Q1// A coil spring lies alone $r(t) = \cos 4t\, i + \sin 4t\, j + tk, \quad 0 \le t \le 2\pi$. The spring density is a constant $\delta(x, y, z) = 1$. Find the spring's mass and center of mass and its moment of inertia and radius of gyration about the $z - axis$.

Q2// A slender metal arch, denser at the bottom than top, lies along the semicircle $y^2 + z^2 = 1, z \ge 0$,in the $yz - plane$. Find the center of the arch's mass if the density at the point (x, y, z) on the arch is $\delta(x, y, z) = 2 - z$.

Q3// Solve the system $u = x - y, v = 2x + y$

For $x\ and\ y$ in terms of $u\ and\ v$. Then find the value of the Jacobian $\frac{\partial(x,y)}{\partial(u,v)}$.

Q4// Use the transformation u= $x + y,\ v = y - 2x$ to evaluate the integral

$$\int_0^1 \int_0^{1-x} \sqrt{x + y}(y - 2x)^2\, dx\, dy.$$

Q5// Applying a transformation to: Evaluate

$$\int_0^4 \int_{x=\frac{y}{2}}^{x=\frac{y}{2}+1} \frac{2x - y}{2}\, dx\, dy.$$

By applying the transformation $u = \frac{2x-y}{2}, v = \frac{y}{2}$ and integrating over an appropriate region in the uv −plane.

Q6// Evaluate

$$\int_0^3 \int_0^4 \int_{\frac{y}{2}}^{\frac{y}{2}+1} \left(\frac{2x - y}{2} + \frac{z}{3}\right) dxdydz$$

By applying the transformation $u = \frac{2x-y}{2}, v = \frac{y}{2}, w = \frac{z}{3}$ and integrate over an appropriate region in $uvw - plane$.

Q7// Give an example of each of the following:

i) A bounded sequence $\{a_n\}$ which is not convergent.

ii) A non-decreasing divergent sequence.

iii) A non-increasing convergent sequence.

Q8// Define a convergent sequence.

Find a formula for the $\boldsymbol{nth}$ term of the sequences:

1) $1, \frac{-1}{4}, \frac{1}{9}, \frac{-1}{16}, \frac{1}{25}, \ldots$ 2) $0,3,8,15,24, \ldots$

Q9// Which of the following sequences $\{a_n\}$conergs and which diverge?Find the limit of each convergent sequence.

1) $a_n = 2 + (0.1)^n$
2) $a_n = \frac{1-2n}{1+2n}$
3) $a_n = \frac{\ln(n+1)}{\sqrt{n}}$
4) $a_n = \left(\frac{3}{n}\right)^{\frac{1}{n}}$
5) $a_n = \left(2 - \frac{1}{2^n}\right)\left(3 + \frac{1}{2^n}\right)$

Q10 // Prove or disprove:

$$\text{If } \lim_{n\to\infty} a_n = 0\,, \text{then } \sum_{n=1}^{\infty} a_n \text{ is convergent.}$$

Q11// Define an infinite series. If $|r| < 1$, then the Geometric series

$$a + ar + ar^2 + ar^3 + \cdots + ar^{n-1} + \cdots = \sum_{n=1}^{\infty} ar^{n-1}$$

Converges to $\frac{a}{1-r}$.

Q12// If $\sum_{n=1}^{\infty} a_n = A$ and $\sum_{n=1}^{\infty} b_n = B$ are convergent series, then $\sum_{n=1}^{\infty} a_n + \sum_{n=1}^{\infty} b_n = \sum_{n=1}^{\infty} a_n + b_n$.

Q13// Let $\{a_n\}$ be a sequence of positive terms. Suppose that $a_n = f(n)$ where f is a continuous, positive, decreasing function of x for all $x \geq N(\,N \text{ is a positive integer})$. Then the series $\sum_{n=N}^{\infty} a_n$ and the integral $\int_N^{\infty} f(x)dx$ both converge or both diverge.

Q14// Suppose that $a_n > 0$ and $b_n > 0$, for all $n > N$ (N is an integer), If $\lim_{n\to\infty} \frac{a_n}{b_n} = c > 0$, then $\sum_{n=1}^{\infty} a_n$ and $\sum_{n=1}^{\infty} b_n$ both are convergent or both are divergent.

Q15// State and proof the Ratio Test.

Q16 // Prove or disprove

$$\text{If } \lim_{n\to\infty} \frac{a_{n+1}}{a_n} = 1\text{, then } \sum_{n=1}^{\infty} a_n \text{ is convergent.}$$

Q17// State the Root Test.

Q18// State and proof the Alternating Series Test(Leibniz's Theorem).

Q19// Define the following statements:

1) Absolutely convergent
2) Conditionally convergent

Q20// Which of the following series are convergent and which are divergent? Justify your answer.

1) $\sum_{n=1}^{\infty} \frac{n}{n^2+1}$

2) $\sum_{n=1}^{\infty} \frac{1}{(n+1)(n+2)}$

3) $\sum_{n=1}^{\infty} \frac{\cos n\pi}{n\sqrt{n}}$ 4) $\sum_{n=1}^{\infty} \frac{2}{3n-1}$

Q21// Find the radius and interval of convergence of each of the following series:

1) $\sum_{n=1}^{\infty} \frac{x^n}{n\sqrt{n}3^n}$ 2) $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}(x+2)^n}{n\,2^n}$.

Q22// Show that $\ln\left(\frac{1+x}{1-x}\right) = 2\sum_{n=0}^{\infty} \frac{x^{2n+1}}{2n+1}$, for $|x| < 1$.

Q23// Find the Taylor expansion of $f(x) = \sin x$ at $a = 0$. Then evaluate the integral.

$$\int \frac{\sin x}{x}\,dx .$$

Q24// Which of the following series are convergent and which are divergent? Justify your answer.

1) $\sum_{n=1}^{\infty} \frac{2^{n+1}}{5^n}$

2) $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n^3+2}}$

3) $\sum_{n=1}^{\infty} \frac{1}{(n+1)(n+2)}$

4) $\sum_{n=1}^{\infty} \frac{(n+3)!}{3!\,n!\,3^n}$

5) $\sum_{n=1}^{\infty} \frac{2}{3n-1}$

6) $\sum_{n=1}^{\infty} (-1)^{n+1} \left(\frac{n}{10}\right)^n$

7) $\sum_{n=1}^{\infty} \frac{n}{n^2+1}$

8) $\sum_{n=1}^{\infty} \frac{\cos n\pi}{n\sqrt{n}}$

Q25//Find the Fourier series associated with the given function

$$f(x) = \begin{cases} 2 & 0 \le x \le \pi \\ -x & \pi < x \le 2\pi \end{cases}$$

Q26// Show that $\ln\left(\frac{1+x}{1-x}\right) = 2\sum_{n=0}^{\infty} \frac{x^{2n+1}}{2n+1}$, for $|x| < 1$.