Salahaddin University-Erbil College of Science Department of Physics 2nd Stage / General		Subject: Analytical Mechanics Period: 60 Minutes Date: 12 Aril 2022 First Examination Second Semester 2021-2022

Q.1/ What are the conservative force field. Write four different points on the conservative force. *[10 Marks]*

Q.2/ For projectile motion with quadratic air resistance: *[12 Marks]*

i) Write the differential equation of motion.

ii) What are the equations for the x, y and z components.

iii) Show that: $v_x = v_{xo} e^{-\gamma s}$

Q.3/ Show that the parametric equations for a particle sliding under gravity in a smooth cycloidal trough are given by: $x = A(2\varphi + \sin 2\varphi) \quad y = A(1 - \cos 2\varphi)$

where A is constant and $\boldsymbol{\varphi}$ is the parameter. *[12 Marks]*

Good Luck *Asst. Prof. Dr. Tahseen G. Abdullah*

Ans. of Q.1:

Conditions for the existence of the Potential Function

$\vec{F} = -\vec{\nabla} V$ → In general: $V(r) = V(x, y, z)$

$$F_x = -\frac{\partial V}{\partial x} = F_x(x,y,z) \;,\; F_y = -\frac{\partial V}{\partial y} = F_y(x,y,z) \quad \& \quad F_z = \ldots$$

$$\frac{\partial F_x}{\partial y} = -\frac{\partial^2 V}{\partial y \partial x} \;,\; \frac{\partial F_y}{\partial x} = -\frac{\partial^2 V}{\partial x \partial y} \qquad \text{but:} \quad \frac{\partial^2 V}{\partial y \partial x} = \frac{\partial^2 V}{\partial x \partial y}$$

A similar argument can be made with the pairs (F_x, F_z) and (F_y, F_z) . Thus we can write:

$$\frac{\partial F_x}{\partial y} = \frac{\partial F_y}{\partial x} \qquad \frac{\partial F_y}{\partial z} = \frac{\partial F_z}{\partial y} \qquad \frac{\partial F_z}{\partial x} = \frac{\partial F_x}{\partial z}$$

- A) Necessary conditions for the existence of the potential energy function.
- B) $\vec{\nabla} \times \vec{F} = 0$
- C) ....$\vec{F} = -\vec{\nabla} V$....
- D) ....$E = T + V(\mathbf{r}) = const$
- E) ....

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Ans. of Q.2:

III. Projectile Motion with Quadratic Air Resistance

- For a projectile with quadratic drag, the projectile experiences both gravity and the drag force, the latter directed in theopposite direction of its motion. Newton's 2nd Law (equation of motion) becomes

$$m\ddot{\mathbf{r}} = \sum Forces \;\rightarrow\; m\ddot{\mathbf{r}} = m\vec{\mathbf{g}} - cv^2\hat{\tau}$$
$$m\ddot{\mathbf{r}} = m\vec{\mathbf{g}} - cv\vec{v}$$

- or

$$m(\hat{i}\,\dot{v}_x + \hat{j}\dot{v}_y + \hat{k}\dot{v}_z) = m(\hat{k}g) - cv(\hat{i}\,v_x + \hat{j}\,v_y + \hat{k}v_z)$$

- This vector equation represents three separate equations for the x, y and z components:

$$m\dot{v}_x = -cvv_x \;\&\; m\dot{v}_y = -cvv_y \qquad where \;\; v = \sqrt{\dot{x}^2 + \dot{y}^2 + \dot{z}^2}$$
$$m\dot{v}_z = mg - cvv_z \qquad\qquad = \sqrt{v_x^2 + v_y^2 + v_z^2}$$

- Notice that the three equations are not of the separable type.
- From the first two equations we obtain:

$$v_x = v_{xo}e^{-\gamma s} \;\;\&\;\; v_y = v_{yo}e^{-\gamma s} \qquad where \;\; \gamma = c/m$$

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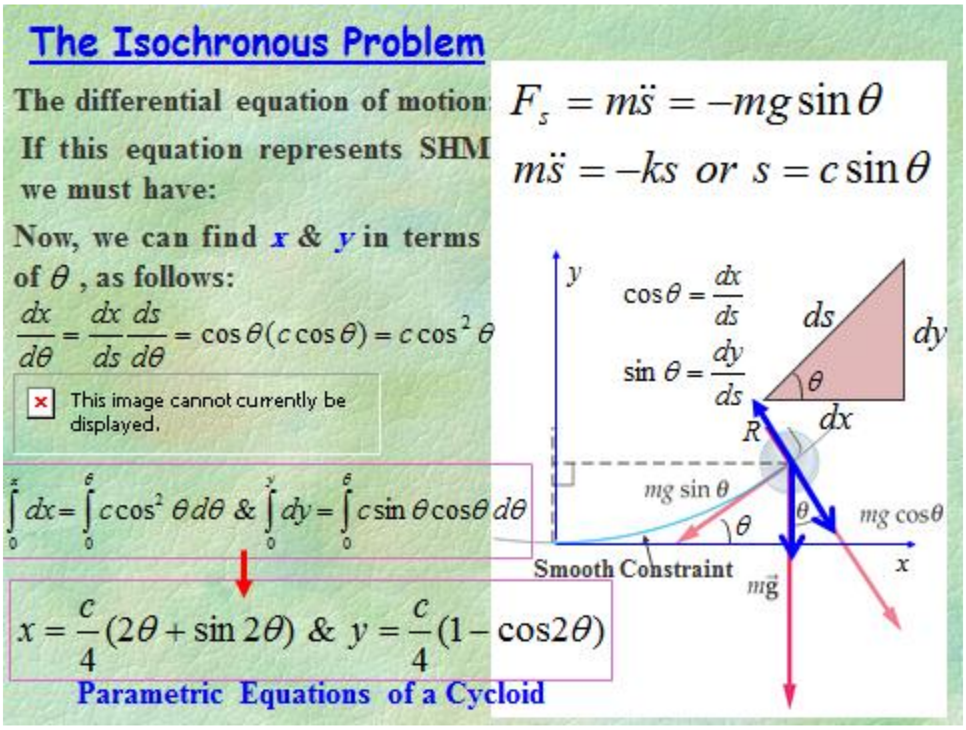
Ans. of Q.3:



The Isochronous Problem
The differential equation of motion: $F_s = m\ddot{s} = -mg\sin\theta$
If this equation represents SHM we must have: $m\ddot{s} = -ks$ or $s = c\sin\theta$
Now, we can find x & y in terms of θ, as follows:
$\frac{dx}{d\theta} = \frac{dx}{ds}\frac{ds}{d\theta} = \cos\theta(c\cos\theta) = c\cos^2\theta$
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$\int_0^x dx = \int_0^\theta c\cos^2\theta\, d\theta$ & $\int_0^y dy = \int_0^\theta c\sin\theta\cos\theta\, d\theta$
$x = \frac{c}{4}(2\theta + \sin 2\theta)$ & $y = \frac{c}{4}(1 - \cos 2\theta)$
Parametric Equations of a Cycloid
$\cos\theta = \frac{dx}{ds}$
$\sin\theta = \frac{dy}{ds}$
ds
dy
dx
θ
R
mg sin θ
mg cosθ
θ
Smooth Constraint
$m\vec{g}$
y
x