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Mathematics II
Chapter seven
Trigonometric Substitutions

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- Trigonometric substitutions can be effective in transforming integrals involving $\sqrt{a^2+x^2}, \sqrt{a^2-x^2}, \sqrt{x^2-a^2}$
- Three basic substitutions
- The most common substitutions are $: x = a\tan\theta, x = a\sin\theta, x = a\sec\theta$

With $x = a\tan\theta$,

$$a^2 + x^2 = a^2 + a^2\tan^2\theta = a^2(1 + \tan^2\theta) = a^2\sec^2\theta.$$

With $x = a\sin\theta$,

$$a^2 - x^2 = a^2 - a^2\sin^2\theta = a^2(1 - \sin^2\theta) = a^2\cos^2\theta.$$

With $x = a\sec\theta$,

$$x^2 - a^2 = a^2\sec^2\theta - a^2 = a^2(\sec^2\theta - 1) = a^2\tan^2\theta.$$

Evaluate

- $\int \frac{du}{a^2+u^2}$
- $\int \frac{du}{\sqrt{a^2-u^2}}$
- $\int \frac{du}{u\sqrt{u^2-a^2}}$

- $\int \frac{du}{a^2+u^2} = \frac{1}{a}\tan^{-1}\frac{u}{a} + c$
- $\int \frac{du}{\sqrt{a^2-u^2}} = \sin^{-1}\frac{u}{a} + c$
- $\int \frac{du}{\sqrt{a^2+u^2}} = \ln\left|\sqrt{a^2+u^2} + u\right| + c$
- $\int \frac{du}{\sqrt{u^2-a^2}} = \ln\left|u + \sqrt{u^2-a^2}\right| + c$

Examples

- $\int \frac{dx}{\sqrt{4+x^2}}$
- $\int \frac{x^2 dx}{\sqrt{9-x^2}}$
- $\int \frac{dx}{\sqrt{25x^2-4}}$
- $\int_0^1 \sqrt{1-x^2}dx$