

**First-Order Linear Equations:**

Solve the differential equations in Exercises 1–14.

1.  $x \frac{dy}{dx} + y = e^x, \quad x > 0$       2.  $e^x \frac{dy}{dx} + 2e^x y = 1$

3.  $xy' + 3y = \frac{\sin x}{x^2}, \quad x > 0$

4.  $y' + (\tan x)y = \cos^2 x, \quad -\pi/2 < x < \pi/2$

5.  $x \frac{dy}{dx} + 2y = 1 - \frac{1}{x}, \quad x > 0$

6.  $(1 + x)y' + y = \sqrt{x}$       7.  $2y' = e^{x/2} + y$

8.  $e^{2x} y' + 2e^{2x} y = 2x$       9.  $xy' - y = 2x \ln x$

10.  $x \frac{dy}{dx} = \frac{\cos x}{x} - 2y, \quad x > 0$

11.  $(t - 1)^3 \frac{ds}{dt} + 4(t - 1)^2 s = t + 1, \quad t > 1$

12.  $(t + 1) \frac{ds}{dt} + 2s = 3(t + 1) + \frac{1}{(t + 1)^2}, \quad t > -1$

13.  $\sin \theta \frac{dr}{d\theta} + (\cos \theta)r = \tan \theta, \quad 0 < \theta < \pi/2$

14.  $\tan \theta \frac{dr}{d\theta} + r = \sin^2 \theta, \quad 0 < \theta < \pi/2$

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Solve the initial value problems in Exercises 15–20.

15.  $\frac{dy}{dt} + 2y = 3, \quad y(0) = 1$

16.  $t \frac{dy}{dt} + 2y = t^3, \quad t > 0, \quad y(2) = 1$

17.  $\theta \frac{dy}{d\theta} + y = \sin \theta, \quad \theta > 0, \quad y(\pi/2) = 1$

18.  $\theta \frac{dy}{d\theta} - 2y = \theta^3 \sec \theta \tan \theta, \quad \theta > 0, \quad y(\pi/3) = 2$

19.  $(x + 1) \frac{dy}{dx} - 2(x^2 + x)y = \frac{e^{x^2}}{x + 1}, \quad x > -1, \quad y(0) = 5$

20.  $\frac{dy}{dx} + xy = x, \quad y(0) = -6$

**First-Order Separable Equations:**

In Exercises 1–6, decide whether the variables in the differential equation can be separated.

1.  $\frac{dy}{dx} = \frac{x}{y+3}$

2.  $\frac{dy}{dx} = \frac{x+1}{x}$

3.  $\frac{dy}{dx} = \frac{1}{x} + 1$

4.  $\frac{dy}{dx} = \frac{x}{x+y}$

5.  $\frac{dy}{dx} = x - y$

6.  $x \frac{dy}{dx} = \frac{1}{y}$

In Exercises 7–26, use separation of variables to find the general solution of the differential equation.

7.  $\frac{dy}{dx} = 2x$

8.  $\frac{dy}{dx} = \frac{1}{x}$

9.  $3y^2 \frac{dy}{dx} = 1$

10.  $\frac{dy}{dx} = x^2y$

11.  $(y+1) \frac{dy}{dx} = 2x$

12.  $(1+y) \frac{dy}{dx} - 4x = 0$

13.  $y' - xy = 0$

14.  $y' - y = 5$

15.  $\frac{dy}{dt} = \frac{e^t}{4y}$

16.  $e^y \frac{dy}{dt} = 3t^2 + 1$

17.  $\frac{dy}{dx} = \sqrt{1-y}$

18.  $\frac{dy}{dx} = \sqrt{\frac{x}{y}}$

19.  $(2 + x)y' = 2y$

21.  $xy' = y$

23.  $y' = \frac{x}{y} - \frac{x}{1 + y}$

25.  $e^x(y' + 1) = 1$

20.  $y' = (2x - 1)(y + 3)$

22.  $y' - y(x + 1) = 0$

24.  $\frac{dy}{dx} = \frac{x^2 + 2}{3y^2}$

26.  $yy' - 2xe^x = 0$

In Exercises 27–32, use the initial condition to find the particular solution of the differential equation.

<i>Differential Equation</i>	<i>Initial Condition</i>
27. $yy' - e^x = 0$	$y = 4$ when $x = 0$
28. $\sqrt{x} + \sqrt{y}y' = 0$	$y = 4$ when $x = 1$
29. $x(y + 4) + y' = 0$	$y = -5$ when $x = 0$
30. $\frac{dy}{dx} = x^2(1 + y)$	$y = 3$ when $x = 0$
31. $dP - 6P dt = 0$	$P = 5$ when $t = 0$
32. $dT + k(T - 70) dt = 0$	$T = 140$ when $t = 0$

**First-Order Exact Equations:**

**In Exercises 1–10, determine whether the differential equation is exact. If it is, find the general solution.**

1.  $(2x - 3y) dx + (2y - 3x) dy = 0$

2.  $ye^x dx + e^x dy = 0$

3.  $(3y^2 + 10xy^2) dx + (6xy - 2 + 10x^2y) dy = 0$

4.  $2 \cos(2x - y) dx - \cos(2x - y) dy = 0$

5.  $(4x^3 - 6xy^2) dx + (4y^3 - 6xy) dy = 0$

6.  $2y^2e^{xy^2} dx + 2xye^{xy^2} dy = 0$

7.  $\frac{1}{x^2 + y^2} (x dy - y dx) = 0$

8.  $e^{-(x^2+y^2)}(x dx + y dy) = 0$

9.  $\frac{1}{(x - y)^2} (y^2 dx + x^2 dy) = 0$

10.  $e^y \cos xy [y dx + (x + \tan xy) dy] = 0$

**First-Order Homogeneous Equations:**

In Problems 1–4, find the general solution of the given differential equation.

1  $y' + 5y = 0$

2  $y' - 2y = 0$

3  $y' + \frac{y}{1+t^2} = 0$

4  $y' + t^2y = 0$

In Problems 5–12, find the particular solution of the initial value problem.

5  $y' + y = 0, \quad y(0) = 4$

6  $y' - 3y = 0, \quad y(1) = -2$

7  $y' + y \sin t = 0, \quad y(\pi) = 1$

8  $y' + ye^t = 0, \quad y(0) = e$

9  $y' + y\sqrt{1+t^4} = 0, \quad y(0) = 0$

10  $y' + y \cos(e^t) = 0, \quad y(0) = 0$

11  $ty' - 2y = 0, \quad y(1) = 4, \quad t > 0$

12  $t^2y' + y = 0, \quad y(1) = -2, \quad t > 0$

**First-Order Bernoulli Equations:**

Solve:

$$yy' = y^2 + e^x$$

$$x y' = x^2 y^3 - y$$

$$y' + xy = xy^4$$

## Second-Order Homogeneous Linear Equations

In Exercises 1–30, find the general solution of the given equation.

1.  $y'' - y' - 12y = 0$

2.  $3y'' - y' = 0$

3.  $y'' + 3y' - 4y = 0$

4.  $y'' - 9y = 0$

5.  $y'' - 4y = 0$

6.  $y'' - 64y = 0$

7.  $2y'' - y' - 3y = 0$

8.  $9y'' - y = 0$

9.  $8y'' - 10y' - 3y = 0$

10.  $3y'' - 20y' + 12y = 0$

11.  $y'' + 9y = 0$

12.  $y'' + 4y' + 5y = 0$

13.  $y'' + 25y = 0$

14.  $y'' + y = 0$

15.  $y'' - 2y' + 5y = 0$

16.  $y'' + 16y = 0$

17.  $y'' + 2y' + 4y = 0$

18.  $y'' - 2y' + 3y = 0$

19.  $y'' + 4y' + 9y = 0$

20.  $4y'' - 4y' + 13y = 0$

21.  $y'' = 0$

22.  $y'' + 8y' + 16y = 0$

23.  $\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 4y = 0$

24.  $\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 9y = 0$

25.  $\frac{d^2y}{dx^2} + 6\frac{dy}{dx} + 9y = 0$

26.  $4\frac{d^2y}{dx^2} - 12\frac{dy}{dx} + 9y = 0$

27.  $4\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + y = 0$

28.  $4\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + y = 0$

29.  $9\frac{d^2y}{dx^2} + 6\frac{dy}{dx} + y = 0$

30.  $9\frac{d^2y}{dx^2} - 12\frac{dy}{dx} + 4y = 0$



## Second-Order Nonhomogeneous Linear Equations

Solve the equations in Exercises 1–16 by the method of undetermined coefficients.

1.  $y'' - 3y' - 10y = -3$
2.  $y'' - 3y' - 10y = 2x - 3$
3.  $y'' - y' = \sin x$
4.  $y'' + 2y' + y = x^2$
5.  $y'' + y = \cos 3x$
6.  $y'' + y = e^{2x}$
7.  $y'' - y' - 2y = 20 \cos x$
8.  $y'' + y = 2x + 3e^x$
9.  $y'' - y = e^x + x^2$
10.  $y'' + 2y' + y = 6 \sin 2x$
11.  $y'' - y' - 6y = e^{-x} - 7 \cos x$
12.  $y'' + 3y' + 2y = e^{-x} + e^{-2x} - x$
13.  $\frac{d^2y}{dx^2} + 5\frac{dy}{dx} = 15x^2$
14.  $\frac{d^2y}{dx^2} - \frac{dy}{dx} = -8x + 3$
15.  $\frac{d^2y}{dx^2} - 3\frac{dy}{dx} = e^{3x} - 12x$
16.  $\frac{d^2y}{dx^2} + 7\frac{dy}{dx} = 42x^2 + 5x + 1$



**Euler Equation**

In Exercises 1–24, find the general solution to the given Euler equation. Assume  $x > 0$  throughout.

1.  $x^2y'' + 2xy' - 2y = 0$
2.  $x^2y'' + xy' - 4y = 0$
3.  $x^2y'' - 6y = 0$
4.  $x^2y'' + xy' - y = 0$
5.  $x^2y'' - 5xy' + 8y = 0$
6.  $2x^2y'' + 7xy' + 2y = 0$
7.  $3x^2y'' + 4xy' = 0$
8.  $x^2y'' + 6xy' + 4y = 0$
9.  $x^2y'' - xy' + y = 0$
10.  $x^2y'' - xy' + 2y = 0$
11.  $x^2y'' - xy' + 5y = 0$
12.  $x^2y'' + 7xy' + 13y = 0$
13.  $x^2y'' + 3xy' + 10y = 0$
14.  $x^2y'' - 5xy' + 10y = 0$
15.  $4x^2y'' + 8xy' + 5y = 0$
16.  $4x^2y'' - 4xy' + 5y = 0$
17.  $x^2y'' + 3xy' + y = 0$
18.  $x^2y'' - 3xy' + 9y = 0$
19.  $x^2y'' + xy' = 0$
20.  $4x^2y'' + y = 0$
21.  $9x^2y'' + 15xy' + y = 0$
22.  $16x^2y'' - 8xy' + 9y = 0$
23.  $16x^2y'' + 56xy' + 25y = 0$
24.  $4x^2y'' - 16xy' + 25y = 0$

## Double Integrals in Cartesian Coordinates

1. Compute  $\int_0^2 \int_0^4 1 + x \, dy \, dx. \Rightarrow$
2. Compute  $\int_{-1}^1 \int_0^2 x + y \, dy \, dx. \Rightarrow$
3. Compute  $\int_1^2 \int_0^y xy \, dx \, dy. \Rightarrow$
4. Compute  $\int_0^1 \int_{y^2/2}^{\sqrt{y}} dx \, dy. \Rightarrow$
5. Compute  $\int_1^2 \int_1^x \frac{x^2}{y^2} \, dy \, dx. \Rightarrow$
6. Compute  $\int_0^1 \int_0^{x^2} \frac{y}{e^x} \, dy \, dx. \Rightarrow$
7. Compute  $\int_0^{\sqrt{\pi/2}} \int_0^{x^2} x \cos y \, dy \, dx. \Rightarrow$
8. Compute  $\int_0^{\pi/2} \int_0^{\cos \theta} r^2 (\cos \theta - r) \, dr \, d\theta. \Rightarrow$
9. Compute:  $\int_0^1 \int_{\sqrt{y}}^1 \sqrt{x^3 + 1} \, dx \, dy. \Rightarrow$
10. Compute:  $\int_0^1 \int_{y^2}^1 y \sin(x^2) \, dx \, dy. \Rightarrow$
11. Compute:  $\int_0^1 \int_{x^2}^1 x \sqrt{1 + y^2} \, dy \, dx \Rightarrow$
12. Compute:  $\int_0^1 \int_0^y \frac{2}{\sqrt{1 - x^2}} \, dx \, dy \Rightarrow$
13. Compute:  $\int_0^1 \int_{3y}^3 e^{x^2} \, dx \, dy \Rightarrow$
14. Compute  $\int_{-1}^1 \int_0^{1-x^2} x^2 - \sqrt{y} \, dy \, dx. \Rightarrow$
15. Compute  $\int_0^{\sqrt{2}/2} \int_{-\sqrt{1-2x^2}}^{\sqrt{1-2x^2}} x \, dy \, dx. \Rightarrow$

16. Evaluate  $\iint x^2 dA$  over the region in the first quadrant bounded by the hyperbola  $xy = 16$  and the lines  $y = x$ ,  $y = 0$ , and  $x = 8$ .  $\Rightarrow$
17. Find the volume below  $z = 1 - y$  above the region  $-1 \leq x \leq 1$ ,  $0 \leq y \leq 1 - x^2$ .  $\Rightarrow$
18. Find the volume bounded by  $z = x^2 + y^2$  and  $z = 4$ .  $\Rightarrow$
19. Find the volume in the first octant bounded by  $y^2 = 4 - x$  and  $y = 2z$ .  $\Rightarrow$
20. Find the volume in the first octant bounded by  $y^2 = 4x$ ,  $2x + y = 4$ ,  $z = y$ , and  $y = 0$ .  $\Rightarrow$

**Double Integrals in Polar Coordinates**

1. Find the volume above the  $x$ - $y$  plane, under the surface  $r^2 = 2z$ , and inside  $r = 2$ .  $\Rightarrow$
2. Find the volume inside both  $r = 1$  and  $r^2 + z^2 = 4$ .  $\Rightarrow$
3. Find the volume below  $z = \sqrt{1 - r^2}$  and above the top half of the cone  $z = r$ .  $\Rightarrow$
4. Find the volume below  $z = r$ , above the  $x$ - $y$  plane, and inside  $r = \cos \theta$ .  $\Rightarrow$
5. Find the volume below  $z = r$ , above the  $x$ - $y$  plane, and inside  $r = 1 + \cos \theta$ .  $\Rightarrow$
6. Find the volume between  $x^2 + y^2 = z^2$  and  $x^2 + y^2 = z$ .  $\Rightarrow$
7. Find the area inside  $r = 1 + \sin \theta$  and outside  $r = 2 \sin \theta$ .  $\Rightarrow$
8. Find the area inside both  $r = 2 \sin \theta$  and  $r = 2 \cos \theta$ .  $\Rightarrow$
9. Find the area inside the four-leaf rose  $r = \cos(2\theta)$  and outside  $r = 1/2$ .  $\Rightarrow$
10. Find the area inside the cardioid  $r = 2(1 + \cos \theta)$  and outside  $r = 2$ .  $\Rightarrow$
11. Find the area of one loop of the three-leaf rose  $r = \cos(3\theta)$ .  $\Rightarrow$

**Surface Area by Double Integrals**

1. Find the area of the surface of a right circular cone of height  $h$  and base radius  $a$ .  $\Rightarrow$
2. Find the area of the portion of the plane  $z = mx$  inside the cylinder  $x^2 + y^2 = a^2$ .  $\Rightarrow$
3. Find the area of the portion of the plane  $x + y + z = 1$  in the first octant.  $\Rightarrow$
4. Find the area of the upper half of the cone  $x^2 + y^2 = z^2$  inside the cylinder  $x^2 + y^2 - 2x = 0$ .  $\Rightarrow$
5. Find the area of the upper half of the cone  $x^2 + y^2 = z^2$  above the interior of one loop of  $r = \cos(2\theta)$ .  $\Rightarrow$
6. Find the area of the upper hemisphere of  $x^2 + y^2 + z^2 = 1$  above the interior of one loop of  $r = \cos(2\theta)$ .  $\Rightarrow$
7. The plane  $ax + by + cz = d$  cuts a triangle in the first octant provided that  $a, b, c$  and  $d$  are all positive. Find the area of this triangle.  $\Rightarrow$
8. Find the area of the portion of the cone  $x^2 + y^2 = 3z^2$  lying above the  $xy$  plane and inside the cylinder  $x^2 + y^2 = 4y$ .  $\Rightarrow$

## Triple Integrals

1. Evaluate  $\int_0^1 \int_0^x \int_0^{x+y} 2x + y - 1 \, dz \, dy \, dx. \Rightarrow$
2. Evaluate  $\int_0^2 \int_{-1}^{x^2} \int_1^y xyz \, dz \, dy \, dx. \Rightarrow$
3. Evaluate  $\int_0^1 \int_0^x \int_0^{\ln y} e^{x+y+z} \, dz \, dy \, dx. \Rightarrow$
4. Evaluate  $\int_0^{\pi/2} \int_0^{\sin \theta} \int_0^{r \cos \theta} r^2 \, dz \, dr \, d\theta. \Rightarrow$
5. Evaluate  $\int_0^\pi \int_0^{\sin \theta} \int_0^{r \sin \theta} r \cos^2 \theta \, dz \, dr \, d\theta. \Rightarrow$
6. Evaluate  $\int_0^1 \int_0^{y^2} \int_0^{x+y} x \, dz \, dx \, dy. \Rightarrow$
7. Evaluate  $\int_1^2 \int_y^{y^2} \int_0^{\ln(y+z)} e^x \, dx \, dz \, dy. \Rightarrow$
8. Compute  $\int_0^\pi \int_0^{\pi/2} \int_0^1 z \sin x + z \cos y \, dz \, dy \, dx. \Rightarrow$

## Reduced Row Echelon Form

Instructions: Find the reduced row echelon form of each of the following matrices

1.

$$\begin{bmatrix} -1 & 1 \\ -1 & 0 \\ 0 & -1 \\ -1 & -2 \end{bmatrix}$$

2.

$$\begin{bmatrix} -1 & 1 \\ -1 & 2 \\ -3 & 2 \end{bmatrix}$$

3.

$$\begin{bmatrix} 1 & 3 & 2 & 1 \\ 2 & -3 & 0 & -2 \end{bmatrix}$$

4.

$$\begin{bmatrix} -1 & -3 \\ 3 & -3 \\ -3 & -3 \\ 2 & 0 \end{bmatrix}$$

5.

$$\begin{bmatrix} 1 & 3 \\ -1 & 3 \\ 3 & 0 \\ 1 & -3 \end{bmatrix}$$

6.

$$\begin{bmatrix} 1 & 2 & 0 \\ 1 & 3 & 3 \\ -1 & 0 & -1 \\ -3 & 0 & 0 \end{bmatrix}$$

7.

$$\begin{bmatrix} 3 & -3 & 2 \\ -2 & 2 & -2 \\ 2 & 1 & 1 \\ 3 & 1 & 1 \end{bmatrix}$$

8.

$$\begin{bmatrix} -1 & 1 & 0 & 1 \\ -2 & -3 & -1 & -2 \\ -3 & -1 & -2 & -1 \end{bmatrix}$$

9.

$$\begin{bmatrix} 0 & 1 & 3 \\ -1 & -3 & 3 \\ 1 & -3 & 0 \end{bmatrix}$$

10.

$$\begin{bmatrix} 3 & -2 & -3 & 3 \\ 2 & 3 & 3 & 2 \end{bmatrix}$$

11.

$$\begin{bmatrix} 0 & -3 & 1 & -1 \\ -2 & 1 & 0 & 3 \end{bmatrix}$$

12.

$$\begin{bmatrix} -2 & -3 & -2 \\ 3 & -2 & -2 \\ 3 & -2 & -1 \\ -1 & -1 & -2 \end{bmatrix}$$

13.

$$\begin{bmatrix} 1 & -1 & 2 \\ 3 & 0 & 2 \\ 1 & -3 & -1 \end{bmatrix}$$

14.

$$\begin{bmatrix} 3 & -3 & -2 & -1 \\ 0 & 2 & -3 & -3 \\ 3 & 3 & 2 & -3 \end{bmatrix}$$

15.

$$\begin{bmatrix} -1 & -2 & 2 \\ 2 & -2 & -2 \\ -3 & 0 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

16.

$$\begin{bmatrix} 1 & -2 & -3 \\ -2 & 3 & -1 \end{bmatrix}$$

17.

$$\begin{bmatrix} 1 & 3 & -3 & -3 \\ 0 & 0 & -2 & 2 \end{bmatrix}$$

18.

$$\begin{bmatrix} 3 & -3 & -2 \\ -2 & 1 & 1 \\ -2 & 1 & -2 \\ -1 & 1 & 2 \end{bmatrix}$$

19.

$$\begin{bmatrix} 2 & -3 & -2 & 3 \\ -2 & 2 & -2 & 0 \\ -1 & 3 & 3 & 2 \\ -3 & -2 & -2 & 2 \end{bmatrix}$$

20.

$$\begin{bmatrix} -1 & 1 & -1 & -2 \\ 2 & 0 & 0 & 3 \\ -1 & 0 & -1 & -2 \\ -3 & 0 & 2 & -1 \end{bmatrix}$$

21.

$$\begin{bmatrix} -3 & -1 & 3 & -3 \\ 0 & -2 & -1 & 1 \\ 3 & 2 & -3 & 0 \end{bmatrix}$$

22.

$$\begin{bmatrix} 2 & 1 \\ -1 & 1 \\ 0 & -2 \\ 1 & 3 \end{bmatrix}$$

23.

$$\begin{bmatrix} 0 & -3 & 2 & -2 \\ 0 & 2 & 2 & -2 \end{bmatrix}$$

24.

$$\begin{bmatrix} -2 & -3 \\ -1 & -1 \\ -3 & -3 \end{bmatrix}$$

25.

$$\begin{bmatrix} -3 & 3 & -2 \\ 2 & 2 & 3 \\ 1 & 1 & 0 \end{bmatrix}$$



**Determinant of Matrix**

For #1-6, compute the determinant of the given matrix.

1.)

$$\begin{pmatrix} 2 & 1 \\ 1 & 1 \end{pmatrix}$$

2.)

$$\begin{pmatrix} 1 & \pi \\ 0 & 1 \end{pmatrix}$$

3.)

$$\begin{pmatrix} -4 & 2 \\ 2 & -1 \end{pmatrix}$$

4.)

$$\begin{pmatrix} 3 & 0 & 0 \\ 107 & 1 & 0 \\ \sqrt{2} & 2 & 6 \end{pmatrix}$$

5.)

$$\begin{pmatrix} 1 & 1 & 1 \\ 2 & 2 & 2 \\ 3 & 3 & 3 \end{pmatrix}$$

6.)

$$\begin{pmatrix} 1 & 2 & 1 \\ 2 & 1 & 2 \\ 1 & 3 & 4 \end{pmatrix}$$