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Mathematics II

Transcendental Function

Chapter Six

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6.4. Natural Logarithm

- The natural logarithm of a positive number x , written as $\ln x$, is the value of an integral.

• Definition:

$$\ln x = \int_1^x \frac{1}{t} dt, \quad x > 0$$

- If $x > 1$, then $\ln x$ is the area under the curve $y=1/t$ from $t=1$ to $t=x$. For $0 < x < 1$, $\ln x$ gives the negative of the area under the curve from x to 1. The function is not defined for $x \leq 0$.

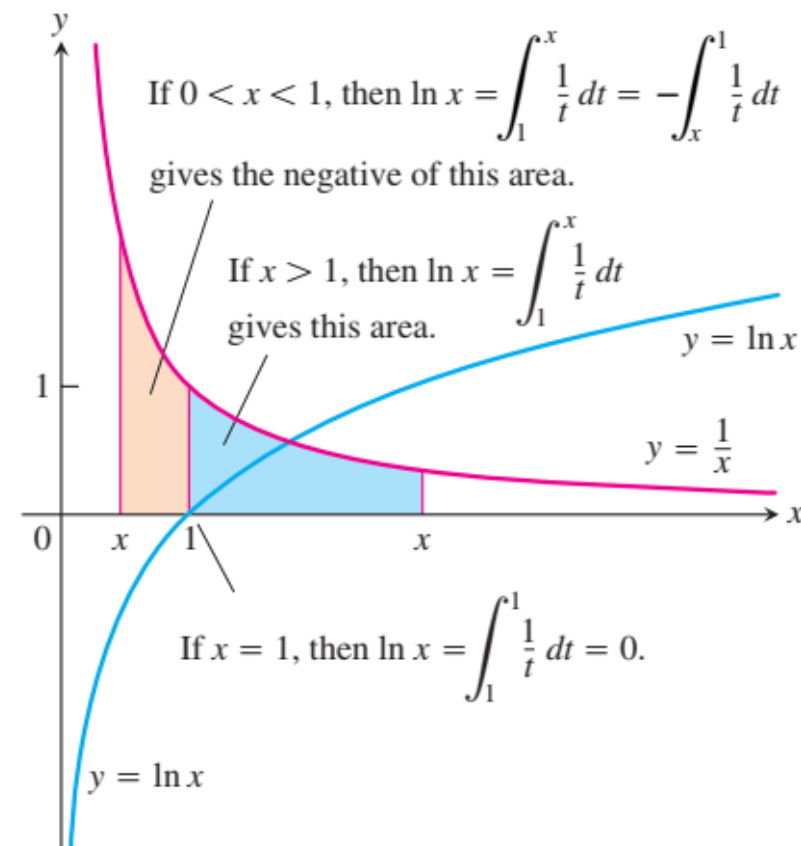
- The Zero Width Interval Rule for definite integrals

$$\ln 1 = \int_1^1 \frac{1}{t} dt = 0$$

- Notice that we show the graph of $y=1/x$ but use $y=1/t$ in the integral.

Using x for everything would have us writing

$$\ln x = \int_1^x \frac{1}{x} dx$$



- Definition The Number e

- The number e is that number in the domain of the natural logarithm satisfying

$$\ln(e) = 1$$

The Derivative of $y = \ln x$

$$\frac{d}{dx} \ln x = \frac{d}{dx} \int_1^x \frac{1}{t} dt = \frac{1}{x}$$

$$\frac{d}{dx} \ln x = \frac{1}{x}$$

$$\frac{d}{dx} \ln u = \frac{1}{u} \frac{du}{dx} \quad u > 0$$

Examples: Derivatives of Natural Logarithms

1. $\ln 2x$
2. $\ln (x^2+3)$

Properties of Logarithms

- For any number $a > 0$ and $x > 0$, the natural logarithm satisfies the following rules:

1. Product Rule:

$$\ln ax = \ln a + \ln x$$

2. Quotient Rule:

$$\ln \frac{a}{x} = \ln a - \ln x$$

3. Reciprocal Rule:

$$\ln \frac{1}{x} = -\ln x \quad \text{Rule 2 with } a=1$$

4. Power Rule:

$$\ln x^r = r \ln x \quad r \text{ rational}$$

Examples: Interpreting the properties of logarithms

a) $\ln 6$

b) $\ln 4 - \ln 5$

c) $\ln \frac{1}{8}$

d) $\ln 4 + \ln \sin x$

e) $\ln \frac{x+1}{2x-3}$

f) $\ln \sec x$

g) $\ln \sqrt[3]{x+1}$

- Proof that $\ln ax = \ln a + \ln x$

The Integral $\int (1/u) du$

- When u is a positive differentiable function

$$\int \frac{1}{u} du = \ln u + C$$

- But if u is negative

$$\int \frac{1}{u} du = \int \frac{1}{-u} d(-u) = \ln(-u) + C$$

- We can combine both equations for +ve and -ve
- If u is a differentiable function that is never zero ($u \neq 0$)

$$\int \frac{1}{u} du = \ln|u| + C$$

- $\int u^n du = \frac{u^{n+1}}{n+1} + c \quad n \neq -1$

- Examples:

1. $\int_0^2 \frac{2x}{x^2-5} dx$

2. $\int_{-\pi/2}^{\pi/2} \frac{4 \cos \theta}{3+2 \sin \theta} d\theta$