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Mathematics II
Transcendental Function
Chapter Six
The Exponential Function

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6.5. The Exponential Function

- The Function $y = e^x$

e=2.718281828459045

$e^2 = e.e$, $e^{-2} = \frac{1}{e^2}$, $e^{1/2} = \sqrt{e}$

- When we take logarithm of e:

$$\ln e^r = r \ln e = r.1 = r$$

- Which means that ***ln and exp*** are inverse of each others and

$$\ln e^2 = 2$$

Definition

For every real number x, $e^x = \ln^{-1} x = \exp x$

Inverse Equations for e^x and $\ln x$

$e^{lnx} = x$ all x>0

$\ln(e^x) = x$ all x

Examples

1. $\ln e^2$
2. $\ln e^{-1}$
3. $\ln \sqrt{e}$
4. $\ln e^{sinx}$
5. e^{ln2}
6. $e^{\ln(x^2+1)}$
7. e^{3ln2}

Example:

Find k if $e^{2k} = 10$

General Exponential Function a^x

- Definition
- For any numbers a > 0 and x, the exponential function with base a is

$$a^x = e^{xlna}$$

- Lows of Exponents for e^x

1. $e^{x1}.e^{x2} = e^{x1+x2}$
2. $e^{-x} = \frac{1}{e^x}$
3. $\frac{e^{x1}}{e^{x2}} = e^{x1-x2}$
4. $(e^{x1})^{x2} = e^{x1x2} = (e^{x2})^{x1}$

Examples

a. e^{x+ln2}

b. e^{-lnx}

c. $\frac{e^{2x}}{e}$

d. $(e^3)^x$

Simplify $\ln \frac{e^{2x}}{5}$

Solve:

1. $e^{3y} = 2 + \cos x$
2. $\ln(y-1) - \ln y = 3x$

Remark

1. To remove logarithm from an equation, exponentiate both sides
2. To remove exponentials take logarithm for both sides

The Derivative and Integral of e^x

- Derivative
- If $y = e^x$, Then $$\frac{d}{dx} e^x = e^x$$
- If u is any differentiable function of x, then $$\frac{d}{dx} e^u = e^u \frac{du}{dx}$$
- Integration $$\int e^u du = e^u + c$$

Examples

- Differentiate
- $y = 5e^{x}$
- $y = e^{-x}$
- $y = e^{sinx}$

- Integrate
- $\int_{0}^{\ln 2} e^{3x}\, dx$
- $\int_{0}^{\pi/2} e^{sinx} \cos x\, dx$

- Example: find area under the curve $y = e^{-x}$ for x=0 to x=b where b>0.

- Simplify $e^{ln2+3lnx}$

- Solve $\frac{dy}{dx} = 2xe^{-y}$, for y=0 when x=2