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Mathematics II
Transcendental Function
The function $y = a^x$ and $y = a^u$

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6.6. The Function $y = a^x$ and $y = a^u$

- Since $a = e^{lna}$

and $e^{xlna} = \left(e^{lna}\right)^x = a^x$

Form exponential rules for any +ve number (a)

$$a^{xy} = (a^x)^y = (a^y)^x$$

Example:

- e^{3lnx}
- e^{3ln2}

6.6. The function $y = a^x$ and $y = a^u$

Derivative of $y = a^x$

$$\frac{d}{dx}(a^x) = \frac{d}{dx}\left(e^{xlna}\right)$$

$$= e^{xlna} . \frac{d}{dx}(xln\ a)$$

$$= e^{xlna} . \ln a = a^x . \ln a$$

If a>0 and u is a differentiable function of x, then au is a differentiable function of x and

$$\frac{d}{dx} a^u = a^u \ln a \frac{du}{dx}$$

Example:

a. $\frac{d}{dx}(4^x)$

b. $\frac{d}{dx}3^{-x}$

c. $\frac{d}{dx}3^{\sin x}$

d. $y = x^x$

- Integration Formula

$$\int a^u du = \int \frac{1}{\ln a}.a^u \ln a \, du = \frac{1}{\ln a}.a^u + c$$

Evaluate:

- $\int 2^x dx$
- $\int 2^{\sin x} \cos x \, dx$

Example: Find $\frac{d}{dx}(e^x) = e^x.\ln e = e^x$

6.7. The function $y = \log_a u$

- The function a^x has a differentiable inverse function which is called logarithm of x to base (a) and denoted as

$$y = \log_a x$$

- So $y = a^x$ and $y = \log_a x$ are inverse of each others thus

$\log_a (a)^x = x$ for all x

$a^{\log_a x} = x$ for x>0

Example:

- $\log_a a = 1$
- $\log_5 25 = 2$
- $\log_2 \frac{1}{4} = -2$
- $\log_2 (2^5)$
- $\log_{10} (10^{-7})$
- $10^{\log_{10} 4}$

Evaluation of $\log_a x$

$$\log_a x = \frac{\ln x}{\ln a}$$

- Properties of $\log_a u$

1. Product rule:

$$\log_a xy = \log_a x + \log_a y$$

2. Quotient rule:

$$\log_a \frac{x}{y} = \log_a x - \log_a y$$

3. Reciprocal rule:

$$\log_a \frac{1}{y} = -\log_a y$$

4. Power rule:

$$\log_a x^y = y \log_a x$$

Derivatives and Integrals involving $\log_a x$

- Derivatives

$$\frac{d}{dx}(\log_a u) = \frac{1}{\ln a} \cdot \frac{1}{u}\frac{du}{dx}$$

- Examples:

a. $\frac{d}{dx}\log_{10}(3x+1)$

b. $\int \frac{\log_2 x}{x}\,dx$