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Mathematics II
Transcendental Function
The function $y = a^x$ and $y = a^u$

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6.6. The Function $y = a^x$ and $y = a^u$

- Since $a = e^{lna}$

and $e^{xlna} = \left(e^{lna}\right)^x = a^x$

Form exponential rules for any +ve number (a)

$$a^{xy} = (a^x)^y = (a^y)^x$$

Example:

- e^{3lnx}

- e^{3ln2}

6.6. The function $y = a^x$ and $y = a^u$

Derivative of $y = a^x$

$$\frac{d}{dx}(a^x) = \frac{d}{dx}\left(e^{xlna}\right)$$

$$= e^{xlna}.\frac{d}{dx}(xln\ a)$$

$$= e^{xlna}.\ln a = a^x.\ln a$$

If a>0 and u is a differentiable function of x, then au is a differentiable function of x and

$$\frac{d}{dx}a^u = a^u \ln a \frac{du}{dx}$$

Example:

a. $\dfrac{d}{dx}(4^x)$

b. $\dfrac{d}{dx}3^{-x}$

c. $\dfrac{d}{dx}3^{\sin x}$

d. $y = x^x$

- Integration Formula

$$\int a^u \, du = \int \frac{1}{\ln a} \cdot a^u \ln a \, du = \frac{1}{\ln a} \cdot a^u + c$$

Evaluate:

- $\int 2^x \, dx$

- $\int 2^{\sin x} \cos x \, dx$

Example: Find $\frac{d}{dx}(e^x) = e^x . \ln e = e^x$

Examples

- Find derivatives of these functions:

- $y = 7^{\sec \theta} \ln 7$
- $y = 5^{-\cos 2t}$
- $y = 3^{\tan \theta} \ln 3$

Examples:

- Find Integration for these functions:

- $\int (1.3)^x \, dx$

- $\int_{-2}^{0} 5^{-\theta} \, d\theta$

- $\int_{0}^{\pi/4} \left(\frac{1}{3}\right)^{\tan t} \sec^2 t \, dt$

Class Activity

$$\bullet \int_{1}^{2} \frac{2^{lnx}}{x}\, dx$$