

Q1// Find the domain, rang and level curves of the following functions:

a) $f(x, y) = y - x$

b) $f(x, y) = xy$

c) $f(x, y) = \sqrt{y - x}$

d) $f(x, y) = \frac{y}{x^2}$

e) $f(x, y) = \ln(x^2 + y^2)$.

Q2// Find the boundary of the function's domain of the following:

a) $f(x, y) = 4x^2 + 9y^2$

b) $f(x, y) = e^{-(x^2+y^2)}$.

Q3// Find an equation for the level curve of the function $f(x, y)$ that passes through the given point

a) $f(x, y) = 16 - x^2 - y^2, (2\sqrt{2}, \sqrt{2})$

b) $f(x, y) = \sqrt{x^2 - 1}, (0, 1)$.

Q4// Sketch a typical level surface of the function

a) $f(x, y) = y^2$

b) $f(x, y, z) = x^2 + y^2$.

Q5// Find the limit in a-c

a) $\lim_{(x,y) \rightarrow (0,0)} \frac{3x^2 - y^2 +}{x^2 + y^2 + 2}$

b) $\lim_{(x,y) \rightarrow (0, \frac{\pi}{4})} \sec x \tan y$

c) $\lim_{(x,y) \rightarrow (0, \ln 2)} e^{x-y}$

Q6// At what points (x, y) in the plane are the function in a-c contiuous?

a) $f(x, y) = \frac{x+y}{x-y}$

b) $f(x, y) = \frac{x+y}{2+\cos x}$

c) $f(x, y) = \frac{1}{x^2 - y}$

Q7// Graph $f(x, y) = 49 - x^2 - y^2$ and plot the level curves $f(x, y) = 24$, $f(x, y) = -15$ and $f(x, y) = -51$ in the domain of f in the plane.

Q8// Define $f(0,0)$ in a way that extends f to be continuous at the origin .where

$$f(x, y) = \frac{3x^2y}{x^2+y^2}.$$

Q9// Find f_x and f_y of $f(x, y)$

a) $f(x, y) = 2x^2 - 3y - 4$

b) $f(x, y) = (xy - 1)^2.$

Q10// Let the function $f(x, y) = y^2x^4e^x + \sin xy$, then Find the followings:

a) f_{xyx} b) $f_{yyx}.$

Q11// Verify that $w_{xy} = w_{yx}$

a) $w = x \sin y + y \sin x + xy$

b) $w = \ln(2x + 3y)$

Q12// (a) Express $\frac{dw}{dt}$ as a function of t , both by using the Chain Rule and by expressing w in terms of t and differentiating directly with respect to t .

Then (b) evaluate $\frac{dw}{dt}$ at the given value of t .

a) $w = x^2 + y^2, x = \cos t, y = \sin t; t = \pi$

b) $w = 2ye^x - \ln z, x = \ln(t^2 + 1), y = \tan^{-1} t, z = e^t; t = 1$

Q13// Show that the angle θ between two non-zero vectors $u = \langle u_1, u_2, u_3 \rangle$ and

$$v = \langle v_1, v_2, v_3 \rangle \text{ is given by } \theta = \cos^{-1}\left(\frac{u_1v_1+u_2v_2+u_3v_3}{|u||v|}\right).$$

Q14// Let u and v be differentiable vector function of t . Prove that

$$\frac{d}{dt}[u(t) \cdot v(t)] = u'(t) \cdot v(t) + u(t) \cdot v'(t).$$

Q15// Let $u = 2i + j + k$, $v = -4i + 3j$ and $w = 7j - 4k$. Then

a) Find the angle between u and v .

b) Find the volume of the box determined by u, v and w .

Q16// Find the tangent plane and normal line of the surface

$$x^2 + y^2 - 2xy - x + 3y - z = -4 \text{ at } p_0(2, -3, 18).$$

Q17// Find the parametric equations for the line through $p(5, 3, -2)$ and $Q(-2, 3, 1)$,

and find the point where the line intersects the plane $5x - 2y + 3z = -2$.

Q18// A: Find equations for the tangent plane and normal line at the point $(0, 1, 2)$ of the surface $\cos \pi - x^2y + e^{xz} + yz = 4$.

B: Find all the local maxima, local minima and saddle points of the function

$$f(x, y) = 3 + 2x + 2y - 2x^2 - 2xy - y^2.$$

Q19// Find the local extreme values of the function

$$f(x, y) = 3y^2 - 2y^3 - 3x^2 + 6xy.$$

Q20// Find the greatest and smallest values of the function $f(x, y) = x^2 + y^2$

subject to the constraint $x^2 - 2x + y^2 - 4y = 0$.

Q21// Find the radii of gyration about the y -axis of a thin triangular plate

bounded by the y -axis and lines $y = x$ and $y = 2 - x$

if $\delta(x, y) = x + 2y$.

Q22// Find the area of the surface generated by revolving the right-hand loop of the

lemniscate $r^2 = \cos 2\theta$ about y -axis.

Q23// Evaluate the double integral

$$a) \int_0^3 \int_0^2 (4 - y^2) dy dx$$

$$b) \int_0^{\pi} \int_0^x x \sin y dy dx$$

$$c) \int_1^{\ln 8} \int_0^{\ln y} e^{x+y} dx dy.$$

Q24// Change the Cartesian integral into an equivalent polar integral. Then evaluate the polar integral.

$$\int_{-1}^1 \int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} \frac{2}{(1+x^2+y^2)^2} dy dx$$

Q25// Evaluate the integral

$$\int_0^{\frac{\pi}{4}} \int_0^{\sec u} \int_{-\infty}^{2t} e^x dx dt du$$