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Mathematics II
Transcendental Function
Chapter Six

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Introduction

- Functions that are ***not algebraic*** are called transcendental.
- The trigonometric, exponential, logarithmic, and hyperbolic functions are transcendental, as are their inverses.
- Transcendental functions occur frequently in many calculus settings and applications, including growths of populations, vibrations and waves, efficiencies of computer algorithms, and the stability of engineered structures.

6.1. The Inverse Function

Definition

- Suppose that f is a function on a domain D with range R. The inverse function f^{-1} is defined by

$$f^{-1}(a) = b \quad if \quad f(b) = a$$

The domain of f^{-1} is R and the range of f^{-1} is D

- The Domains and Ranges of f and f^{-1} are interchanged.

The process of passing from f to f^{-1} can be summarized as a two-step process:

1. Solve the equation for *x*. This gives a formula where *x* is expressed as a function of *y*.

2. Interchange *x* and *y*, obtaining a formula where is expressed in the conventional format with *x* as the independent variable and *y* as the dependent variable.

Example:

Find an inverse function for the following functions:

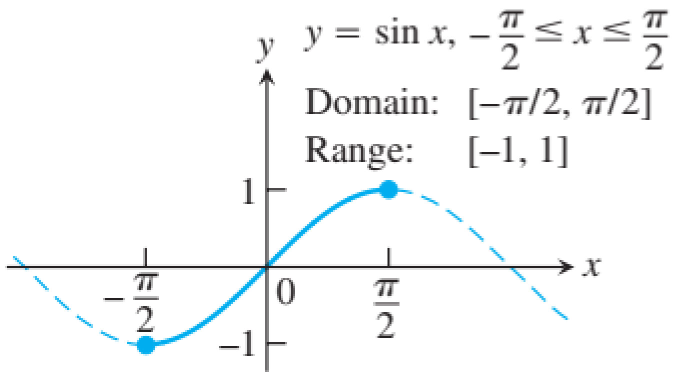
1. $y = \sqrt{x}$

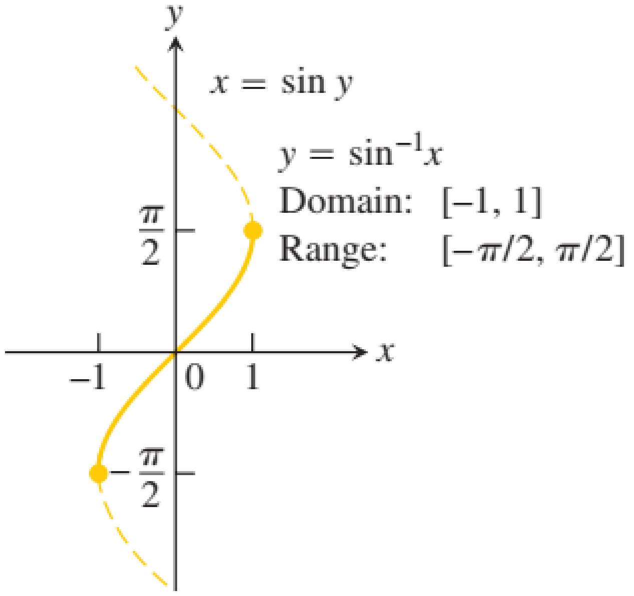
2. $y = \frac{1}{2}x + 1$

3. $y = 8x^3$

6.2. The inverse of Trigonometric Function

- The Arcsine ($\sin^{-1}$)





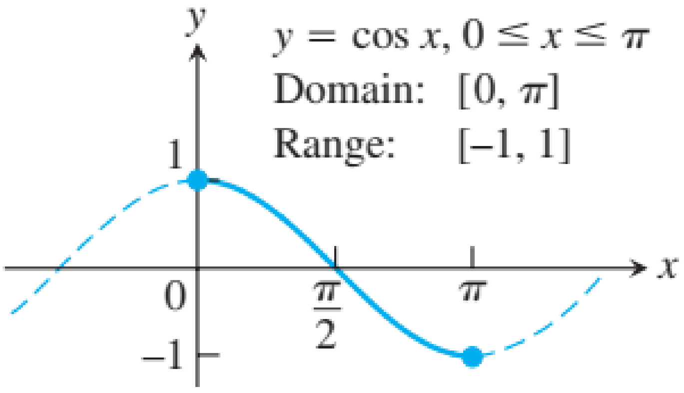
Common values of $\sin^{-1}$

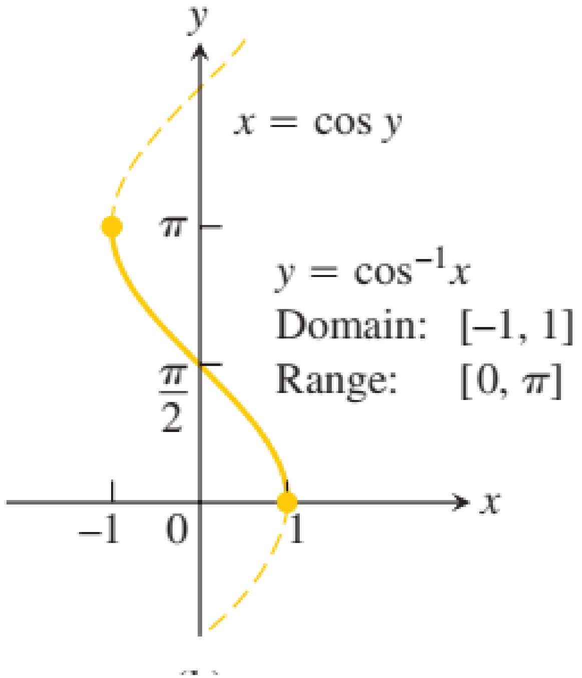
- $\sin^{-1} 0 = 0$
- $\sin^{-1} \frac{\sqrt{3}}{2} = \frac{\pi}{3}$
- $\sin^{-1} 1 = \frac{\pi}{2}$
- $\sin^{-1} \frac{\sqrt{2}}{2} = -\frac{\pi}{4}$

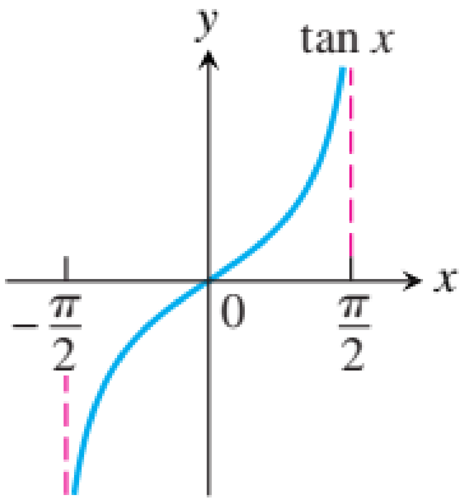
6.2. The inverse of Trigonometric Function(Cont.)

- The Arccosine ($\cos^{-1}$)

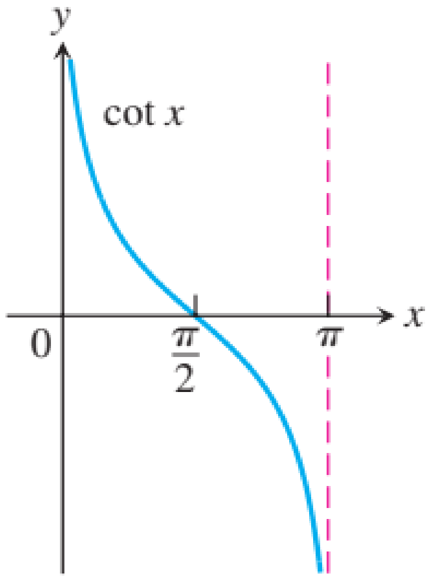
- Common values of $\cos^{-1}$
- $\cos^{-1}\frac{\pi}{6} = \frac{\sqrt{3}}{2}$
- $\cos^{-1}\frac{1}{\sqrt{2}} = \frac{\pi}{4}$
- $\cos^{-1}\left(-\frac{1}{2}\right) = \frac{2}{3}\pi$



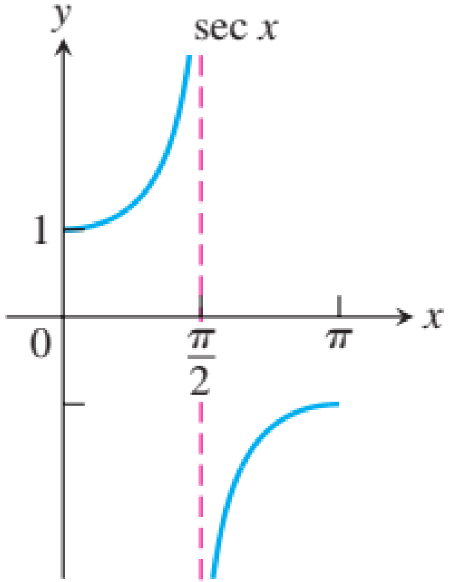




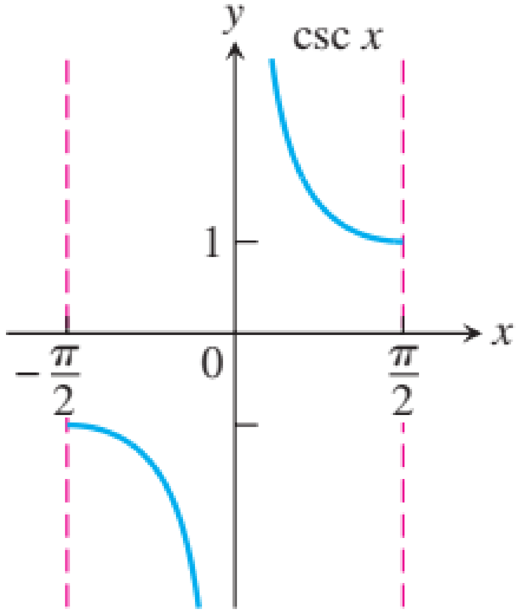
tan x D(-π/2,π /2) R(-∞,∞)



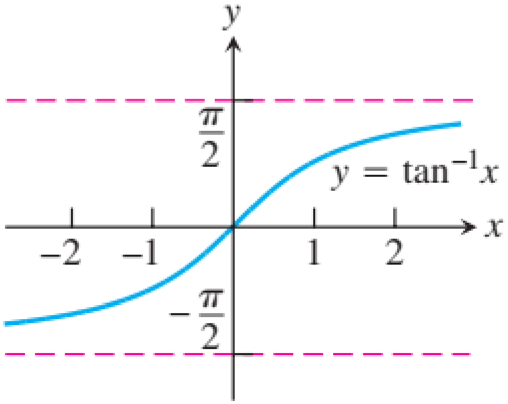
cot x D(0,π) R(-∞,∞)



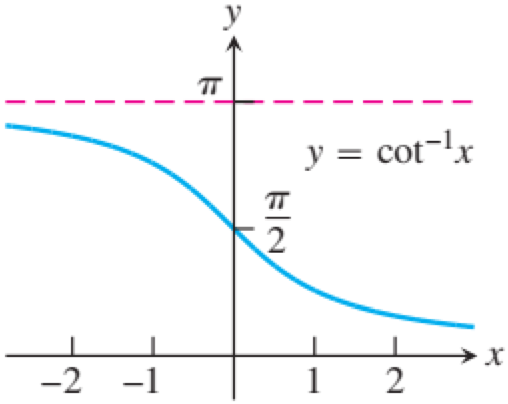
sec x D[0, π /2)U(π/2,π] R(-∞,-1]U[1,∞)



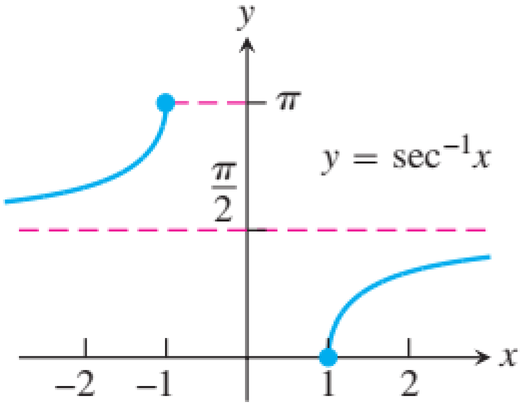
csc x D[-π /2,0)U(0,π/2] R(-∞,-1]U[1,∞)



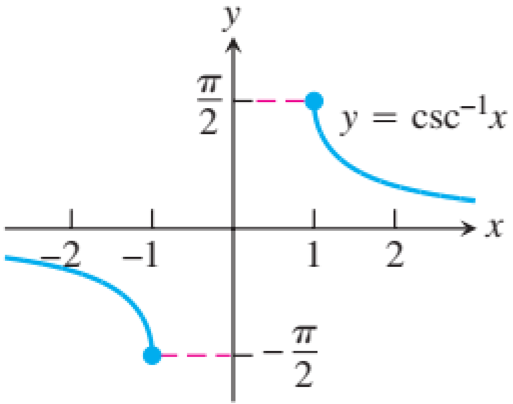
$\tan^{-1} x$ D(-∞,∞) R(-π/2,π /2)



$\cot^{-1} x$ D (-∞,∞) R(0,π)



$\sec^{-1} x$ D (-∞,-1]U[1,∞) R [0, π /2)U(π/2,π]



$\csc^{-1} x$ D (-∞,-1]U[1,∞) R [-π /2,0)U(0,π/2]

$$\sin^{-1} x + \cos^{-1} x = \pi/2$$

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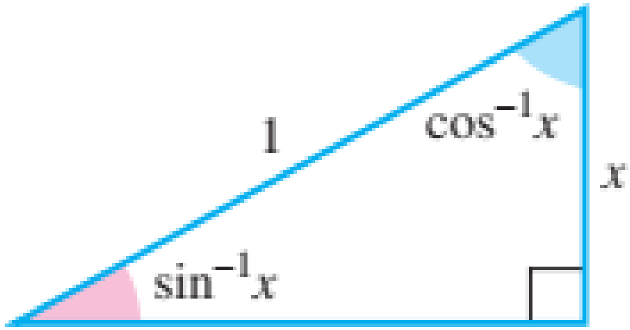

- There is no general agreement about how to define $\sec^{-1} x$ for negative values of x, so:

$$\sec^{-1} x = \cos^{-1} \frac{1}{x}$$
$$\csc^{-1} x = \sin^{-1} \frac{1}{x}$$

For the next lecture we will learn:

- Derivative of Inverse Trigonometric Function