Salahaddin University-Erbil College of Science Department of Physics 2nd Stage/General Secnd Semester 2020-2021	Subject: Analytical Mechanics Period: 2 hours Date: June 2021 Final Examination Second Trial

Q.1/ Choose the ***correct*** answer of the following: *[15 Marks]*

a) The magnitude of the free-fall acceleration at a point that is a distance $2R_e$ above the surface of the Earth, where R_e is the radius of the Earth, is about:
(9.8 m/s², 4.9 m/s², 2.45 m/ s², 1.09 m/ s², None of them)

b) A particle is placed on top of a smooth sphere of radius (3 m). As the particle slides down the side of the sphere, at what point (height) will it leave?
(2 m, 3 m, 4 m, 6 m, None of them)

c) For an elastic head-on collision between two bodies with (v_{1i}=5 m/s, v_{2i}=9 m/s and v_{1f}=7 m/s). What is the value of v_{2f}? *(3 m/s, 5 m/s, 7 m/s, 9 m/s, None of them)*

d) The force for of the potential energy function $V = cxyz + c$ is:
($\vec{F} = -c(\hat{\imath}yz + \hat{\jmath}xy + \hat{k}xz)$, $\vec{F} = -c(\hat{\imath}xz + \hat{\jmath}yz + \hat{k}xy)$, $\vec{F} = -c(\hat{\imath}xy + \hat{\jmath}xz + \hat{k}yz)$, $\vec{F} = -c(\hat{\imath}yz + \hat{\jmath}xz + \hat{k}xy)$, *None of them*)

e) For what values of the constants a, b and c is the force $\vec{F} = \hat{\imath}(ax + by^2) + \hat{\jmath}cxy)$ conservative? *(a=1 b=2 c=3 , a=3 b=2 c=1 , a=1 b=3 c=2 , a=3 b=1 c=2 , None of them)*

Q.2/ What are the physical meanings of the following equations? *[15 Marks]*

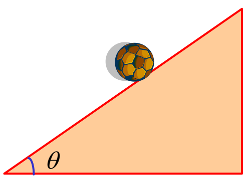
(I) $\mu\ddot{\vec{R}} = f(R)\frac{\vec{R}}{R}$ (II) $F_s = m\ddot{s} = -\frac{dV(s)}{ds}$ (III) $\sum \vec{F}_{ext} = m\vec{a}_{cm}$

(IV) $\vec{F} = q\vec{E} + q(\vec{v} \times \vec{B})$ (V) $m\vec{a} = \vec{F} + \vec{R}$

Q.3/ If two bodies undergo a direct collision, write the kinetic energy of a two particle system and show that the loss in kinetic energy is equal to $Q = \frac{1}{2}\mu v(1 - \epsilon^2)$, where μ is the reduced mass, v is the related speed and ϵ is the coefficient of restitution.
[15 Marks]

Q.4/

Find the acceleration of a ball shown in the figure rolling down a perfectly rough fixed inclined plane by using Lagrange's equations. *[15 Marks]*



Good Luck *Asst. Prof. Dr. Tahseen G. Abdullah*

Ans. of Q.1:

Q.1/ Choose the <u>**correct**</u> answer of the following: *[15 Marks]*

a) The magnitude of the free-fall acceleration at a point that is a distance $2R_e$ above the surface of the Earth, where R_e is the radius of the Earth, is about:

($9.8\ m/s^2$, $4.9\ m/s^2$, $2.45\ m/s^2$, <u>$1.09\ m/s^2$</u>, None of them)

b) A particle is placed on top of a smooth sphere of radius (3 m). As the particle slides down the side of the sphere, at what point (height) will it leave?

(<u>2 m</u> , 3 m , 4 m , 6 m , None of them)

c) For an elastic head-on collision between two bodies with (v_{1i}=5 m/s, v_{2i}=9 m/s and v_{1f}=7 m/s). What is the value of v_{2f}? (<u>3 m/s</u> , 5 m/s , 7 m/s , 9 m/s , None of them)

d) The force for of the potential energy function $V = cxyz + c$ is:

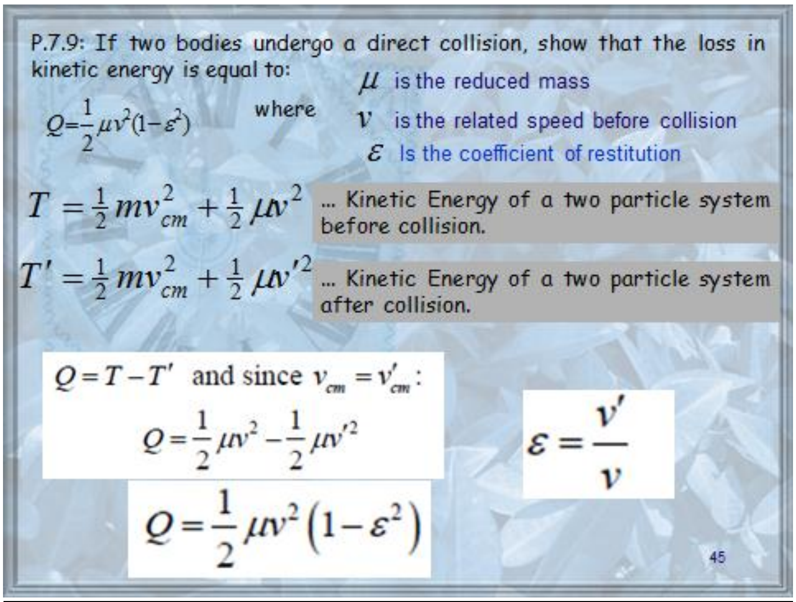
($\vec{F} = -c(\hat{\imath}yz + \hat{\jmath}xy + \hat{k}xz)$, $\vec{F} = -c(\hat{\imath}xz + \hat{\jmath}yz + \hat{k}xy)$, $\vec{F} = -c(\hat{\imath}xy + \hat{\jmath}xz + \hat{k}yz)$, <u>$\vec{F} = -c(\hat{\imath}yz + \hat{\jmath}xz + \hat{k}xy)$</u> , None of them)

e) For what values of the constants a, b and c is the force $\vec{F} = \hat{\imath}(ax + by^2) + \hat{\jmath}cxy$ conservative? (a=1 b=2 c=3 , a=3 b=2 c=1 , a=1 b=3 c=2 , <u>a=3 b=1 c=2</u> , None of them)

Ans. of Q.2:

(I) Newton's Motion of two interacting bodies (Two body problem). Motion of particle1 relative to particle2 (motion of central Field).
(II) Differential Equation for motion of a particle on the curve.
(III) Newton's second law for the system of *N* particles <u>treated as a single particle of mass *m* located at the center of mass</u>.
(IV) Motion of Charged Particle in Electro-Magnetic Field.
(V) Differential Equation for Constrained Motion.

Ans. of Q.3:



Ans. of Q.4:

- *n=1 the system has one degree of freedom*
- *The generalized coordinate is:* $q_1 = x$
- *The kinetic energy for a system is:*

$$T = \tfrac{1}{2} m\dot{x}^2 + \tfrac{1}{2} I w^2 = \tfrac{1}{2} m\dot{x}^2 + \tfrac{1}{2}\left(\tfrac{2}{3} m b^2\right)\frac{\dot{x}^2}{b^2} = \tfrac{5}{6} m\dot{x}^2$$

- *The potential energy for a system is:*

$$V = -mgx\sin\theta$$

- *The Lagrangian function is:*

$$\mathcal{L} = T - V \quad = \tfrac{5}{6} m\dot{x}^2 + mgx\sin\theta$$

$$\frac{d}{dt}\frac{\partial \mathcal{L}}{\partial \dot{x}} = \frac{\partial \mathcal{L}}{\partial x} \Rightarrow \frac{d}{dt}\left[\frac{5}{3} m\dot{x}\right] = mg\sin\theta$$

$$\ddot{x} = \frac{3}{5} g\sin\theta$$