Title of Research Project

زانکۆی سەلاحەدین - هەولێر
Salahaddin University-Erbil

Prepared by: Sumaya M. Abdulqader

College of Education - Mathematics Department

Supervised by : MSc. Suham H. Awla

Introduction

Euler's studies on quadratic residues were further denoted by the French mathematician Adriean Marie Legendre (1752 -1833). Legendre's memoir ' Recherchesd' analyse indetermine (1785) contains an account of the Quadratic Reciprocity Law and its many applications , a sketch of a theory of the representation of an integer as the sum of three squares , In 1978, legendry introduced the symbol $({}^{a}/_{b})$, Legendre mentioned many of number theoretic. Contributions of Euler and Lagrange.

Methodology

Definition 1.5 [2]: Let n be a fixed positive integer. Two integers a and $b$are said to be congruent modulo n, symbolized by $a \equiv b(mod\ n)$ if n divides the difference $a - b$; that is, provided that $a - b = kn$ for some integer k.

Definition [1]: Let p be an odd prime and $gcd(a, p) = 1$. If the quadratic congruence $x^2 \equiv a(mod\ p)$ has a solution, then a said to be a quadratic residue of p. Otherwise, a called a quadratic non residue of p .The point to bear in mind is that if $a \equiv b(mod\ p)$, then a is a quadratic residue of p if and only if b is a quadratic residue of p.

Theorem 2.3 [1]: Euler's criterion. Let p be an odd prime and $gcd\ (a, p) = 1$. Then a is a quadratic residue of p if and only if $a^{(p-1)/2} \equiv 1(mod\ p)$.

Discussion

Definition : Let p be an odd prime and let $gcd\ (a, p) = 1$. The Legendre symbol is denoted by $({}^{a}/_{p})$ or $(\frac{a}{p})$ and defined by

$$({}^{a}/_{p}) = \begin{cases} 1 \ if\ a\ is\ a\ quadratic\ residue\ of\ p \\ -1\ if\ a\ is\ a\ quadratic\ non\ residue\ of\ p \end{cases}.$$

Theorem: Let p be an odd prime and let a and b be integers that are relatively prime to p. Then the Legendre symbol has the following properties:

If $a \equiv b\ (mod\ p)$, then $(a/p) = (b/p)$.

$(a^2/p) = 1$.

Theorem : If p is an odd prime, then $\sum_{a=1}^{p-1}(a/p)$= 0

Hence, there are precisely $\frac{p-1}{2}$ quadratic residues and $\frac{p-1}{2}$ quadratic non-residue of p.

Theorem : Quadratic Reciprocity Law

If p and q are distinct odd primes, then

$$({}^{p}/_{q})({}^{q}/_{p}) = (-1)^{\frac{p-1}{2}\frac{q-1}{2}}.$$

Conclusion

The necessary and sufficient condition is given under which an elements is quadratic residue. Moreover we study Legendre symbol and determine some property of it. Finally we get the calculation of quadratic residue easier by Legendre symbol.

References

1. D.M.Burton (1980) '' **Elementary Number Thorey ", Ellyn and Bacan.**
2. James J.Tattersall.(1999),''**Elementary Number Theory in Nine Chapters''**,Cmbridge Unmiversity press.
3. . K. H. Rosen London (1984), '' **Elementary Number Thorey and its Applications**
4. 4.Titu. Andrees cu, Dorin Andrica and Zuming Feyn (2007), ''**104 Number Theory problems''Birkha user Boston n company.**