

Salahaddin University-Erbil
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Mathematics II

Chapter seven

Trigonometric Substitutions

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- Trigonometric substitutions can be effective in transforming integrals involving $\sqrt{a^2 + x^2}$, $\sqrt{a^2 - x^2}$, $\sqrt{x^2 - a^2}$
- **Three basic substitutions**
- The most common substitutions are : $x = a \tan \theta$, $x = a \sin \theta$, $x = a \sec \theta$

With $x = a \tan \theta$,

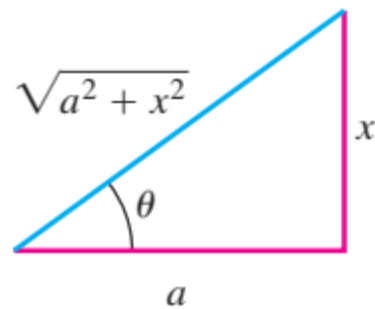
$$a^2 + x^2 = a^2 + a^2 \tan^2 \theta = a^2(1 + \tan^2 \theta) = a^2 \sec^2 \theta.$$

With $x = a \sin \theta$,

$$a^2 - x^2 = a^2 - a^2 \sin^2 \theta = a^2(1 - \sin^2 \theta) = a^2 \cos^2 \theta.$$

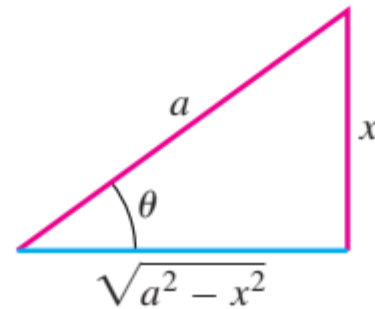
With $x = a \sec \theta$,

$$x^2 - a^2 = a^2 \sec^2 \theta - a^2 = a^2(\sec^2 \theta - 1) = a^2 \tan^2 \theta.$$



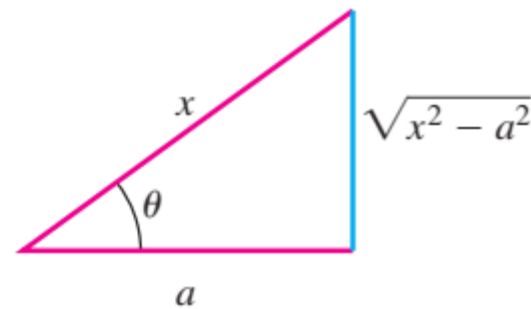
$$x = a \tan \theta$$

$$\sqrt{a^2 + x^2} = a|\sec \theta|$$



$$x = a \sin \theta$$

$$\sqrt{a^2 - x^2} = a|\cos \theta|$$



$$x = a \sec \theta$$

$$\sqrt{x^2 - a^2} = a|\tan \theta|$$

$$\bullet \int \frac{du}{a^2+u^2} = \frac{1}{a} \tan^{-1} \frac{u}{a} + c$$

$$\bullet \int \frac{du}{\sqrt{a^2-u^2}} = \sin^{-1} \frac{u}{a} + c$$

$$\bullet \int \frac{du}{\sqrt{a^2+u^2}} = \ln |\sqrt{a^2+u^2} + u| + c$$

$$\bullet \int \frac{du}{\sqrt{u^2-a^2}} = \ln |u + \sqrt{u^2-a^2}| + c$$

Examples

- $\int \frac{dx}{\sqrt{4+x^2}}$

- $\int \frac{x^2 dx}{\sqrt{9-x^2}}$

- $\int \frac{dx}{\sqrt{25x^2-4}}$