زانکۆی سەلاحەدین- هەولێر

Salahaddin University-Erbil

On Strongly Unit elements in The Group Ring Z_nG

Research Project

Submitted to the department of Mathematic the degree of in partial fulfillment of the requirements for BSc. in Mathematic

By

Shivan Hamad Braym

Supervised by

MSc. Suham H. Awla

2022-2023

Certification of the Supervisors

I certify that this work was prepared under my supervision at the Department of Mathematics/ College of Education /Salahaddin University-Erbil in partial

fulfillment of the requirements for the degree of Bachelor of philosophy of

Signature:

Supervisor : **MSc. Suham H. Awla**

Scientific grade: Assist. Professor

Date:5 /4 /2023

In view of the available recommendations, I forward this work for debate by the examining committee.

Signature :

Name: **Dr. Rashad Rasheed Haje .**

Scientific grade: Assist. Professor

Chairman of the Mathematics Department

Date:5 /4 /2023

Acknowledgment

I express my deep sense of gratitude and thanks to the Almighty ALLAH for providing me with strength, health, faith, patience, willing and self- confidence to accomplish this study.

My sincere thanks and appreciation are extended to the presidency of Salahaddin University, especially the deanery of the College of Education for their facilities to carry out my research work.

I would like to give special thanks to my supervisor "Suham H. Awla" for her constant and valuable guidance and encouragement during my research work. Her attention, support and timely suggestions were useful and the most needed in the preparation of my bachelor thesis.

My deepest thanks go to professor "Dr. Rashad Rasheed Haji" the head of Mathematics Department of the College of Education, further more I wish to thank the staff members of the College of Education, especially the library staff of the College of Education.

Shivan Hamad Braym

2023

Content

Abstract

In this work we study and discuss the concept of strongly unit elements and strongly zero divisors in the group ring $Z_n G$, where G is a cyclic group of order m.

List of symbols

Symbols	Descriptions
$\gcd(a, b)$	the greatest common divisor of a and b
$\forall$	For all
$\in$	belong to
Z_n	The ring of integers modulo n
Z_nG	Group ring
$a \equiv b(mod\ n)$	a is congruent to b module n
$\|G\|$	order of G

Introduction

The study of numbers has always occupied a unique position in the world of mathematics. It may very well be the best subject for a student trying to learn what constitutes a mathematical proof, and to construct the proofs, such as the theories of congruences and prime numbers. Most of the results of this work can be considered as an application of the number theory.

The present work consists of two chapters along with a list of references at the end. The first chapter is deals with some definitions and theorems about ring and number theory, which are needed in our work.

Chapter two includes two sections. In section one we study the concept of strongly unit elements in the group ring Z_2G, where G is a cyclic group of order n. We prove that In the group ring Z_2G , where G is a cyclic group of order $2n$,the element $g + g^2 + g^3 \dots + g^{2n-1}$ is a strongly unit. So we prove that in the group ring Z_2G, where $G$is a cyclic group of order $2^k n$ for $k \geq 2$, the element $1 + g + g^3 + g^5 + \cdots + g^{2^k n-1}$ is a strongly unit. In section two we introduce and study the concept of strongly zero divisor element in the group ring Z_2G, where G is a cyclic group of order n, moreover we prove In the group ring Z_2G , where G is a cyclic group of order $2^k n$, $k \geq 2$ and $n = p_1^{\alpha_1} \dots . p_2^{\alpha_2}$,$\alpha_i \geq 0$,the element $1 + g^2 + g^4 \dots g^{2^k n-2}$ is a strongly zero divisor .

Chapter one

Background

In this chapter we take some known definitions and results that we are needed in our work.

Definition 1.1: [3] Let a and b be given integers, with at least one of them different from zero. The greatest common divisor of a and b denoted by $gcd(a,b)$, is the positive integer d satisfying the following:

(a) $d|a$ and $d|b$.
(b) If $c|a$ and $c|b$, then $c \leq d$.

Example 1.2: let $a = 10$ and $b = 5$ then $gcd(10,5) = 5$.

Definition 1.3: [1] Let R be a ring with unity. An element a in R is a unit of R if it has a multiplicative inverse in R.

Example 1.4: Let $R = \mathrm{Z}_5$ be a ring. Then $2 \in \mathrm{Z}_5$ is a unit since 3 is the inverse of 2. Then $3.2 \equiv 1(mod\ 5)$.

Definition 1.5:[1] If two non-zero elements of a ring R such that $a\,.\,b = 0$, then a and b are divisors of zero (or zero divisors).

Example 1.6: Let Z_4 be a ring, then 2 and 2 are different from zero and $2.2 \equiv 0(mod\ 4)$. Then a and b are zero divisors.

Theorem 1.7:[3] The linear congruence $ax \equiv b(modn)$ has a solution if and only if $d|b$ where $d = gcd(a,n)$ If $d|b$, then it has d mutually incongruent solutions modulo n .

Definition 1.8:[2] let R be a commutative ring with unit 1 and G be a multiplicative group. The group ring RG of the group G over the ring R consists of all finite formal sums of the form $\sum_i \alpha_i g_i$ (i-runs over a finite number) where $\alpha_i \in R\ and\ g_i \in G$ satisfying the following condition:

i. $\sum_{i=1}^{n} \alpha_i \, g_i = \sum_{i=1}^{n} \beta_i \, g_i \Leftrightarrow \alpha_i = \beta_i \;\; for \;\; i = 1,2, \dots , n, g_i \epsilon G.$

ii. $(\sum_{i=1}^{n} \alpha_i \, g_i) + (\sum_{i=1}^{n} \beta_i \, g_i) = \sum_{i=1}^{n} (\alpha_i + \beta_i) \, g_i \; ; \; g_i \epsilon G.$

iii. $(\sum_i \alpha_i \, g_i)\big(\sum_j \beta_i \, g_i\big) = \sum_k \gamma_k m_k \;\; where \, \gamma_k = \sum \alpha_i \beta_i \;\; , g_i b_j = m_k \; .$

iv. $r_i m_i = m_i r_i \;\; for \, all \, r_i \in R \; and \; m_i \in G.$

v. $r \sum_{i=1}^{n} r_i g_i = \sum_{i=1}^{n} (r r_i) \, g_i \, for \;\; r_i, r \in R \; and \; \sum r_i g_i \in RG.$

RG is a ring with $0 \in R$ as its additive identity. Since $1 \in R$ we have $G = 1.G \subset G$ and $Re = R \subseteq RG$ where e is the identity of G. clearly if we replace the group G by a semi group S we say RS is the semi group ring of the semi group S over the ring R.

Example 1.9: Let $Z_2 = \{0,1\}$ be the ring and $G = \langle g | g^3 = 1 \rangle$ then the group ring $Z_2 G = \{0, 1, g, g^2, 1 + g, 1 + g^2, g + g^2, 1 + g + g^2\}$ and the order of group ring is $|Z_2 G| = |Z_2|^{|G|} = 2^3 = 8$.

Definition 1.10:Let R be a ring, an element x in R is said to be a strongly unit if there exists an element $y \in R$ such that $xy = 1$ and there are $a, b \in R \backslash \{0, x, y\}$ such that

1. $ax = b$ or $xa = b$
2. $ay = b$ or y$a = b$.

Definition 1.11: Let R be a ring, an element x in R is said to be a strongly zero divisor if there exist an element $y \in R$ such that $xy = 0$ and there are $a, b \in R \backslash \{0, x, y\}$ such that

1. $ax = b$ or $xa = b$
2. $ay = b$ or y$a = b$.

Chapter two

Section one

Strongly Unit Elements in the Group Ring Z_nG

In this chapter we study some elements in a group ring Z_nG that is satisfy the definition of strongly unit or not.

Here we prove a result in any ring R that we need in our work:

Proposition 2.1.1: Let R be a ring. A unit element $x \in R$ such that $x^2 = 1$. Then $\boldsymbol{x}$ is strongly unit.

Proof: Suppose that x is a unit such that $y = x$, then $xy = x^2 = 1$, consider the linear congruence $xa \equiv b(modn) \ldots\ (1)$.

Since x is a unit, then $gcd(x,n) = 1$, then by Theorem 1.7, the linear congruence (1) has a unique solution let be s. hence x is strongly unit of a ring R.

Proposition 2.1.2: In the group ring Z_2G, where G is a cyclic group of order n , the element g^k is strongly unit.

Proof: let $x = g^k$, $y = g^{n-k}$

$$xy = g^k g^{n-k} = g^n = 1.$$

For $a = b = 1 + g + g^2 + \cdots + g^{n-1}$, we have

$$xa = g^k(1 + g + g^2 + \cdots + g^{n-1}) =$$

$$= 1 + g + g^2 + \cdots + g^{n-1} = b$$

$$ya = g^{n-k}(1 + g + g^2 + \cdots + g^{n-1})$$

$$= 1 + g + g^2 + \cdots + g^{n-1} = b.$$

Then by Definition 1.10, we see that the element x is a strongly unit of Z_2G.

Proposition 2.1.3: For $x = y =$ we have $x^2 = 1$

and for $a = b = 1 + g^2 + g^3 + \cdots + g^{2^k n-1}$

We have $\qquad ax = b$

Therefore by Definition 1.10, x is a stronly unit.

Proposition 2.1.4: In the group ring Z_2G , where G is a cyclic group of order $2n$,the element $g + g^2 + g^3 \ldots + g^{2n-1}$ is a strongly unit.

Proof: Let $a = g + g^2 + g^3 \ldots + g^{2n-1}$ .Then

$$a^2 = \begin{pmatrix} g^2 & g^3 & g^4 & g^5 & g^6 & g^7 & \cdots & g^{2n-1} & 1 \\ g^3 & g^4 & g^5 & g^6 & g^7 & g^8 & \cdots & 1 & g \\ g^4 & g^5 & g^6 & g^7 & g^8 & g^9 & \cdots & g & g^2 \\ g^5 & g^6 & g^7 & g^8 & g^9 & g^{10} & \cdots & g^2 & g^3 \\ g^6 & g^7 & g^8 & g^9 & g^{10} & g^{11} & \cdots & g^3 & g^4 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 1 & g & g^2 & g^3 & g^4 & g^5 & \cdots & g^{2n-3} & g^{2n-2} \end{pmatrix} = 1$$

Then by Proposition 2.1.1, the element x is strongly unit of Z_2G.

Example: Let Z_2G be a group ring where G is cyclic group of order 10. Then $g + g^2 + g^3 + g^4 + \cdots + g^9$ is a strongly unit of Z_2G.

Solution: let $x = g + g^2 + g^3 + g^4 + \cdots + g^9$

$$x^2 = \begin{pmatrix} g^2 & g^3 & g^4 & g^5 & g^6 & g^7 & g^8 & g^9 & 1 \\ g^3 & g^4 & g^5 & g^6 & g^7 & g^8 & g^9 & 1 & g \\ g^4 & g^5 & g^6 & g^7 & g^8 & g^9 & 1 & g & g^2 \\ g^5 & g^6 & g^7 & g^8 & g^9 & 1 & g & g^2 & g^3 \\ g^6 & g^7 & g^8 & g^9 & 1 & g & g^2 & g^3 & g^4 \\ g^7 & g^8 & g^9 & 1 & g & g^2 & g^3 & g^4 & g^5 \\ g^8 & g^9 & 1 & g & g^2 & g^3 & g^4 & g^5 & g^6 \\ g^9 & 1 & g & g^2 & g^3 & g^4 & g^5 & g^6 & g^7 \\ 1 & g & g^2 & g^3 & g^4 & g^5 & g^6 & g^7 & g^8 \end{pmatrix} = 1$$

Then by Proposition 2.1.1, the element $x$is a strongly unit of Z_2G.

Proposition 2.1.5: In the group ring Z_2G, where G is a cyclic group of order $2^k n$,$k \geq 2$ the element $g^2+g^4+g^6+g^8 \ldots + g^{2^k n-2}$ is a strongly unit .

Proof: Let $x = g^2+g^4+g^6+g^8 \ldots + g^{2^k n-2}$ Then

$$x^2 = \begin{pmatrix} g^4 & g^6 & g^8 & g^{10} & \cdots\cdots & g^{10} \\ g^6 & g^8 & g^{10} & g^{12} & \ldots 1 & g^2 \\ g^8 & g^{10} & g^{12} & g^{14} & \cdots g^2 & g^4 \\ g^{10} & g^{12} & g^{14} & g^{16} & \cdots g^4 & g^6 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 1 & g^2 & g^4 & g^6 & \cdots\cdots & g^{2^k n-4} \end{pmatrix} = 1$$

Then by Proposition 2.1.1, the element $x$is a strongly unit of Z_2G.

Example: Let Z_2G be a group ring, where |G|=16. Then

$x = g^2+g^4+g^6+g^8 + g^{10}+g^{12}+g^{14}$

$$x^2 = \begin{pmatrix} g^4 & g^6 & g^8 & g^{10} & g^{12} & g^{14} & 1 \\ g^6 & g^8 & g^{10} & g^{12} & g^{14} & 1 & g^2 \\ g^8 & g^{10} & g^{12} & g^{14} & 1 & g^2 & g^4 \\ g^{10} & g^{12} & g^{14} & 1 & g^2 & g^4 & g^6 \\ g^{12} & g^{14} & 1 & g^2 & g^4 & g^6 & g^8 \\ g^{14} & 1 & g^2 & g^4 & g^6 & g^8 & g^{10} \\ 1 & g^2 & g^4 & g^6 & g^8 & g^{10} & g^{12} \end{pmatrix} = 1$$

Then by Proposition 2.1.1, the element $x$is a strongly unit of Z_2G.

Proposition 2.1.6: In the group ring Z_2G, where $G$is a cyclic group of order 2^kn for $k \geq 2$, the element $1 + g + g^3+g^5 + \cdots + g^{2^kn-1}$ is a strongly unit.

Proof: Let $x = 1 + g + g^3+g^5 + \cdots + g^{2^kn-1}$

$$x^2 = \begin{pmatrix} 1 & g & g^3 & g^5 & g^7 & \dots & g^{2^kn-3} & g^{2^kn-1} \\ g & g^2 & g^4 & g^6 & g^8 & \dots & 1 & g^2 \\ g^3 & g^4 & g^6 & g^8 & g^{10} & \dots & g^2 & g^4 \\ g^5 & g^6 & g^8 & g^{10} & g^{12} & \dots & g^4 & g^6 \\ g^7 & g^8 & g^{10} & g^{12} & g^{14} & \dots & g^6 & g^8 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \dots & g^8 & g^{10} \\ g^{2^kn-3} & g^{2^kn-2} & 1 & g^2 & g^4 & \dots & g^{2^kn-6} & g^{2^kn-4} \\ g^{2^kn-1} & 1 & g^2 & g^4 & g^6 & \dots & g^{2^kn-4} & g^{2^kn-2} \end{pmatrix} = 1$$

Then by Proposition 2.1.1, the element $x$is a strongly unit of Z_2G.

Example: Let Z_2G be a group ring, where |G|=24. Then the element

$1 + g + g^3+g^5+g^7+g^9 + g^{11}+g^{13}+g^{15}+g^{17}+g^{19}+g^{21} + g^{23}$ is a strongly unit.

Solution: Let $x = 1 + g + g^3+g^5+g^7+g^9 + g^{11}+g^{13}+g^{15}+g^{17}+g^{19}+g^{21} + g^{23}$. Then

$$x^2 = \begin{pmatrix} 1 & g & g^3 & g^5 & g^7 & g^9 & g^{11} & g^{13} & g^{15} & g^{17} & g^{19} & g^{21} & g^{23} \\ g & g^2 & g^4 & g^6 & g^8 & g^{10} & g^{12} & g^{14} & g^{16} & g^{18} & g^{20} & g^{22} & 1 \\ g^3 & g^4 & g^6 & g^8 & g^{10} & g^{12} & g^{14} & g^{16} & g^{18} & g^{20} & g^{22} & 1 & g^2 \\ g^5 & g^6 & g^8 & g^{10} & g^{12} & g^{14} & g^{16} & g^{18} & g^{20} & g^{22} & 1 & g^2 & g^4 \\ g^7 & g^8 & g^{10} & g^{12} & g^{14} & g^{16} & g^{18} & g^{20} & g^{22} & 1 & g^2 & g^4 & g^6 \\ g^9 & g^{10} & g^{12} & g^{14} & g^{16} & g^{18} & g^{20} & g^{22} & 1 & g^2 & g^4 & g^6 & g^8 \\ g^{11} & g^{12} & g^{14} & g^{16} & g^{18} & g^{20} & g^{22} & 1 & g^2 & g^4 & g^6 & g^8 & g^{10} \\ g^{13} & g^{14} & g^{16} & g^{18} & g^{20} & g^{22} & 1 & g^2 & g^4 & g^6 & g^8 & g^{10} & g^{12} \\ g^{15} & g^{16} & g^{18} & g^{20} & g^{22} & 1 & g^2 & g^4 & g^6 & g^8 & g^{10} & g^{12} & g^{14} \\ g^{17} & g^{18} & g^{20} & g^{22} & 1 & g^2 & g^4 & g^6 & g^8 & g^{10} & g^{12} & g^{16} & g^{18} \\ g^{19} & g^{20} & g^{22} & 1 & g^2 & g^4 & g^6 & g^8 & g^{10} & g^{12} & g^{14} & g^{16} & g^{18} \\ g^{21} & g^{22} & 1 & g^2 & g^4 & g^6 & g^8 & g^{10} & g^{12} & g^{14} & g^{16} & g^{18} & g^{20} \\ g^{23} & 1 & g^2 & g^4 & g^6 & g^8 & g^{10} & g^{12} & g^{14} & g^{16} & g^{18} & g^{20} & g^{22} \end{pmatrix} = 1$$

Then by Proposition 2.1.1, the element $x$is a strongly unit of Z_2G.

Section two

On Strongly Zero Divisor in Group Rings Z_nG

In this section we study the elements that is a strongly zero divisor in Z_nG, where G is a cyclic group of order m.

Proposition 2.2.1: In the group ring Z_2G, where $G$is a cyclic group of order 2^kn, for $n = p_1^{\alpha_1} \ldots . p_2^{\alpha_2}$, $\alpha_i \geq 0$,the element $g + g^3 + g^5 \ldots + g^{2^kn-1}$ is a strongly zero divisor.

Proof: Let $x = g + g^3 + g^5 \ldots + g^{2n-1}$ .Then

$$a^2 = \begin{pmatrix} g^2 & g^4 & g^6 & g^8 & g^{10} & g^{12} & \cdots & g^{2^kn-2} & 1 \\ g^4 & g^6 & g^8 & g^{10} & g^{12} & g^{14} & \cdots & 1 & g \\ g^6 & g^8 & g^{10} & g^{12} & g^{14} & g^{16} & \cdots & g & g^2 \\ g^8 & g^{10} & g^{12} & g^{14} & g^{16} & g^{18} & \cdots & g^2 & g^4 \\ g^{10} & g^{12} & g^{14} & g^{16} & g^{18} & g^{20} & \cdots & g^4 & g^6 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 1 & g^2 & g^4 & g^6 & g^8 & g^{10} & \cdots & g^{2^kn-4} & g^{2^kn-2} \end{pmatrix} = 0$$

Let $a = g$, and $b = 1 + g^2 + g^4 \ldots + g^{2^kn-2}$. Then $a, b \in Z_2G \backslash \{0, x\}$, such that

$$xa = (g + g^3 + g^5 + \cdots + g^{2^kn-1})(g)$$

$$= 1 + g^2 + g^4 + \cdots + g^{2^kn-2}$$

$$= b$$

by Definition 1.11, x is a strongly zero divisor of Z_2G.

Example: Consider the group ring $Z_2, G,$ where $|\mathrm{G}| = 16$. Then for

$x = g + g^3 + g^5 + g^7 + g^9 + g^{11} + g^{13} + g^{15}$, we have

$$x^2 = \begin{pmatrix} g^2 & g^4 & g^6 & g^8 & g^{10} & g^{12} & g^{14} & 1 \\ g^4 & g^6 & g^8 & g^{10} & g^{12} & g^{14} & 1 & g^2 \\ g^6 & g^8 & g^{10} & g^{12} & g^{14} & 1 & g^2 & g^4 \\ g^8 & g^{10} & g^{12} & g^{14} & 1 & g^2 & g^4 & g^6 \\ g^{10} & g^{12} & g^{14} & 1 & g^2 & g^4 & g^6 & g^8 \\ g^{12} & g^{14} & 1 & g^2 & g^4 & g^6 & g^8 & g^{10} \\ g^{14} & 1 & g^2 & g^4 & g^6 & g^8 & g^{10} & g^{12} \\ 1 & g^2 & g^4 & g^6 & g^8 & g^{10} & g^{12} & g^{14} \end{pmatrix} = 0$$

For $a = g$ and $b = 1 + g^2 + g^4 + g^6 + g^{14}$, we have $a, b \in Z_2G\backslash\{0, x\}$ and

$$xa = (g + g^3 + g^5 + g^7 + g^9 + g^{11} + g^{13} + g^{15})g$$

$$= 1 + g^2 + g^4 + g^6 + g^{14} = b$$

by Definition 1.11, x is a strongly zero divisor of Z_2G.

Proposition 2.2.2: In the group ring Z_2G , where G is a cyclic group of order $2^k n$, $k \geq 2$ and $n = p_1^{\alpha_1} \ldots . p_2^{\alpha_2}$,$\alpha_i \geq 0$,the element $1 + g^2 + g^4 \ldots g^{2^k n-2}$ is a strongly zero divisor .

Proof: Let $x = 1 + g^2 + g^4 + g^6 \ldots + g^{2^k n-2}$ .Then

$$x^2 = \begin{pmatrix} 1 & g^2 & g^4 & g^6 & g^8 & g^{10} & \cdots & g^{2^k n-4} & g^{2^k n-2} \\ g^2 & g^4 & g^6 & g^8 & g^{10} & g^{12} & \cdots & g^{2^k n-2} & 1 \\ g^4 & g^6 & g^8 & g^{10} & g^{12} & g^{14} & \cdots & 1 & g^2 \\ g^6 & g^8 & g^{10} & g^{12} & g^{14} & g^{16} & \cdots & g^2 & g^4 \\ g^9 & g^{10} & g^{12} & g^{14} & g^{16} & g^{18} & \cdots & g^4 & g^6 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ g^{2^k n-2} & 1 & g^2 & g^4 & g^6 & g^8 & \cdots & g^{2^k n-6} & g^{2^k n-4} \end{pmatrix} = 0$$

Then for $a = g$ and $b = g + g^3 + g^5 \ldots + g^{2^k n-1}$. we have $a, b \in Z_2G\backslash\{0, x\}$ and

$$xa = \left(1 + g^2 + g^4 \ldots + g^{2^k n-2}\right) g$$

$$= g + g^3 + g^5 \ldots + g^{2^k n-1} = b$$

Hence by Definition 1.11, x is a strongly zero divisor of Z_2G.

Example: Let Z_2G be a group ring, where G is a cyclic group of order 12 and

$x = 1 + g^2 + g^4 + g^6 + g^8 + g^{10}$. Then

$$x^2 = \begin{pmatrix} 1 & g^2 & g^4 & g^6 & g^8 & g^{10} \\ g^2 & g^4 & g^6 & g^8 & g^{10} & 1 \\ g^4 & g^6 & g^8 & g^{10} & 1 & g^2 \\ g^6 & g^8 & g^{10} & 1 & g^2 & g^4 \\ g^8 & g^{10} & 1 & g^2 & g^4 & g^6 \\ g^{10} & 1 & g^2 & g^4 & g^6 & g^8 \end{pmatrix} = 0$$

Then for $a = g$ and $b = g + g^3 + g^5 \dots + g^{11}$. we have $a, b \in Z_2G \backslash \{0, x\}$ and

$$xa = (1 + g^2 + g^4 \dots + g^{10})g$$

$$= g + g^3 + g^5 \dots + g^{11} = b$$

Hence by Definition 1.11, x is a strongly zero divisor of Z_2G.

References

1. B.Fraleigh, John. *A first course in abstract algebra.* USA: Pearson Education india, 2003.

2. Kandasamy, W.b.Vasantha. *Smarandache rings*. USA: Infinite Study, 2002.

3. M.Burton, David. *Elementary number theory*. USA: McGraw Hill, 2010.

پوختە

لەم ئیشەدا ئێمە بەدواداچونمان کردووە بۆ دانەی یەکانەی بەهێز وە لە دانەی دابەشی سفری بەهێز و گفتوگۆمان لەسەر کردوە لە گرووپی ئاڵقەیی Z_nG و کاتێ G گرووپێکي خولیە لە ئۆردەري m