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Mathematics II
Chapter seven
Techniques of Integration

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7.1. Introduction

All the following types of functions can be integrated directly with short tables:

1. $\int du = u + C$
2. $\int k\,du = ku + C \qquad (\text{any number } k)$
3. $\int (du + dv) = \int du + \int dv$
4. $\int u^n\,du = \frac{u^{n+1}}{n+1} + C \qquad (n \neq -1)$
5. $\int \frac{du}{u} = \ln|u| + C$
6. $\int \sin u\,du = -\cos u + C$
7. $\int \cos u\,du = \sin u + C$
8. $\int \sec^2 u\,du = \tan u + C$
9. $\int \csc^2 u\,du = -\cot u + C$
10. $\int \sec u \tan u\,du = \sec u + C$
11. $\int \csc u \cot u\,du = -\csc u + C$
12. $\int \tan u\,du = -\ln|\cos u| + C = \ln|\sec u| + C$
13. $\int \cot u\,du = \ln|\sin u| + C = -\ln|\csc u| + C$
14. $\int e^u\,du = e^u + C$
15. $\int a^u\,du = \frac{a^u}{\ln a} + C \qquad (a > 0, a \neq 1)$
16. $\int \sinh u\,du = \cosh u + C$
17. $\int \cosh u\,du = \sinh u + C$
18. $\int \frac{du}{\sqrt{a^2 - u^2}} = \sin^{-1}\left(\frac{u}{a}\right) + C$
19. $\int \frac{du}{a^2 + u^2} = \frac{1}{a}\tan^{-1}\left(\frac{u}{a}\right) + C$
20. $\int \frac{du}{u\sqrt{u^2 - a^2}} = \frac{1}{a}\sec^{-1}\left|\frac{u}{a}\right| + C$
21. $\int \frac{du}{\sqrt{a^2 + u^2}} = \sinh^{-1}\left(\frac{u}{a}\right) + C \qquad (a > 0)$
22. $\int \frac{du}{\sqrt{u^2 - a^2}} = \cosh^{-1}\left(\frac{u}{a}\right) + C \qquad (u > a > 0)$

But there are some common types of functions that are not included in this table that can be solved by another methods

7.2 Integration by Part

- This method depends on the product rule of derivatives, as
- $\frac{d}{dx}(u.v) = u.\frac{dv}{dx} + v.\frac{du}{dx}$

- Integration by parts formula

$$\int udv = uv - \int vdu$$

7.2 Integration by Part

- Example
- $\int x \cos x \, dx$
- $\int \ln x \, dx$
- $\int x^2 e^x \mathrm{dx}$
- $\int e^x \cos x \, dx$
- $\int \tan^{-1} x \, dx$
- $\int x^2 \cos ax \, dx$
- $\int \sin(\ln x) \, dx$

• Evaluating Definite Integrals by parts

Definite integrals by part formula:

$$\int_a^b f(x)g'(x)dx = f(x)g(x)]_a^b - \int_a^b f'(x)g(x)\,dx$$

- Example Finding Area
- Find the area of the region bounded by the curve. $y = xe^{-x}$ and the x-axis from x=0 to x=4

- Integrate

$$\int_1^e \frac{lnx}{x}\,dx$$