Determinants of Matrices

Determinant is a real number associated with a square matrix

2×2 Determinant

- If $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$
- then
- $\det(A) = |A| = \begin{vmatrix} a & b \\ c & d \end{vmatrix}$
- $= ad - bc$
- Down product – up product

Find the determinant of $\begin{vmatrix} 1 & 2 \\ 3 & 4 \end{vmatrix}$

$$\begin{vmatrix} 1 & 2 \\ 3 & 4 \end{vmatrix}$$
$$= 1(4) - 3(2)$$
$$= 4 - 6$$
$$= -2$$

3×3 Determinant

- Copy 1st two columns after matrix
- + products of downs − products of ups

Find the determinant of $\begin{vmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{vmatrix}$

$$\begin{vmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{vmatrix} \begin{matrix} 1 & 2 \\ 4 & 5 \\ 7 & 8 \end{matrix}$$

$$= 1 \cdot 5 \cdot 9 + 2 \cdot 6 \cdot 7 + 3 \cdot 4 \cdot 8 - 7 \cdot 5 \cdot 3 - 8 \cdot 6 \cdot 1 - 9 \cdot 4 \cdot 2$$

$$= 45 + 84 + 96 - 105 - 48 - 72$$

$$= 0$$

Cramer's Rule

- Used to solve systems of equations
- $x_1 = \frac{|A_1|}{|A|}$ $\qquad$ $x_2 = \frac{|A_2|}{|A|}$
- A = coefficient matrix
- A_n = coefficient matrix with column n replaced with constants
- If |A| = 0, then no solution or many solutions

Use Cramer's Rule

$$\begin{cases} 2x + y + z = 6 \\ -x - y + 3z = 1 \\ y - 2z = -3 \end{cases}$$

$$x = \frac{\begin{vmatrix} 6 & 1 & 1 \\ 1 & -1 & 3 \\ -3 & 1 & -2 \end{vmatrix} \begin{matrix} 6 & 1 \\ 1 & -1 \\ -3 & 1 \end{matrix}}{\begin{vmatrix} 2 & 1 & 1 \\ -1 & -1 & 3 \\ 0 & 1 & -2 \end{vmatrix} \begin{matrix} 2 & 1 \\ -1 & -1 \\ 0 & 1 \end{matrix}}$$

$$= \frac{12 + (-9) + 1 - 3 - 18 - (-2)}{4 + 0 + (-1) - 0 - 6 - 2} = -\frac{15}{-5} = 3$$

$$y = \frac{\begin{vmatrix} 2 & 6 & 1 \\ -1 & 1 & 3 \\ 0 & -3 & -2 \end{vmatrix} \begin{matrix} 2 & 6 \\ -1 & 1 \\ 0 & -3 \end{matrix}}{-5}$$

$$= \frac{-4 + 0 + 3 - 0 - (-18) - 12}{-5} = \frac{5}{-5} = -1$$

$$z = \frac{\begin{vmatrix} 2 & 1 & 6 \\ -1 & -1 & 1 \\ 0 & 1 & -3 \end{vmatrix} \begin{matrix} 2 & 1 \\ -1 & -1 \\ 0 & 1 \end{matrix}}{-5}$$

$$= \frac{6 + 0 + (-6) - 0 - 2 - 3}{-5} = -\frac{5}{-5} = 1$$

(3, -1, 1)