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Mathematics II
Transcendental Function
Chapter Six
The Exponential Function

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6.5. The Exponential Function

- The Function $\qquad y = e^x$

e=2.718281828459045

$$e^2 = e.e \qquad , e^{-2} = \frac{1}{e^2} \qquad , e^{1/2} = \sqrt{e}$$

- When we take logarithm of e:

$$\ln e^r = r \ln e = r.1 = r$$

- Which means that **ln and exp** are inverse of each others and

$$\ln e^2 = 2$$

Definition

For every real number x, $\quad e^x = \ln^{-1} x = \exp x$

Inverse Equations for e^x and $\ln x$

$e^{\ln x} = x$ all x>0

$\ln(e^x) = x$ all x

Examples

1. $\ln e^2$

2. $\ln e^{-1}$

3. $\ln \sqrt{e}$

4. $\ln e^{\sin x}$

5. $e^{\ln 2}$

6. $e^{\ln(x^2+1)}$

7. $e^{3\ln 2}$

Example:

Find k if $e^{2k} = 10$

General Exponential Function a^x

- Definition
- For any numbers a > 0 and x, the exponential function with base a is

$$a^x = e^{xlna}$$

- Lows of Exponents for e^x

1. $e^{x1} \cdot e^{x2} = e^{x1+x2}$

2. $e^{-x} = \dfrac{1}{e^x}$

3. $\dfrac{e^{x1}}{e^{x2}} = e^{x1-x2}$

4. $(e^{x1})^{x2} = e^{x1x2} = (e^{x2})^{x1}$

Examples

a. e^{x+ln2}

b. e^{-lnx}

c. $\dfrac{e^{2x}}{e}$

d. $\left(e^3\right)^x$

Simplify $\quad \ln\dfrac{e^{2x}}{5}$

Solve:

1. $e^{3y} = 2 + \cos x$

2. $\ln(y - 1) - \ln y = 3x$

Remark

1. To remove logarithm from an equation, exponentiate both sides

2. To remove exponentials take logarithm for both sides

The Derivative and Integral of e^x

- Derivative

- If $y = e^x$, Then
$$\frac{d}{dx}e^x = e^x$$

- If u is any differentiable function of x, then
$$\frac{d}{dx}e^u = e^u\frac{du}{dx}$$

- Integration
$$\int e^u du = e^u + c$$

Examples

- Differentiate
- $y = 5e^x$
- $y = e^{-x}$
- $y = e^{sinx}$

- Integrate
- $\int_0^{\ln 2} e^{3x} \, dx$
- $\int_0^{\pi/2} e^{sinx} \cos x \, dx$

- Example: find area under the curve $y = e^{-x}$ for x=0 to x=b where b>0.

- Simplify $e^{ln2+3lnx}$

- Solve $\frac{dy}{dx} = 2xe^{-y}$, for y=0 when x=2