Salahaddin University-Erbil
College of Science
Department of Physics
2nd Stage/General
Secnd Semester 2020-2021

Subject: Analytical Mechanics
Period: 2 hours
Date: June 2021
Final Examination
First Trial

Q.1/ Choose the ***correct*** answer of the following: *[15 Marks]*

a) The magnitude of the free-fall acceleration at a point that is a distance R_e above the surface of the Earth, where R_e is the radius of the Earth, is about:

(9.8 m/s^2 , 4.9 m/s^2 , 2.45 m/ s^2 , 1.09 m/ s^2, None of them)

b) A particle is placed on top of a smooth sphere of radius (6 m). As the particle slides down the side of the sphere, at what point (height) will it leave?

(2 m , 3 m , 4 m , 6 m , None of them)

c) For an elastic head-on collision between two bodies with (v_{1i}=5 m/s, v_{2i}=9 m/s and v_{2f}=3 m/s). What is the value of v_{1f}? *(3 m/s , 5 m/s , 7 m/s , 9 m/s , None of them)*

d) The force for of the potential energy function $V = cxyz + c$ is:

($\vec{F} = -c(\hat{\imath}yz + \hat{\jmath}xy + \hat{k}xz)$, $\vec{F} = -c(\hat{\imath}xz + \hat{\jmath}yz + \hat{k}xy)$, $\vec{F} = -c(\hat{\imath}xy + \hat{\jmath}xz + \hat{k}yz)$, $\vec{F} = -c(\hat{\imath}yz + \hat{\jmath}xz + \hat{k}xy)$, *None of them*)

e) For what values of the constants a, b and c is the force $\vec{F} = \hat{\imath}(ax + by^2) + \hat{\jmath}cxy$ conservative? *(a=1 b=1 c=2 , a=2 b=1 c=1 , a=1 b=2 c=1 , a=1 b=1 c=1 , None of them)*

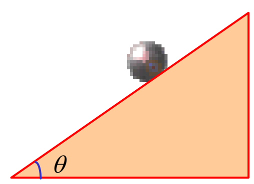
Q.2/ Find the kinetic energy for motion of the individual particles relative to the center of mass for the following system of 3-particles: *[15 Marks]*

$$\vec{v}_1 = 3\hat{i} \quad , \quad \vec{v}_2 = \hat{i} + 2\hat{j} \quad \& \quad \vec{v}_3 = 2\hat{i} + \hat{j} + 3\hat{k} \qquad m_1 = m_2 = m_3 = 1$$

Q.3/ Consider a particle sliding under gravity in a smooth cycloidal trough (The Isochronous Problem). If the particle performs simple harmonic motion find the parametric equations of a cycloid. *[15 Marks]*

Q.4/

Find the acceleration of a solid uniform sphere shown in the figure rolling down a perfectly rough fixed inclined plane by using Lagrange's equations. *[15 Marks]*



Good Luck *Asst. Prof. Dr. Tahseen G. Abdullah*

Ans. of Q.1:

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e) For what values of the constants a, b and c is the force $\vec{F} = \hat{\imath}(ax + by^2) + \hat{\jmath}cxy)$ conservative? ***(a=1 b=1 c=2 , a=2 b=1 c=1 , a=1 b=2 c=1 , a=1 b=1 c=1 , None of them)***

Ans. of Q.2:

$$T = \tfrac{1}{2} m v_{cm}^2 + \sum_{i=1}^{n} \tfrac{1}{2} m_i v_i'^2 \qquad m = \sum m_i = 1+1+1 = 3$$

$$\vec{v}_{cm} = \frac{1}{m}\sum m_i \vec{v}_i$$

$$\vec{v}_{cm} = \frac{1}{3}(\vec{v}_1 + \vec{v}_2 + \vec{v}_3) = \frac{1}{3}(3\hat{i} + \hat{i} + 2\hat{j} + 2\hat{i} + \hat{j} + 3\hat{k}) = \frac{6}{3}\hat{i} + \frac{3}{3}\hat{j} + \frac{3}{3}\hat{k} = 2\hat{i} + \hat{j} + \hat{k}$$

$$T_{cn} = \frac{1}{2} m v_{cm}^2 = \frac{1}{2} \times 3 \times (4+1+1) = 9$$

$$T = \sum_i \tfrac{1}{2} m_i v_i^2 = \tfrac{1}{2}(v_1^2 + v_2^2 + v_3^2) = \tfrac{1}{2}(9+5+4+1+9) = 14$$

$$\boxed{T' = T - T_{cn} = 14 - 9 = 5}$$

2nd way:

$$T' = \sum_i \tfrac{1}{2} m_i v_i'^2 = \tfrac{1}{2}(v_1'^2 + v_2'^2 + v_3'^2) \qquad \boxed{\vec{v}_i = \vec{v}_{cm} + \vec{v}_i'}$$

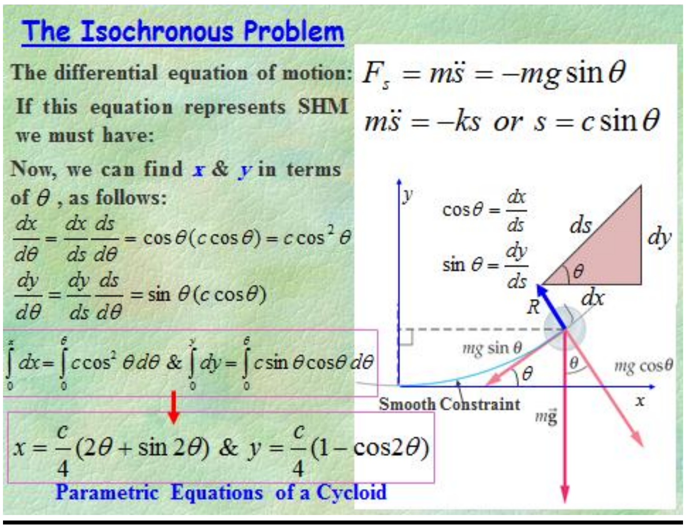
$$\vec{v}_1' = \vec{v}_1 - \vec{v}_{cm} = 3\hat{i} - (2\hat{i} + \hat{j} + \hat{k}) = \hat{i} - \hat{j} - \hat{k} \quad \Rightarrow v_1'^2 = 1+1+1 = 3$$

$$\vec{v}_2' = \vec{v}_2 - \vec{v}_{cm} = \hat{i} + 2\hat{j} - (2\hat{i} + \hat{j} + \hat{k}) = -\hat{i} + \hat{j} - \hat{k} \quad \Rightarrow v_2'^2 = 1+1+1 = 3$$

$$\vec{v}_3' = \vec{v}_3 - \vec{v}_{cm} = (2\hat{i} + \hat{j} + 3\hat{k}) - (2\hat{i} + \hat{j} + \hat{k}) = 2\hat{k} \quad \Rightarrow v_3'^2 = 4$$

$$T' = \tfrac{1}{2}(v_1'^2 + v_2'^2 + v_3'^2) = \tfrac{1}{2}(3+3+4) = \tfrac{1}{2}(10) = 5$$

Ans. of Q.3:



Ans. of Q.4:

Example15: Find the acceleration of a solid uniform sphere rolling down a perfectly rough fixed inclined plane.

$\omega = \frac{\dot{x}}{a}$ $\quad$ $n=1$ $\quad$ $q_1=x$

$$T = \frac{1}{2}m\dot{x}^2 + \frac{1}{2}I\omega^2 = \frac{1}{2}m\dot{x}^2 + \frac{1}{2}\left(\frac{2}{5}ma^2\right)\left(\frac{\dot{x}}{a}\right)^2 = \frac{7}{10}m\dot{x}^2$$

For $V = 0$ at the initial position of the sphere, Reference Level

$$V = -mgx\sin\theta$$

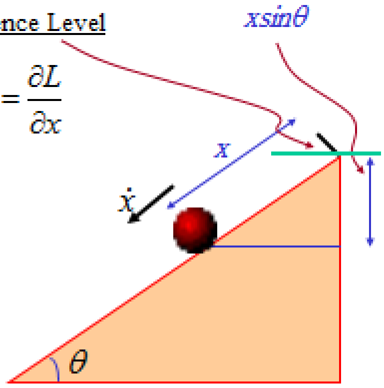
$$L = T - V = \frac{7}{10}m\dot{x}^2 + mgx\sin\theta \rightarrow \frac{d}{dt}\frac{\partial L}{\partial \dot{x}} = \frac{\partial L}{\partial x}$$

$$\frac{\partial L}{\partial \dot{x}} = \frac{7}{5}m\dot{x}, \qquad \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}}\right) = \frac{7}{5}m\ddot{x}$$

$$\frac{\partial L}{\partial x} = mg\sin\theta$$

$$\frac{7}{5}m\ddot{x} = mg\sin\theta$$

$$\ddot{x} = \frac{5}{7}g\sin\theta$$



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