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Mathematics II
Transcendental Function
Chapter Six

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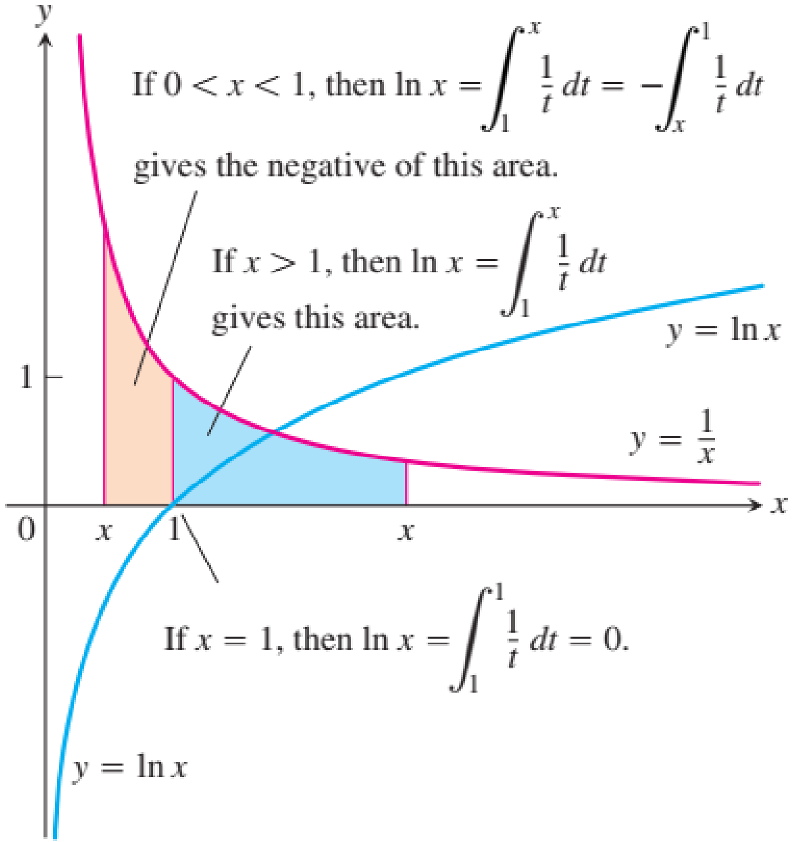
6.4. Natural Logarithm

- The natural logarithm of a positive number *x*, written as *ln x*, is the value of an integral.
- Definition:

$$\ln x = \int_1^x \frac{1}{t} dt \quad , \qquad x>0$$

- If x >1, then ln x is the area under the curve y=1/t from t=1 to t=x . For 0 < x < 1, ln *x* gives the negative of the area under the curve from *x* to1. The function is not defined for x≤ 0.



- The Zero Width Interval Rule for definite integrals

$$\ln 1 = \int_1^1 \frac{1}{t} dt = 0$$

- Notice that we show the graph of y=1/x but use y=1/t in the integral.

Using *x* for everything would have us writing

$$\ln x = \int_1^x \frac{1}{x} dx$$

- Definition The Number e

- The number e is that number in the domain of the natural logarithm satisfying

$$\ln(e) = 1$$

The Derivative of $y = \ln x$

$$\frac{d}{dx}\ln x = \frac{d}{dx}\int_1^x \frac{1}{t}\,dt = \frac{1}{x}$$

$$\frac{d}{dx}\ln x = \frac{1}{x}$$

$$\frac{d}{dx}\ln u = \frac{1}{u}\frac{du}{dx}$$

u>0

Examples: Derivatives of Natural Logarithms

1. Ln 2x
2. Ln (x^2+3)

Properties of Logarithms

- For any number a > 0 and x > 0, the natural logarithm satisfies the following rules:

1. Product Rule: $\ln ax = \ln a + \ln x$
2. Quotient Rule: $\ln \frac{a}{x} = \ln a - \ln x$
3. Reciprocal Rule: $\ln \frac{1}{x} = -\ln x$ Rule 2 with a=1
4. Power Rule: $\ln x^r = r \ln x$ r rational

Examples: Interpreting the properties of logarithms

a) $Ln\, 6$
b) $Ln\, 4$ – $\ln 5$
c) $\ln \frac{1}{8}$
d) $\ln 4 + \ln \sin x$
e) $\ln \frac{x+1}{2x-3}$
f) $Ln \sec x$
g) $\ln \sqrt[3]{x+1}$

- Proof that $\ln ax = \ln a + \ln x$

The Integral $\int (1/u)\, du$

- When u is a positive differentiable function

$$\int \frac{1}{u} du = \ln u + C$$

- But if u is negative

$$\int \frac{1}{u} du = \int \frac{1}{-u} d(-u) = \ln(-u) + C$$

- We can combine both equations for +ve and –ve
- If u is a differentiable function that is never zero ($u \neq 0$)

$$\int \frac{1}{u} du = \ln|u| + C$$

- $\int u^n\, du = \frac{u^{n+1}}{n+1} + c \quad n \neq -1$

- Examples:

1. $\int_0^2 \frac{2x}{x^2-5}\,dx$

2. $\int_{-\pi/2}^{\pi/2} \frac{4\cos\theta}{3+2\sin\theta}\,d\theta$