Salahaddin University-Erbil
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Mathematics II
Transcendental Function
The function $y = \log_a u$

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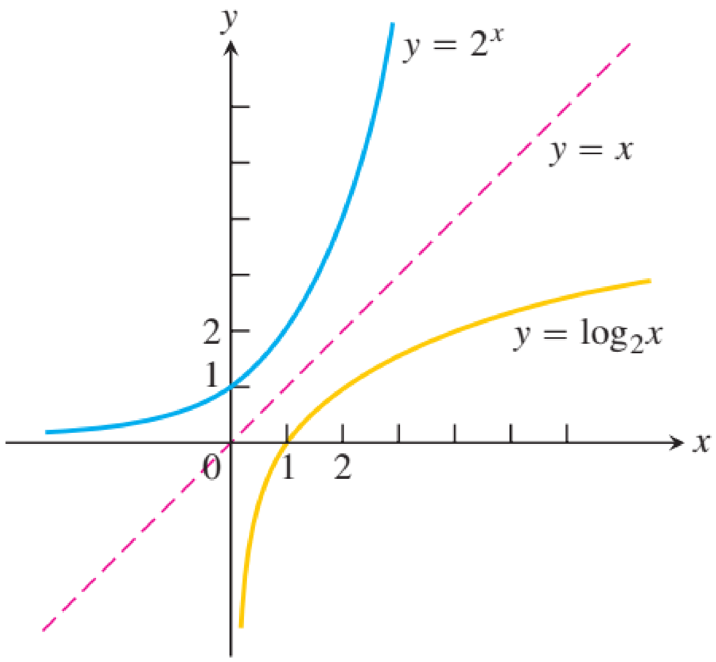
6.7. The function $y = \log_a u$

- The function a^x has a differentiable inverse function which is called logarithm of x to base (a) and denoted as

$$y = \log_a x$$



DEFINITION

For any positive number $a \neq 0$

$\log_a x$ is the inverse function of a^x

- So $y = a^x$ and $y = \log_a x$ are inverse of each others thus

$\log_a (a)^x = x$ for all x

$a^{\log_a x} = x$ for x>0

6.7. The function $\boldsymbol{y = \log_a u}$

Example:

- $\log_a a = 1$
- $\log_5 25 = 2$
- $\log_2 \frac{1}{4} = -2$
- $\log_2(2^5)$
- $\log_{10}(10^{-7})$
- $10^{\log_{10} 4}$

Evaluation of $\log_a x$

$$\log_a x = \frac{\ln x}{\ln a}$$

Example:

$$\log_{10} 2$$

- Properties of $\log_a u$

1. Product rule:

$$\log_a xy = \log_a x + \log_a y$$

2. Quotient rule:

$$\log_a \frac{x}{y} = \log_a x - \log_a y$$

3. Reciprocal rule:

$$\log_a \frac{1}{y} = -\log_a y$$

4. Power rule:

$$\log_a x^y = y \log_a x$$

Derivatives and Integrals involving $\log_a x$

- Derivatives and Integrals
- To find derivatives or integrals involving base *a* logarithms, we convert them to natural logarithms.
  If *u* is a positive differentiable function of *x*, then
- $\frac{d}{dx}(\log_a u) = \frac{d}{dx}\left(\frac{\ln u}{\ln a}\right) = \frac{1}{\ln a}\frac{d}{dx}(\ln u) = \frac{1}{\ln a}.\frac{1}{u}\frac{du}{dx}$

$$\frac{d}{dx}(\log_a u) = \frac{1}{\ln a}.\frac{1}{u}\frac{du}{dx}$$

Derivatives and Integrals involving $\log_a x$

- Examples:

a. $\dfrac{d}{dx}\log_{10}(3x+1)$

b. $\displaystyle\int \frac{\log_2 x}{x}\,dx$

c. $y = \log_3(1 + \theta \ln 3)$

d. $y = \log_2(8t^{\ln 2})$

e. $\displaystyle\int \frac{dx}{x \log_{10} x}$

f. $\displaystyle\int_2^3 \frac{2\log_2(x-1)}{x-1}\,dx$