) = \frac{ds}{dt} = \lim_{\Delta t \to 0} \frac{f(t + \Delta t) - f(t)}{\Delta t}$$

- Speed

Definition: speed is the absolute value of velocity

$$speed = |v(t)| = \left|\frac{ds}{dt}\right|$$

- Acceleration

Definitions: acceleration is the derivative of velocity with respect to time. If a body's position at time t is x=f(x), then the body's acceleration at time t is

$$a(t) = \frac{dv}{dt} = \frac{d^2 s}{dt^2}$$

Near the surface of the earth all bodies fall with the same constant acceleration (when air resistance is absent and the only force acting on a falling body in the force due to gravity). Galileo's experiments with free fall lead to the equation

$$s = \frac{1}{2} g t^2$$

Where s=distance, g=acceleration and its (32.2ft/sec^2) or (9.81m/sec^2)

Examples

Example 1: Modeling free fall

A heavy ball fall from rest at time t=0 sec.

a) How many meters does the ball fall in the first 2 sec.?

b) What is its velocity, speed, and acceleration then?

Example 2: Modeling vertical motion

A dynamic blast blows a heavy rock straight up with a launch velocity of 160 ft/sec.. It reaches a height of $s=160t-16t^2$ ft after t sec.

a) How high does the rock go?

b) What are the velocity and speed of the rock when it is 256ft above the ground on the way up? On the way down?

c) What is the acceleration of the rock at any time t during its flight (after the blast)?

d) When does the rock hit the ground again?

Derivative of Trigonometric Function

- Rules

$$\frac{d}{dx}(\sin x) = \cos x$$

$$\frac{d}{dx}(\cos x) = -sin\, x$$

$$\frac{d}{dx}(\tan x) = sec^2 x$$

$$\frac{d}{dx}(cot\, x) = -csc^2 x$$

$$\frac{d}{dx}(sec\, x) = \sec x . \tan x$$

$$\frac{d}{dx}(csc\, x) = -\csc x . \cot x$$

Examples:

Ex.1: Find the equation of lines that are tangent and normal to the curve $y = tanx$ at pt(π/4, 1).

Ex.: Differentiate the following functions

$$y = \frac{\sin x}{x}$$

$$y = \sin x \cos x$$

$$y = \sec x \tan x$$

$$y = \sin(1 + \tan x)$$

$$y = \tan\left(\frac{1}{x}\right)$$

$$2y = x^2 + \sin y$$

Examples

- Drive the followings:

- $p = \frac{\tan q}{1+\tan q}$
- $s = \frac{\sin t}{1-\cos t}$
- $r = 4 - \theta^2 sin\theta$
- $p = 5 + \frac{1}{\cot q}$