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Mathematics II
Chapter seven
Products of sine and cosine

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Powers of Sine and Cosine

- For integral $\int \sin^m x \cos^n x \, dx$, and n & m are non-negative integral (positive and zero)
- We have three cases:
- Case 1: if m is odd use the identity $\sin^2 x = 1 - \cos^2 x$
- Then we combine the single sin x with dx in the integral and set sin x dx equal -d(cos x)
- Case 2: if m is even and n is odd use the identity $\cos^2 x = 1 - \sin^2 x$
- Then we combine the single cos x with dx in the integral and set cos x dx equal d(sin x)
- Case 3: if both m and n are even, we substitute
- $\sin^2 x = \frac{1-\cos 2x}{2}$, $cos^2 x = \frac{1+\cos 2x}{2}$

Examples illustrating each cases

1. $\int \sin^3 x \cos^2 x \, dx$
2. $\int \cos^5 x \, dx$
3. $\int \sin^2 x \cos^4 x \, dx$

Products of sine and cosine

Products of Sines and Cosines

The integrals

$$\int \sin mx \sin nx\, dx, \qquad \int \sin mx \cos nx\, dx, \qquad \text{and} \qquad \int \cos mx \cos nx\, dx$$

Rules:

$$\sin mx \sin nx = \frac{1}{2}[\cos (m - n)x - \cos (m + n)x],$$

$$\sin mx \cos nx = \frac{1}{2}[\sin (m - n)x + \sin (m + n)x],$$

$$\cos mx \cos nx = \frac{1}{2}[\cos (m - n)x + \cos (m + n)x].$$

Example:

- $\int \sin 3x \cos 5x \, dx$